Best-Effort Interference Alignment for K Users Quasi-Static MIMO Interference Channels

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Abstract

Interference alignment (IA) has been a powerful approach to achieve the maximum degree of freedom (DoF) for K users multiple-input-multiple-output (MIMO) interference channels. However, due to the feasibility constraint, aligning all the interference signals at each receiver is impractical for large K without symbol extension. In this paper, we propose two best-effort interference alignment (BEIA) schemes that the network selects the maximum number of interfering transmitters to align their signals given the feasibility conditions when each transmitter-receiver pair has a constant number of data streams. Besides, in case of not all interfering signals aligned at each receiver, an upper bound of the average throughput is derived. Simulation results show that the proposed schemes have superiority over the traditional methods, such as time division multiple access (TDMA) and cluster IA(CIA), in low and moderate signal-to-noise ratio (SNR) region in terms of average user throughput. In addition, the proposed max-min relative interference distance alignment scheme outperforms the proposed scheme of equal interfering transmitters number alignment in terms of both average user throughput and minimum user throughput.

Keywords: Interference Alignment (IA), best-effort, degree of freedom (DoF), throughput, quasi-static MIMO interference channels

1. Introduction

Due to the explosive demand of wireless networks in nowadays, wireless communication technology has been evolving at a booming rate. Therefore, the bottleneck of limited wireless resource need to be broken in this aspect. Network densification is an instrumental mechanism to improve the wireless network capacity. However, the densification leads to the dilemma of high interference in networks. Interference is the major and eternal challenge in multiple-input-multiple-output (MIMO) interference channels.

1.1 Backgrounds and Motivation

In retrospect, the traditional approaches, including time division multiple access (TDMA), frequency division multiple access (FDMA) and code division multiple access (CDMA), deal with interference by giving every user an exclusive use of a fraction of resources [1]. Obviously, these orthogonal access methods are unable to make full use of valuable communication resources to cope with the high demanding communication requirements nowadays. Therefore, interference alignment (IA) has been proposed for combating interference in an effective way for MIMO X channels [2],[3] and K users MIMO interference channels [4],[5],[6] and cooperative networks [7]. The idea of IA is that by jointly designing precoding and decoding matrices, the interference signals are aligned in a lower subspace at each receiver while the remaining interference-free subspace is reserved for the desired signals.

For K users $M \times N$ MIMO interference channels, the authors of [8] show that the achieved degree of freedom (DoF) scales linearly with the number of the users K, which is $KR\min(M,N)/(R+1)$, where $R = \max(M,N)/\min(M,N)$. However, the DoF mentioned above is obtained by infinite symbol extension, which is hard to implement in the real-world systems. In this paper, IA is studied in quasi-static MIMO interference channels without symbol extension. In [9], the authors prove the feasibility conditions of IA in quasi-static MIMO channels. Specifically, for K users $M \times N$ MIMO interference channels and each transmitter-receiver pair having L data streams, the inequality $(M+N)/L-1 \ge K$ should be satisfied in order to align all interference signals at each receiver [9],[10]. As a result, it is not possible to align all $K-1$ interfering users in $N-L$ subspace at each receiver node by pure interference alignment for large K. To fully align all the interference signals in this case, an approach called cluster IA (CIA) is proposed in [11],[12],[13]. The idea of CIA is that nearby users are clustered into a group given the feasibility constraint using IA to cancel intra-cluster interference and every cluster uses orthogonal resources, such as slots [11],[12] or subchannels [13], to avoid inter-cluster interference. However, CIA may also not use the communication resources adequately because the clusters use orthogonal communication resources.

In view of the fact that the interfering transmitters far away from the desired receiver contribute less interference to the receiver due to path loss, they will be given less priority in selecting nodes to align given the feasibility constraint. In this paper, two best-effort IA (BEIA) schemes in which each receiver node selects the interfering transmitters to align until one more transmitter may break the feasibility constraint, are proposed to reduce interference in K users MIMO networks. The concept of BEIA proposed in [14] is different from the one in this paper. The BEIA signifies that when the dimensionality condition for perfect alignment is not satisfied the residual interference at each receiver is minimized via designing the precoding

and decoding matrices iteratively by best-effort in [14]. However, in this paper, the BEIA means that when IA feasibility constraint is unsatisfied, a maximum number of the interfering transmitters is selected to align. Besides, in [14] the design of the precoding and decoding matrices needs global channel state information (CSI) and takes a long time to converge by iterations, which brings huge calculation burden. However, in this paper, interfering transmitters are selected to be aligned only by distance and only the CSI of the interfering links selected to be aligned is needed. In simulations, we find that the proposed schemes have obvious advantages over [14] in terms of minimum user throughput at the cost of a slight reduction in average user throughput at high signal-to-noise ratios (SNRs) region. Interfering transmitters selection is also researched in [15], where the interfering transmitters are selected aiming at optimizing average symbol error rate by partial IA and interference detection for MIMO interference channels with discrete constellation inputs. The idea is different from the research topic of this paper, in which our selection criterion focuses on the throughput achieved by IA only and aligns interfering transmitters by best-effort given feasibility conditions for general alignment set newly developed in [16],[17].

1.2 Major Contributions

- Two different BEIA schemes are proposed and their throughputs are evaluated and compared. The proposed schemes focus on the selection of interfering transmitters to be aligned. One is that the number of interfering transmitters aligned at each receiver is equal. The other one ensures that the minimum relative distance between the unaligned interfering transmitters and the typical receiver is maximized. Simulation results verify the superiority of the proposed schemes at moderate and low SNRs.
- A throughput upper bound is derived when there are some residual interference signals are unaligned at each receiver, which gives a new insight into the floor effect of throughput for two proposed schemes. To our best knowledge, this new theory result is firstly developed in this paper.

1.3 Paper Organization

The reminder of this paper is organized as follows. In Section 2, the system model is described and the problem is formulated. Two BEIA schemes are proposed in Section 3. In Section 4, an upper bound of average throughput for two proposed schemes is derived. Section 5 provides the simulation results and discussions. Concluding remarks are given in Section 6.

Notations: Throughout the paper, upper and lower bold-face letters denote the matrices and vectors, respectively. X^*, X^T and X^H denote the conjugate, transpose and Hermitian transpose of matrix X, respectively. vec(X) operator stacks the columns of the matrix X into a long column vector. $\|\mathbf{x}\|$ is the Euclidean norm of the complex vector $\mathbf{x}$. E[•] stands for the expectation operator. $C^{M \times N}$ is the set of complex matrices with M rows and N columns. $CN(0,1)$ denotes a circularly symmetric complex normal distribution with zero mean and unit variance. $a \otimes b$ is the Kronecker product of vector a and b.

2. System Model and Problem Description

A symmetric K user quasi-static MIMO interference channel is considered in this paper. Each transmitter equipped with M antennas sends L data streams to its corresponding receiver with N antennas. In this paper, we consider the case of $L = 1$ for simplicity and the analysis can be extended to multiple data streams as future work. The wireless channels are characterized by path-loss and small-scale fading. Then, the post-processed signal y_i at receiver i can be

expressed as

$$y_i = \sqrt{P} d_{ii}^{-a/2} \boldsymbol{u}_i^H \boldsymbol{H}_{ii} \boldsymbol{v}_i x_i + \sum_{j \neq i} \sqrt{P} d_{ij}^{-a/2} \boldsymbol{u}_i^H \boldsymbol{H}_{ij} \boldsymbol{v}_j x_j + \boldsymbol{u}_i^H \boldsymbol{n}_i , \ i \in \{1,2,\ldots,K\} \tag{1}$$

In (1), P is the transmit power, $\boldsymbol{H}_{ij} \in C^{N \times M}$ is the fading coefficients of the channel from transmitter j to receiver i with its elements being i.i.d. $CN(0,1)$, d_{ij} is the propagation distance between the transmitter j to receiver i and a is the path-loss exponent, $\boldsymbol{u}_i \in C^{M \times 1}$ and $\boldsymbol{v}_j \in C^{N \times 1}$ denote the precoder at transmitter i and decorrelator at receiver j, respectively, with $\|\boldsymbol{u}_i\|^2 = 1$ and $\|\boldsymbol{v}_j\|^2 = 1$, x_i is the data stream of transmitter i and $\boldsymbol{n}_i$ denotes the circularly symmetric Additive White Gaussian Noise (AWGN) vector with $E\left(\boldsymbol{n}_i \boldsymbol{n}_i^H\right) = \sigma^2 \boldsymbol{I}_N$.

Based on the feasibility conditions in [9],[10], we know that the inequality $M + N - 1 \geq K$ should hold to align all the K-1 interfering signals in the N-1 dimension subspace at each receiver, which is not possible for large K without symbol extension. In the proposed BEIA schemes, each receiver node selects the transmitters to align until one more transmitter may lead to the feasibility constraints unsatisfied. Let the arrow lines denote the transmitter signal to be aligned at receiver i as shown in **Fig. 1** and S_i denote the interfering transmitters set to be aligned at the receiver i.
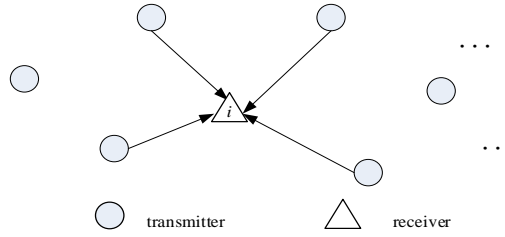

Fig. 1. Alignment model

Then, interference alignment conditions can be expressed as

$$\boldsymbol{u}_i^H \boldsymbol{H}_{ij} \boldsymbol{v}_j = 0, \ j \in S_i, \ i \in \{1,2,\ldots,K\} \tag{2}$$

$$\boldsymbol{u}_i^H \boldsymbol{H}_{ii} \boldsymbol{v}_i \neq 0, i \in \{1,2,\ldots,K\} \tag{3}$$

For the general channel matrices without special structure, (3) is automatically satisfied [18]. Based on the theory of algebraic geometry used in [9], (2) is solvable if and only if the number of variables is not less than the number of equations. That is, the following inequality should be satisfied when each user transmits one data stream.

$$\sum_{i=1}^{K} (M + N - 2) \geq \sum_{i=1}^{K} |S_i| \tag{4}$$

where $|S_i|$ is the cardinality of set S_i. Based on the results of Corollary 3.4("the divisible case") in [16], for the case of $L = 1$, equality (2) has solutions almost surely if and only if (4) is satisfied. The computation of $\{\boldsymbol{u}_i, \boldsymbol{v}_i, \forall i\}$ is not the focus of this paper. In this paper, we focus on designing different BEIA schemes by selecting maximum number of interfering signals to align. Besides, we will evaluate the effect on the system throughput by the selection

of S_i. Let C denote the average throughput of users. Then, the studied problem can be expressed as

$$C = f(S_1, S_2, \ldots, S_K) \tag{5}$$

In the next section, we analyze the throughput of users under two different schemes.

3. Two Different BEIA SCHEMES

In this paper, the expression "aligning $|S_i|$ interfering transmitters signals" means that the signals of the nearest $|S_i|$ interfering transmitters to receiver i are aligned into $N-1$ dimension interference subspace at receiver i.

(A) Scheme 1: equal interference user number assignment

Firstly, we discuss a simple case where equal number of transmitters is selected to be aligned

at each receiver, $|S_1| = |S_2| = \ldots = |S_K|$. Then, the cardinality of the set is given as $|S_i| = M + N - 2, \forall i$. The signal-to-interference-plus-noise ratio (SINR) of user i can be expressed as

$$SINR_i = \frac{Pd_{ii}^{-a} \left| \boldsymbol{u}_i^H \boldsymbol{H}_{ii} \boldsymbol{v}_i \right|^2}{\sum\limits_{j \notin S_i} Pd_{ij}^{-a} \left| \boldsymbol{u}_i^H \boldsymbol{H}_{ij} \boldsymbol{v}_j \right|^2 + \sigma^2} = \frac{\rho \left| \boldsymbol{u}_i^H \boldsymbol{H}_{ii} \boldsymbol{v}_i \right|^2}{\sum\limits_{j \notin S_i} \rho r_{ij}^{-a} \left| \boldsymbol{u}_i^H \boldsymbol{H}_{ij} \boldsymbol{v}_j \right|^2 + 1} \tag{6}$$

where σ^2 is the noise power, $\rho = Pd_{ii}^{-a}/\sigma^2$ is the signal-to-noise ratio (SNR) and $r_{ij} = d_{ij}/d_{ii}$ is the relative distance between transmitter j and receiver i. In this case, the average throughput can be expressed as

$$C = E\left[\frac{1}{K} \sum_i \log_2 \left(1 + SINR_i\right) \right] \tag{7}$$

(B) Scheme 2: max-min relative interference distance alignment

Due to the random distribution of users, the propagation distances of the interfering signals are different for each receiver. There maybe exist one case where some receivers have many interfering transmitters nearby while the interfering transmitters of some receiver nodes are far away. Then, an equal number of aligned transmitters may lead to a large difference in the individual throughput. So, a further research of a fair scheme to maximize the minimum throughput is proposed, which can be expressed as

$$\max \min_i \ \log_2 \left(1 + SINR_i\right)$$
$$s.t. \quad \sum_{i=1}^K (M + N - 2) \geq \sum_{i=1}^K |S_i| \tag{8}$$
$$|S_i| \ is \ \text{non-negative integer}$$

From (6), we find SINR$_i$ is influenced by $\boldsymbol{u}_i, \boldsymbol{v}_i$ and S_i. Besides, for different S_i, the design of $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ is also different. Let $x \to y$ denote that y is influenced by x. **Fig. 2** illustrates the relation among the design of $\boldsymbol{u}_i$, $\boldsymbol{v}_i$, the selection of S_i, and the quantity of *SINR$_i$*. There are

two loops in the relation map. The optimal solution not only requires a brute force search over all possible set selections, but also demands full CSI to compute the precoding and decoding vectors in every case, thereby defeating the purpose of (8) by huge overhead.
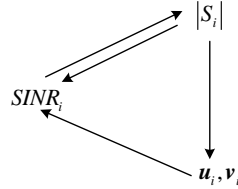

Fig. 2. The relation among S_i, $\boldsymbol{u}_i$, $\boldsymbol{v}_i$ and SINR$_i$

Therefore, an approach based on the nearest interfering transmitter distance is proposed to approximate this scheme. The new scheme ensures that the minimum relative distance between the interfering transmitters may not aligned and the corresponding receiver is maximized, which can be expressed as

$$\max \ \min_i \ r_{\min}^i$$
$$s.t. \quad \sum_{i=1}^{K}(M+N-2) \geq \sum_{i=1}^{K}|S_i| \qquad (9)$$
$$|S_i| \ is \ \text{non-negative integer}$$

where $r_{\min}^i$ is the minimum relative distance between the interfering transmitters that are not aligned and the receiver i. In this paper, a low complexity algorithm is proposed to solve (9). The main idea of the algorithm is to cancel the interference link with the smallest relative distance sequentially. The details of the algorithm are shown as follows.

Step 1   Initialization: Each receiver has 0 transmitters to align and the relative interference distance set of receiver i is R_i:

$|S_i| = 0$,

$R_i = \{r_{i1}, r_{i2}, \ldots, r_{i,i-1}, r_{i,i+1}, \ldots, r_{iK}\}, \forall i$

Step 2   Select the minimum distance between receiver i and the interfering transmitter unaligned:

$r_{\min}^i = \min\{R_i\}, \forall i$,

$R_{\min} = \{r_{\min}^1, r_{\min}^2, \ldots, r_{\min}^K\}$

Step 3   Find the receiver index with the minimum distance of $R_{\min}$:

$n = \arg\min_i R_{\min}$

Step 4   Allocate one link to be aligned to the receiver with the minimum distance:

$|S_n| = |S_n| + 1$

Step 5   Remove the minimum distance from interference distance set R_n with the minimum distance:

$R_n = R_n \setminus r_{\min}^n$

Repeat **Step 2**-**Step 5** until $\sum_i |S_i| = K(M+N-2)$.

4. An Upper Bound of Average Throughput

In this section, we point out that the throughput achieved is upper bounded by a constant for the scenario where K transmitters and receivers are distributed in a $l \times b$ m^2 area randomly and the minimum distance between the transmitter and its desired receiver is d_0, which is used for simulation in the next section.

Theorem 1: The average throughput achieved with the unaligned interference signals at the receiver for the scenario above is bounded as the SNR is taken to infinity

$$C \leq \log_2 \left(1 + \frac{\pi \left(l^2 + b^2 \right)^{a/2+1}}{2lb(a+2)d_0^a} \frac{MN}{MN-1} \right) + \log_2(MN-1) + \log_2 e \qquad (10)$$

Proof: Recall (6) and $SINR_i$ can be rewritten as

$$SINR_i = \frac{\rho \left| \boldsymbol{u}_i^H \boldsymbol{H}_{ii} \boldsymbol{v}_i \right|^2}{\sum\limits_{j \notin S_i} \rho r_{ij}^{-a} \left| \boldsymbol{u}_i^H \boldsymbol{H}_{ij} \boldsymbol{v}_j \right|^2 + 1} = \frac{\rho \left\| \boldsymbol{h}_{ii} \right\|^2 \left| \hat{\boldsymbol{h}}_{ii}^T \cdot \left(\boldsymbol{v}_i \otimes \boldsymbol{u}_i^* \right) \right|^2}{\sum\limits_{j \notin S_i} \rho r_{ij}^{-a} \left\| \boldsymbol{h}_{ij} \right\|^2 \left| \hat{\boldsymbol{h}}_{ij}^T \cdot \left(\boldsymbol{v}_j \otimes \boldsymbol{u}_i^* \right) \right|^2 + 1} = \frac{\rho \left\| \boldsymbol{h}_{ii} \right\|^2 X_{ii}}{\sum\limits_{j \notin S_i} \rho r_{ij}^{-a} \left\| \boldsymbol{h}_{ij} \right\|^2 Y_{ij} + 1}$$

(11)

where $\boldsymbol{h}_{ij} = vec\left(\boldsymbol{H}_{ij} \right) = \left\| \boldsymbol{h}_{ij} \right\| \hat{\boldsymbol{h}}, \forall i, j$, $X_{ii} = \left| \hat{\boldsymbol{h}}_{ii}^T \cdot \left(\boldsymbol{v}_i \otimes \boldsymbol{u}_i^* \right) \right|^2$ and $Y_{ij} = \left| \hat{\boldsymbol{h}}_{ij}^T \cdot \left(\boldsymbol{v}_j \otimes \boldsymbol{u}_i^* \right) \right|^2$. Due to the fact that the elements of $\boldsymbol{H}_{ij}$ are i.i.d. $CN(0,1)$, $\left\| \boldsymbol{h}_{ij} \right\|^2$ follows χ_{2MN}^2. Note that the design of $\boldsymbol{u}_i$ and $\boldsymbol{v}_j$ actually focuses on condition (2) and does not involve the direct channel $\boldsymbol{H}_{ii}$ and the interference channel $\boldsymbol{H}_{ij}$, $j \notin S_i$. Thus, $\boldsymbol{u}_i$ *and* $\boldsymbol{v}_i$ are independent of $\boldsymbol{H}_{ii}$, *and* $\boldsymbol{u}_i$ *and* $\boldsymbol{v}_j$ are independent of $\boldsymbol{H}_{ij}$, $j \notin S_i$ [19]. Then, $\hat{\boldsymbol{h}}_{ij}\left(j \notin S_i \right)$ and $\boldsymbol{v}_j \otimes \boldsymbol{u}_i^*$ are i.i.d. isotropic unit vectors in the MN-dimensional space, the quantities $X_{ii}, Y_{ij}, \forall i, j \neq i$ are $B(1,MN\text{-}1)$ distributed, where $B(1,MN\text{-}1)$ is beta distributed with parameters 1 and $MN\text{-}1$. Besides, the random variables $\left\| \boldsymbol{h}_{ii} \right\|^2, \left\| \boldsymbol{h}_{ij} \right\|^2, X_{ii}, Y_{ij}, \forall i, j \neq i$ are all independent. Let $E_{\|\boldsymbol{h}\|^2}[\cdot]$ denotes $E_{\|\boldsymbol{h}_{ii}\|^2, \|\boldsymbol{h}_{ij}\|^2}[\cdot]$, then the average throughput can be expressed as

$$C = E_{\|\boldsymbol{h}\|^2, X_{ii}, Y_{ij}, r_{ij}} \left[\log_2 \left(1 + \frac{\rho \left\| \boldsymbol{h}_{ii} \right\|^2 X_{ii}}{\sum\limits_{j \notin S_i} \rho r_{ij}^{-a} \left\| \boldsymbol{h}_{ij} \right\|^2 Y_{ij} + 1} \right) \right]$$

$$\overset{(a)}{\leq} E_{\|\boldsymbol{h}\|^2, X_{ii}, Y_{ij}, r_{ij}} \left[\log_2 \left(1 + \frac{\left\| \boldsymbol{h}_{ii} \right\|^2 X_{ii}}{r_{ij}^{-a} \left\| \boldsymbol{h}_{ij} \right\|^2 Y_{ij}} \right) \right]$$

$$= E_{X_{ii}, Y_{ij}} \left[E_{\|\boldsymbol{h}\|^2, r_{ij}} \left[\log_2 \left(1 + \frac{\left\| \boldsymbol{h}_{ii} \right\|^2 X_{ii}}{r_{ij}^{-a} \left\| \boldsymbol{h}_{ij} \right\|^2 Y_{ij}} \right) \right] \right]$$

$$\overset{(b)}{\leq} E_{X_{ii},Y_{ij}} \left[\log_2 \left(1 + E_{r_{ij}} \left(r_{ij}^a \right) E_{\|\boldsymbol{h}\|^2} \left(\frac{\|\boldsymbol{h}_{ii}\|^2}{\|\boldsymbol{h}_{ij}\|^2} \right) \frac{X_{ii}}{Y_{ij}} \right) \right]$$

$$\overset{(c)}{\leq} E_{X_{ii},Y_{ij}} \left[\log_2 \left(1 + \frac{\pi \left(l^2 + b^2 \right)^{a/2+1}}{2lb(a+2)d_0^a} \frac{MN}{MN-1} \frac{X_{ii}}{Y_{ij}} \right) \right]$$

$$= E_{X_{ii},Y_{ij}} \left[\log_2 \left(Y_{ij} + \frac{\pi \left(l^2 + b^2 \right)^{a/2+1}}{2lb(a+2)d_0^a} \frac{MN}{MN-1} X_{ii} \right) - \log_2 \left(X_{ii} \right) \right]$$

$$\overset{(d)}{\leq} \log_2 \left(1 + \frac{\pi \left(l^2 + b^2 \right)^{a/2+1}}{2lb(a+2)d_0^a} \frac{MN}{MN-1} \right) + E_{X_{ii}} \left[-\log_2 \left(X_{ii} \right) \right]$$

$$\overset{(e)}{=} \log_2 \left(1 + \frac{\pi \left(l^2 + b^2 \right)^{a/2+1}}{2lb(a+2)d_0^a} \frac{MN}{MN-1} \right) + \left(\log_2 e \right) \sum_{k=1}^{MN-1} \frac{1}{k}$$

$$\leq \log_2 \left(1 + \frac{\pi \left(l^2 + b^2 \right)^{a/2+1}}{2lb(a+2)d_0^a} \frac{MN}{MN-1} \right) + \log_2 (MN-1) + \log_2 e$$

where (*a*) holds because only one of the interference terms is considered and (*d*) uses the fact that X_{ii} and Y_{ij} are beta distributed, then $X_{ii}, Y_{ij} \leq 1$. Please see Appendix 1 for the derivation of (*b*), (*c*) and (*e*).

Remark 1: Though the derivation is conducted in a rectangular area, the average throughput is also upper bounded by a different constant upper limit for generalized area shapes. Besides, although the derived upper bound is loose, it does correctly predict the floor effect of average user throughput, which will be shown in Section 5.

5. Simulation Results

In this section, we evaluate the performance of the proposed schemes via simulations and three existed schemes are adopted for performance comparison. The details of three baseline algorithms are described as follows. TDMA: Each user is allocated $1/K$ time resources to transmit and every slot only has one transmitting user. For fair comparison, the power used by every transmitter is KP (not in *dB*) and the waterfilling algorithm is applied for power allocation. CIA: Assign maximum number of users whose signals can be aligned fully into one cluster and different clusters use different slots. And the power of each transmitter also scales linearly with the number of clusters for fairness concern. BEIA proposed in [14]: The throughput is averaged over 10 groups of $\{ d_{ij}^{a/2} H_{ij}, \forall i, j \}$ and at each channel group the precoding and interference subspace matrices are designed by 5000 iterations aiming at minimizing sum interference power leakage as the constrained optimization problem (5) in [14].

In the simulations, we consider a MIMO interference networks with antennas number $M = N = 3$, data streams $L = 1$ and the exponent of path loss $a = 3$. All transmitters nodes are distributed in a $100m \times 100m$ rectangular area randomly as discussed in Section 4,

and the distance between the transmitter and its desired receiver is assumed to be a constant d_c ($d_c = 10m$) without loss of generality. This model is widely used in wireless communications, for example, femtocell networks.

Fig. 3 shows the average user throughput for different SNR ρ with $K = 10$. It is shown that the average throughput of proposed schemes is larger than two traditional schemes (TDMA and CIA) when SNR<32dB. Hence, the proposed schemes have superior performance compared with CIA and TDMA in low and moderate SNRs in terms of average throughput. The reason is that more total data streams are transmitted in the proposed schemes and the gain achieved by more data streams is larger than that without interference when SNR is not large. Besides, the performance of Scheme 2 outperforms that of Scheme 1 which is at the cost of higher complexity. However, the throughputs of proposed schemes converge to an upper limit as expected with the SNR increasing. That's because both the useful signal and residual interference power scale linearly with SNR. This result also verifies the validity of Theorem 1. Please note that although more streams are transmitted in the proposed schemes, this interference-limited behavior makes CIA and TDMA more preferable at high SNRs. In addition, it can be seen that the proposed schemes has similar performance compared with BEIA in terms of average throughput due to the fact that the interference mainly comes from the nearby interfering transmitters aligned in the proposed schemes. In detail, we find that BEIA has a slight improvement of average throughput when SNR>30dB. However, this slight improvement is at the cost of huge computational burden, while in this paper only (2) needs to be solved. Besides, the BEIA is a centralized iterative algorithm and global CSI is needed, while in this paper only the CSI of links to be aligned is required.
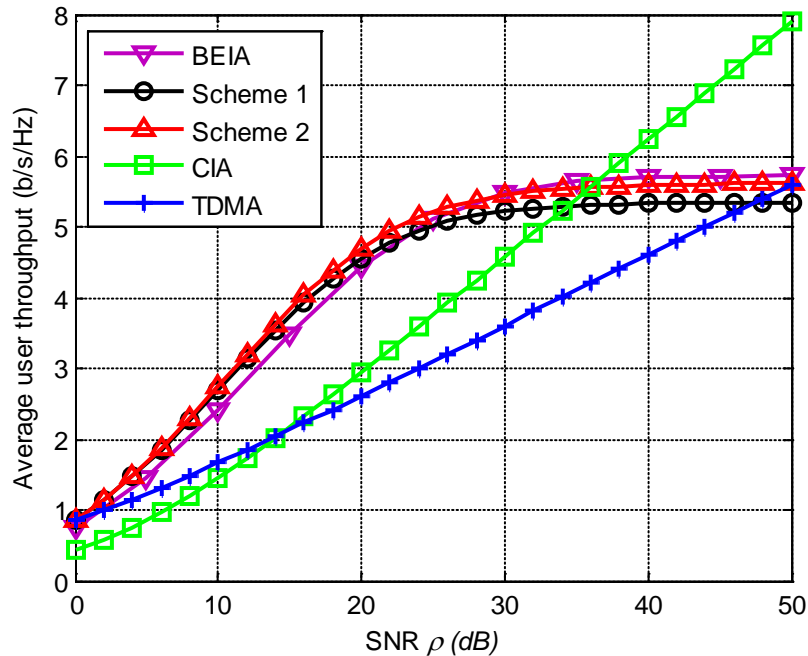

Fig. 3. Average user throughput versus SNR ρ

Fig. 4 plots the average user throughput for different user numbers with SNR=10 and 20*dB*, respectively. It can be seen that the average throughput of all schemes decreases as the number of users increases, which is obvious because more residual interference power is brought forth in the proposed schemes and less time resource is allocated to every user for transmission in CIA and TDMA due to more users. In addition, the throughput of Scheme 2 is superior to Scheme 1 while the superiority fades with *K*. That's because dense user distribution leads to little difference in the radius of interference-free area for two schemes. Combining **Fig. 3** and **Fig. 4**, the throughput of TDMA is larger than CIA when *ρ*=10*dB* and the result is opposite when *ρ*=20*dB*, because the gain of SNR and of data streams is more dominant for throughput in the low and high SNR region, respectively. Also, it is shown that the average throughput of proposed schemes approaches that of BEIA but with low complexity.
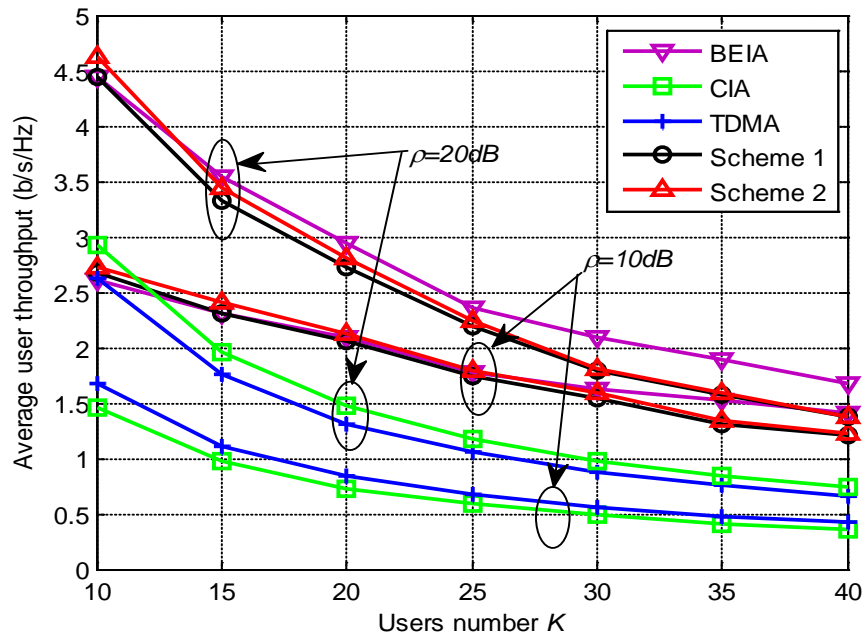

Fig. 4. Average user throughput versus users number *K*

Fig. 5 displays the minimum user throughput for different SNR with *K*=10, 20 and 30. It is shown that the minimum throughput of Scheme 2 outperforms Scheme 1 as expected and the advantage is more obvious as SNR increases. The reason is that the radius of the interference-free area is nearly equal in Scheme 2, while for Scheme 1 the receiver surrounded by denser interference transmitters may have a less radius of no interference area leading to more throughput loss. Also, the floor effect exists in the minimum throughput for two proposed schemes, which can be explained by the upper bound theory. Besides, as *K* increases, the SNR region where the proposed schemes is superior to CIA is decreasing, which is also true for the average user throughput. Besides, from the figure, we can see that the proposed schemes have obvious advantages over the BEIA algorithm in terms of minimum user throughput, which verifies more fairness of the proposed schemes. The reason is that BEIA aims at minimizing the total interference power leakage, which may be with the sacrifice of individual user throughput.
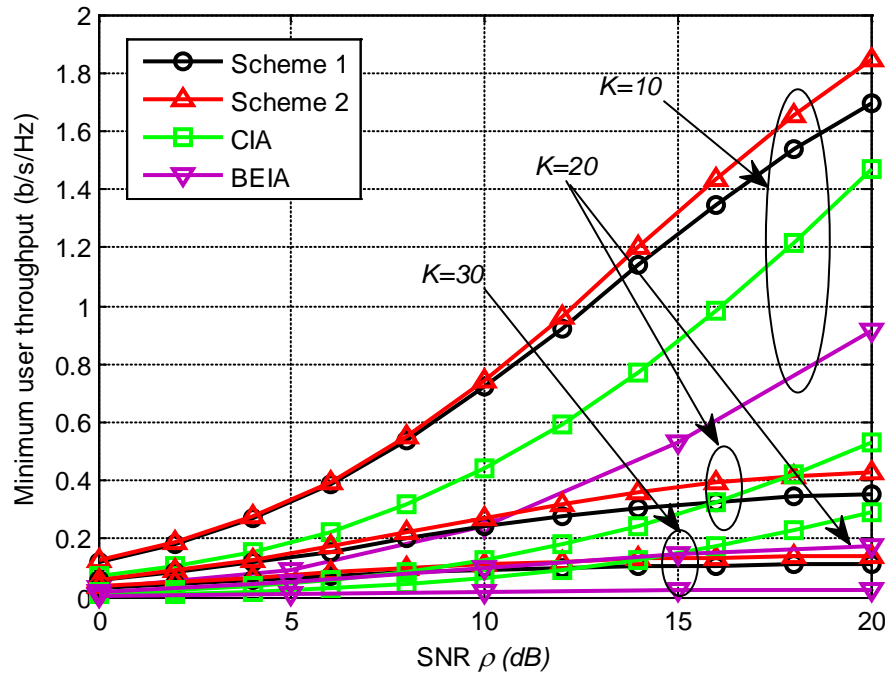
Fig. 5. Minimum user throughput versus SNR

6. Conclusion

In this paper, we investigate the throughput of BEIA when some interference signals cannot be aligned in N-1 subspace at each receiver under K users MIMO interference channels without symbol extension. Two schemes for selecting the interference transmitter set to be aligned have been proposed and their throughput performance has been analyzed and compared via simulations. Besides, a loose upper bound of throughput has been derived for two proposed schemes when at least one interference signal is not aligned at each receiver. Simulation results show that the proposed schemes have superiority over the traditional methods (TDMA, CIA) in low and moderate SNR region in terms of average user throughput and achieve higher minimum user throughput than the existed BEIA at the cost of slight reduction in average throughput but with rather low computation complexity. In addition, the proposed scheme of max-min relative interference distance alignment outperforms the proposed scheme of equal interfering transmitters number alignment in terms of user throughput.

Appendix 1

The proof of (b)
By Jensen's inequality, we can get
$$f(X) \leq g(X) \tag{12}$$

where $f(X) = E_{\|\boldsymbol{h}\|^2, r_{ij}} \left[\log_2 \left(1 + \dfrac{\|\boldsymbol{h}_{ii}\|^2}{r_{ij}^{-a} \|\boldsymbol{h}_{ij}\|^2} \dfrac{X_{ii}}{Y_{ij}} \right) \right]$,

$$g(X) = \log_2\left(1 + E_{r_{ij}}\left(r_{ij}^a\right)E_{\|h\|^2}\left(\frac{\|h_{ii}\|^2}{\|h_{ij}\|^2}\right)\frac{X_{ii}}{Y_{ij}}\right).$$

For a variable $X = X_{ii}/Y_{ij} > 0$ and $f(X) \leq g(X)$,

$$E_X\left[g(X)\right] - E_X\left[f(X)\right] = E_X\left[g(X) - f(X)\right] \geq 0 \tag{13}$$

The proof of (c)

Since $\|h_{ii}\|^2$ and $\|h_{ij}\|^2$ are χ^2_{2MN} distributed and independent, $\dfrac{\|h_{ii}\|^2}{\|h_{ij}\|^2} = \dfrac{\|h_{ii}\|^2/2MN}{\|h_{ij}\|^2/2MN}$ follows

$F(2MN, 2MN)$, where $F(2MN, 2MN)$ is an F-distribution with parameters $2MN$, $2MN$ [20]. Then, we can get

$$E_{\|h\|^2}\left(\frac{\|h_{ii}\|^2}{\|h_{ij}\|^2}\right) = \frac{MN}{MN-1}, \text{ for } MN > 1. \tag{14}$$

Next, we discuss the expectation of r_{ij}^a

$$E_{r_{ij}}\left(r_{ij}^a\right) = E_{r_{ij}}\left(d_{ij}^a/d_{ii}^a\right) \leq E_{d_{ij}}\left(d_{ij}^a/d_0^a\right) = E_{d_{ij}}\left(d_{ij}^a\right)/d_0^a \tag{15}$$

We know that for transmitters and receivers distributed in a rectangular area, the receiver located at a corner has the maximum expectation of interference distance. Let Z denote the distance between the interfering transmitter and the receiver at the corner. From **Fig. 6**, we can get the upper bound of PDF of Z for the receiver located at the corner.

$$f_Z(z) = F_Z'(z) = \lim_{r \to 0}\frac{F(z+r) - F(z)}{r}$$
$$= \lim_{r \to 0}\frac{the\ area\ of\ shadow}{l \times b \times r} \leq \lim_{r \to 0}\frac{\pi(z+r)^2 - \pi z^2}{4lbr} = \frac{\pi z}{2lb} \tag{16}$$

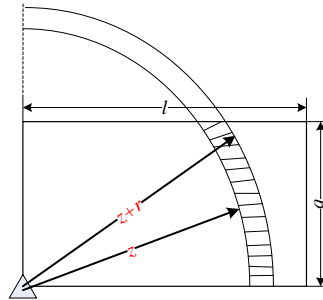**Fig. 6.** The distance between the receiver at the corner and possible interfering transmitters

Then, (15) can be rewritten as

$$E_{d_{ij}}\left(d_{ij}^a\right)/d_0^a \leq E_Z\left(Z^a\right)/d_0^a = \int_0^{\sqrt{a^2+b^2}} z^a f_z(z)dz/d_0^a$$
$$\leq \frac{\pi}{2lb(a+2)}z^{a+2}/d_0^a\Big|_0^{\sqrt{a^2+b^2}} \leq \frac{\pi\left(l^2+b^2\right)^{a/2+1}}{2lb(a+2)d_0^a} \tag{17}$$

Since all the variables are positive and log2(•) increases monotonously, we can get that

$$\log_2\left(1+E_{r_{ij}}\left(r_{ij}^a\right)E_{\|\boldsymbol{h}\|^2}\left(\frac{\|\boldsymbol{h}_{ii}\|^2}{\|\boldsymbol{h}_{ij}\|^2}\right)\frac{X_{ii}}{Y_{ij}}\right)\le\log_2\left(1+\frac{\pi\left(l^2+b^2\right)^{a/2+1}}{2lb(a+2)d_0^a}\frac{MN}{MN-1}\frac{X_{ii}}{Y_{ij}}\right). \tag{18}$$

The remaining proof is the same as (13) in the proof of (b).

The proof of (e):

$$E_{X_{ii}}\left[-\log_2\left(X_{ii}\right)\right]=\left(\log_2 e\right)\sum_{k=1}^{MN-1}\frac{1}{k}\le\log_2(MN-1)+\log_2 e \tag{19}$$

when X_{ii} is $B(1,MN\text{-}1)$ distributed in Lemma 3 of [21].

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