Resource Allocation in Multi-User MIMO-OFDM Systems with Double-objective Optimization

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Abstract

A resource allocation algorithm is proposed in this paper to simultaneously minimize the total system power consumption and maximize the system throughput for the downlink of multi-user multiple input multiple output-orthogonal frequency division multiplexing (MIMO-OFDM) systems. With the Lagrange dual decomposition method, we transform the original problem to its convex dual problem and prove that the duality gap between the two problems is zero, which means the optimal solution of the original problem can be obtained by solving its dual problem. Then, we use convex optimization method to solve the dual problem and utilize bisection method to obtain the optimal dual variable. The numerical results show that the proposed algorithm is superior to traditional single-objective optimization method in both the system throughput and the system energy consumption.

Keywords: Multi-user MIMO-OFDM, joint optimization, resource optimization, ZF-LBD

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1. Introduction

With the increasing demand for system throughput in modern communications, improving system throughput via allocating wireless resources, e.g., time, frequency and power, is becoming more and more important. In multi-user multiple input multiple output-orthogonal frequency division multiplexing (MIMO-OFDM) systems used by 5G, resources are doubled on allocable degrees of freedom owing to the introduction of space division multiple access (SDMA), which makes the resource allocation problem more complex than ever. In order to reduce the complexity, most literatures impose some restrictions on the degrees of freedom when allocating resources [1]-[5]. For example, authors in [2] and [4] both have the restriction that each subcarrier can carry only one user, which means they give up the resource gain brought by SDMA; authors in [5] limit that power can only be allocated uniformly to each subcarrier, however, due to different channel gain, uniform allocation of power makes the channels with good condition cannot make full use of power resources to achieve greater transmission rate.

Meanwhile, due to the pursuit of green communications, minimizing total system power given certain system throughput is gradually becoming an optimization target of 5G systems. Since power optimization and throughput optimization are conflicting, most of the existing literatures only investigate single-objective optimization strategy, that is, minimizing total system power based on the condition that the minimum total system rate is given [2], [4]; or maximizing total system rate on the basis that total system power is given [6], [7], [8]. As far as we know, there are no literatures that simultaneously optimize power and rate in multi-user MIMO-OFDM systems.

The ultimate causes of the phenomenon that power and rate are hard to be optimized simultaneously can be boiled down to the following aspects: 1. In the presence of multiple allocable degrees of freedom of resources, there will be nonlinear constraints that lead the optimization problem to be non-convex and hard to be solved; 2. Since power and rate have function relationship, the traditional greedy algorithm cannot be used when selecting users for subcarriers, and thereby cannot obtain the suboptimal solution. Therefore, in general, the simultaneous optimization of the two objectives can only be achieved by exhaustion method. However, it is infeasible to implement exhaustive method owing to the large degrees of freedom of resources in multi-user MIMO-OFDM systems.

In order to settle the aforementioned problems, this paper proposes a method for simultaneously optimizing power and rate. Specifically, we integrate the user selection process of subcarriers into power and rate optimization and thereby reducing the problem complexity brought by the degrees of freedom of resources. We establish a double-objective optimization model of power and rate. Due to the existence of nonlinear constraints, we propose to utilize the dual problem of the original problem such that the original problem is converted to a convex optimization problem that can be solved. Meanwhile, we prove that the duality gap between the original problem and its dual problem is 0, which means the original problem shares the same optimal solution with its dual problem. In solving the dual problem, a decomposition and iterative method is proposed to obtain the optimal dual variables.

The main contributions of this paper are as follows:

1. We first propose to optimize power and rate simultaneously.

2. We propose to solve the dual problem of the original problem in order to settle the non-convexity of the original problem. Moreover, we prove that the duality gap between the

original problem and its dual problem is 0, which means the optimal solution of the original problem can be obtained by exploring the optimal solution of its dual problem.

The rest of the paper is organized as follows: The related works are introduced in Section 2. The system model is proposed in Section 3. In Section 4, a double-objective optimization model is established and the solution is presented. Simulation results and discussions are implemented in Section 5. Section 6 concludes the paper.

2. Related Work

Authors in [9] put forward a resource allocation scheme in multi-user MIMO-OFDM downlink systems by adopting cooperative game algorithm. The algorithm can compromise on system performance and quality, but it does not consider the improvement of overall system throughput. What is more, the algorithm requires timely channel feedback information and has great overheads, which makes it difficult to apply to users with high demand for real-time and rate. Authors in [10] propose a resource sharing and allocation scheme based on utility maximization. The scheme can jointly optimize the subcarrier allocation and antenna allocation for every single user, and utilize frequency and spatial diversity gain of multiple users to increase system throughput and improve spectrum efficiency. However, they do not consider the total system power consumption. Authors in [11] propose a distributed architecture for ultra-dense cellular networks with single and multiple gateways in 5G cellular scenarios. They investigate the impact of different numbers of small cell BSs on the backhaul network capacity and the backhaul energy efficiency of ultra-dense cellular networks. Nevertheless, they just analyze the parameters in the proposed network architecture but do not propose a certain optimization method to optimize them. Authors in [12] investigate energy and cost efficiency optimization solutions for 5G wireless communication systems with a large number of antennas and RF chains. They propose an energy efficient hybrid precoding with the minimum number of RF chains (EEHP-MRFC) algorithm to reduce the cost of RF circuits. They also develop the critical number of antennas searching (CNAS) and user equipment number optimization (UENO) algorithms to optimize the energy efficiency. They reach the conclusion that the maximum energy efficiency can be improved compared with the conventional zero-forcing (ZF) precoding algorithm. Authors in [13] put forward a kind of beamforming algorithms for multi-user successive maximum ratio transmission (MS-MRT) to optimize video quality for multi-user MIMO-OFDM systems. The algorithm optimizes the quality of video experience by maximizing system throughput. Compared with previous algorithms, its performance improves significantly, yet it can only be applied to unicast systems. Authors in [14] minimize transmission power of base station by weighing between power control and discontinuous transmission. They first estimate the delay and antenna configuration in sleep mode by using average channel condition, and then obtain timely channel state information at the sending end; according to these information, they finally find the optimal antenna configuration, resource allocation scheme and the number of time slots using discontinuous transmission. Authors in [15] introduce a power optimization algorithm in femtocell networks. They use the zero-forcing beamforming to remove interference among users, and then utilize genetic algorithm for power allocation in order to improve signal-to-noise ratio (SNR) of the system to a certain extent. However, the algorithm is based on the assumptions that users are in femtocell networks and are covered by macro base station, and it cannot guarantee the excellent performance of the system when users are in other kinds of situation, thus the applicable scope of the algorithm is narrow.

According to the above analysis, most of the existing literatures only investigate single-objective optimization strategy in resource allocation. Different from these previous works, this paper proposes to optimize power and rate simultaneously. As far as we know, there are no literatures that simultaneously optimize power and rate in multi-user MIMO-OFDM systems.

3. System Model

In this section, we will derive the constraint on the number of users sharing the same subcarrier and the rate of users on the subcarrier in multi-user MIMO-OFDM systems.

The number of transmitting antennas of system base station is denoted as N_T and the number of receiving antennas of each terminal is denoted as n_r. The system has K users. Denoting the number of users who reuse subcarrier m ($m=1,2,3...N$) as K_m, then the total number of receiving antennas of K_m terminals are $N_R = K_m n_r$. Generally, $N_T \geq N_R$. The interference among K_m users can be eliminated by the zero-forcing linear block diagonalization (ZF-LBD) technique [16]. Denoting the pre-coding matrix of user k $(k=1,2\cdots K_m)$ on subcarrier m as $\boldsymbol{T}_{k,m}$ and the transmission data of user k as $\boldsymbol{b}_{k,m}$, the received signal of user i $(i=1,2\cdots K_m)$ on subcarrier m can be expressed as:

$$\boldsymbol{y}_{i,m} = \boldsymbol{H}_{i,m} \sum_{k=1,k\neq i}^{K_m} \boldsymbol{T}_{k,m}\boldsymbol{b}_{k,m} + \boldsymbol{H}_{i,m}\boldsymbol{T}_{i,m}\boldsymbol{b}_{i,m} + \boldsymbol{n}_{i,m} \tag{1}$$

where $\boldsymbol{H}_{i,m}$ denotes the channel gain matrix of user i on subcarrier m and $\boldsymbol{n}_{i,m}$ denotes Gaussian white noise on this channel. Apparently, in order to eliminate the interference from other users, the first part of the right-hand-side of (1) should be set to be $\boldsymbol{0}$, i.e.,

$$\boldsymbol{H}_{i,m} \sum_{k=1,k\neq i}^{K_m} \boldsymbol{T}_{k,m}\boldsymbol{b}_{k,m} = \boldsymbol{0} \tag{2}$$

Since the transmission data $\boldsymbol{b}_{i,m}$ is positive, we have

$$\boldsymbol{H}_{i,m} \sum_{k=1,k\neq i}^{K_m} \boldsymbol{T}_{k,m} = \boldsymbol{0} \tag{3}$$

Similarly, the product of the channel gain matrix of any other users and the pre-coding matrix of user i should be $\boldsymbol{0}$, that is,

$$[\boldsymbol{H}_{1,m}^T, \cdots, \boldsymbol{H}_{i-1,m}^T, \boldsymbol{H}_{i+1,m}^T, \cdots, \boldsymbol{H}_{K_m,m}^T]^T \boldsymbol{T}_{i,m} = \boldsymbol{0} \tag{4}$$

where $[\boldsymbol{H}_{1,m}^T, \cdots, \boldsymbol{H}_{i-1,m}^T, \boldsymbol{H}_{i+1,m}^T, \cdots, \boldsymbol{H}_{K_m,m}^T]^T$ is the joint matrix of interference users, denoted as $\tilde{\boldsymbol{H}}_{i,m}$. The dimension of $\tilde{\boldsymbol{H}}_{i,m}$ is $\sum_{k=1,k\neq i}^{K_m} n_r \times N_T$. Supposing $\tilde{\boldsymbol{H}}_{i,m}$ is a full rank matrix with the rank being $r(\tilde{\boldsymbol{H}}_{i,m}) = \min(\sum_{k=1,k\neq i}^{K_m} n_r, N_T) = \sum_{k=1,k\neq i}^{K_m} n_r = N_R - n_r$, we perform singular value decomposition (SVD) on it [16], and do the following transformation

$$\tilde{\boldsymbol{H}}_{i,m} = \tilde{\boldsymbol{U}}_{i,m} \tilde{\boldsymbol{S}}_{i,m} \tilde{\boldsymbol{V}}_{i,m}^H = \tilde{\boldsymbol{U}}_{i,m}[\boldsymbol{\Sigma}_1, \boldsymbol{0}][\tilde{\boldsymbol{V}}_{i,m}^{(1)}, \tilde{\boldsymbol{V}}_{i,m}^{(0)}]^H \tag{5}$$

where $\tilde{\boldsymbol{U}}_{i,m}$ and $\tilde{\boldsymbol{V}}_{i,m}^H$ are unitary matrices satisfying $\tilde{\boldsymbol{U}}_{i,m}^H \tilde{\boldsymbol{U}}_{i,m} = \boldsymbol{I}$ and $\tilde{\boldsymbol{V}}_{i,m}^H \tilde{\boldsymbol{V}}_{i,m} = \boldsymbol{I}$. Since vectors of each column of a unitary matrix are orthogonal, the product of any two columns of a unitary matrix is 0. In (5), we pre-multiply $\tilde{\boldsymbol{H}}_{i,m}$ by $\tilde{\boldsymbol{U}}_{i,m}^{H}$ and post-multiply the right-hand-side of the formula by $[\tilde{\boldsymbol{V}}_{i,m}^{(1)},\tilde{\boldsymbol{V}}_{i,m}^{(0)}]$, we can get

$$\tilde{\boldsymbol{U}}_{i,m}^{H}\tilde{\boldsymbol{H}}_{i,m}[\tilde{\boldsymbol{V}}_{i,m}^{(1)},\tilde{\boldsymbol{V}}_{i,m}^{(0)}]=[\boldsymbol{\Sigma}_{I},\boldsymbol{0}] \tag{6}$$

According to the nature of matrix multiplication, we have

$$\tilde{\boldsymbol{U}}_{i,m}^{H}\tilde{\boldsymbol{H}}_{i,m}\tilde{\boldsymbol{V}}_{i,m}^{(0)}=\boldsymbol{0} \tag{7}$$

Pre-multiplying both sides of (7) by $\tilde{\boldsymbol{U}}_{i,m}$ yields

$$\tilde{\boldsymbol{H}}_{i,m}\tilde{\boldsymbol{V}}_{i,m}^{(0)}=\boldsymbol{0} \tag{8}$$

Therefore, the pre-coding matrix satisfies

$$\boldsymbol{T}_{i,m}=\tilde{\boldsymbol{V}}_{i,m}^{(0)} \tag{9}$$

where $\tilde{\boldsymbol{V}}_{i,m}^{(0)}$ denotes the right singular value vector corresponding to the zero singular value of $\tilde{\boldsymbol{H}}_{i,m}$, which is called the zero space of $\tilde{\boldsymbol{H}}_{i,m}$. That is, for user i, $\tilde{\boldsymbol{V}}_{i,m}^{(0)}$ can eliminate the interference from other users.

Next, we will obtain the maximum number of users sharing the same subcarrier by deriving solvability conditions for (8).

In (5), the dimension of $\tilde{\boldsymbol{U}}_{i,m}$ is $(N_R-n_r)\times(N_R-n_r)$. $[\boldsymbol{\Sigma}_1,\boldsymbol{0}]$ is a diagonal matrix consists of N_R-n_r non-zero singular values of $\tilde{\boldsymbol{H}}_{i,m}$, having the same dimension as $\tilde{\boldsymbol{H}}_{i,m}$, that is, $(N_R-n_r)\times N_T$. The dimension of $\tilde{\boldsymbol{V}}_{i,m}^{H}$, i.e., $[\tilde{\boldsymbol{V}}_{i,m}^{(1)},\tilde{\boldsymbol{V}}_{i,m}^{(0)}]^H$ is $N_T\times N_T$, where $\tilde{\boldsymbol{V}}_{i,m}^{(1)}$ is the left singular value vector corresponding to the N_R-n_r non-zero singular values of $\tilde{\boldsymbol{H}}_{i,m}$ with the dimension being $N_T\times(N_R-n_r)$ and $\tilde{\boldsymbol{V}}_{i,m}^{(0)}$ is the right singular value vector corresponding to the $N_T-N_R+n_r$ zero singular values of $\tilde{\boldsymbol{H}}_{i,m}$ with the dimension being $N_T\times(N_T-N_R+n_r)$. The multiplication of $\tilde{\boldsymbol{H}}_{i,m}$ and $\tilde{\boldsymbol{V}}_{i,m}^{(0)}$ is the multiplication of a $(N_R-n_r)\times N_T$ -dimensional matrix and a $N_T\times(N_T-N_R+n_r)$ -dimensional matrix, which is equivalent to $N_T-N_R+n_r$ systems of homogeneous linear equations with N_T unknown numbers, where each system of equations are the product of a $(N_R-n_r)\times N_T$ -dimensional matrix and a $N_T\times 1$- dimensional unknown vector. The necessary and sufficient condition of a system of homogeneous linear equations $\boldsymbol{Ax}=\boldsymbol{0}$ with n unknown numbers having solution is $r(\boldsymbol{A})<n$. Since $r(\tilde{\boldsymbol{H}}_{i,m})$ = N_R-n_r, the necessary and sufficient condition of every single system of homogeneous linear equations with N_T unknown numbers having solution is

$$N_R-n_r<N_T \tag{10}$$

Divide both sides of (10) by n_r, then, the maximum number of users on each subcarrier satisfies the following constraint

$$K_m < \frac{N_T}{n_r} + 1 \tag{11}$$

K_m in (11) is the maximum number of users sharing the same subcarrier. Only when this constraint is satisfied, the interference among users can be eliminated and users can share the same subcarrier. When we select users for each subcarrier in latter section, the maximum number of users should satisfy (11).

Defining $\hat{\boldsymbol{H}}_{i,m} = \boldsymbol{H}_{i,m}\tilde{\boldsymbol{V}}^{(0)}_{i,m}$ and performing SVD on it yields

$$\hat{\boldsymbol{H}}_{i,m} = \boldsymbol{H}_{i,m}\tilde{\boldsymbol{V}}^{(0)}_{i,m} = \hat{\boldsymbol{U}}_{i,m}\hat{\boldsymbol{S}}_{i,m}\hat{\boldsymbol{V}}^{H}_{i,m} \tag{12}$$

The pre-coding technique is used at the transmitter and the corresponding processing matrix is used at the receiver to eliminate interference among users. For subcarrier m, the transmitting pre-coding matrix of base station is defined as $\boldsymbol{T}_m = [\boldsymbol{T}_{1,m}, \boldsymbol{T}_{2,m}, \cdots, \boldsymbol{T}_{K_m,m}]$, where

$$\boldsymbol{T}_{i,m} = \tilde{\boldsymbol{V}}^{(0)}_{i,m}\hat{\boldsymbol{V}}_{i,m} \tag{13}$$

The processing matrix at the receiver is defined as $\boldsymbol{W}_m = [\boldsymbol{W}_{1,m}, \boldsymbol{W}_{2,m}, \cdots \boldsymbol{W}_{K_m,m}]$, where $\boldsymbol{W}_{i,m} = \hat{\boldsymbol{U}}_{i,m}$. Pre-multiplying the received signal $\boldsymbol{y}_{i,m}$ in (1) by $\boldsymbol{W}^{H}_{i,m}$ and bringing the pre-coding matrix (13) into it, we have

$$\begin{aligned} &\boldsymbol{W}^{H}_{i,m}(\boldsymbol{H}_{i,m}\boldsymbol{T}_{i,m}\boldsymbol{b}_{i,m} + \boldsymbol{n}_{i,m}) \\ &\Rightarrow \boldsymbol{W}^{H}_{i,m}(\boldsymbol{H}_{i,m}\boldsymbol{T}_{i,m}\boldsymbol{b}_{i,m} + \boldsymbol{n}_{i,m}) \\ &\Rightarrow \hat{\boldsymbol{U}}^{H}_{i,m}\boldsymbol{H}_{i,m}\tilde{\boldsymbol{V}}^{(0)}_{i,m}\hat{\boldsymbol{V}}_{i,m}\boldsymbol{b}_{i,m} + \boldsymbol{W}^{H}_{i,m}\boldsymbol{n}_{i,m} \end{aligned} \tag{14}$$

where $\boldsymbol{H}_{i,m}\tilde{\boldsymbol{V}}^{(0)}_{i,m}\hat{\boldsymbol{V}}_{i,m}$ denotes the equivalent channel gain matrix of user i on subcarrier m.

Pre-multiplying $\hat{\boldsymbol{U}}_{i,m}\hat{\boldsymbol{S}}_{i,m}\hat{\boldsymbol{V}}^{H}_{i,m}$ in (12) by $\hat{\boldsymbol{U}}^{H}_{i,m}$ and post-multiplying it by $\hat{\boldsymbol{V}}_{i,m}$ and bringing it into (14), we can get

$$\hat{\boldsymbol{S}}_{i,m}\boldsymbol{b}_{i,m} + \boldsymbol{W}^{H}_{i,m}\boldsymbol{n}_{i,m} \tag{15}$$

According to (14) and (15), the equivalent channel gain matrix is $\hat{\boldsymbol{S}}_{i,m}$, which is a diagonal matrix composed of singular values of $\boldsymbol{H}_{i,m}\tilde{\boldsymbol{V}}^{(0)}_{i,m}$.

After such processing, interference among users is eliminated. The MU-MIMO channels on each subcarrier are equivalent to multiple independent SU-MIMO channels. All elements in (15) are matrices after block diagonalization, namely, all other elements except diagonal elements are 0.

Denoting $\eta_{i,m}$ as the rank of channel gain diagonal matrix $\hat{\boldsymbol{S}}_{i,m}$, that is, $\hat{\boldsymbol{S}}_{i,m}$ has $\eta_{i,m}$ non-zero singular values, then, the transmission channel can be expressed as $\boldsymbol{y}'_{i,m} = \hat{\boldsymbol{S}}_{i,m}\boldsymbol{b}_{i,m}(i = 1, 2, 3\cdots, K_m, m = 1, 2, 3\cdots N)$. Therefore, the channel for each user on each subcarrier can be equivalent to $\eta_{i,m}$ parallel channels. On subcarrier m, the bandwidth normalized rate of equivalent parallel channel l of user i can be expressed as

$$r_{i,m,l}^{\partial_{i,m}} = \log_2(1 + \frac{p_{i,m,l}(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0}) \quad (16)$$

where $\frac{1}{\Gamma} = -\frac{\ln(5\mathrm{BER})}{1.5}$. Given certain error rate, $\frac{1}{\Gamma}$ is the power loss caused by non-ideal transmission technology [17]. N_o denotes the power of channel noise that satisfies the zero-mean complex Gaussian random variable. $s_{i,m,l}^{\partial_{i,m}}$ denotes the l th diagonal element of $\hat{\mathbf{S}}_{i,m}$, i.e., the channel gain of the l th equivalent parallel channel of user i on subcarrier m. $p_{i,m,l}$ denotes power allocated to this equivalent parallel channel and $\partial_{i,m}$ denotes the selection result of user i on subcarrier m, which is defined as

$$\partial_{(i,m)} = \begin{cases} 1, \textit{user i is on subcarrier m} \\ 0, \textit{user i is not on subcarrier m} \end{cases} \quad (17)$$

Therefore, on subcarrier m, the bandwidth normalized rate of user i can be expressed as

$$r_{i,m}^{\partial_{i,m}} = \sum_{l=1}^{\eta_{i,m}} \log_2(1 + \frac{p_{i,m,l}(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0}) \quad (18)$$

According to the analysis above, the transmission rate of each subcarrier is related to the user selection result on it. The same subcarrier will have different channel gain for different users, so different user selection result will affect total power consumption and total transmission rate on this subcarrier.

4. Establishment and Solution of a Double-objective Optimization Model

Since the same subcarrier has different gain for different users, thus effectively selecting specific users and allocating suitable power for different subcarriers are the key factors affecting the total power and rate of the system. For N subcarriers, we will minimize the transmission power and maximize the total system rate simultaneously while ensuring the total system rate no lower than a certain requirement by finding the optimal user selection and power allocation for different subcarriers. With that, we can achieve power and rate optimization at the same time.

Denote the minimum bandwidth normalized total system rate requirement as R with the unit being bit/s/Hz. Combining (16), (17) and (18), we can establish the following double-objective optimization model

$$\begin{aligned} &\min_{\{p_{i,m,l}\}} \quad P = \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} p_{i,m,l} \\ &\max_{\{\partial_{i,m}\}} \quad \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} r_{i,m,l}^{\partial_{i,m}} \\ &\textit{subject to} \quad \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} r_{i,m,l}^{\partial_{i,m}} \geq R \end{aligned} \quad (19)$$

where N denotes the number of subcarriers, K_m denotes the number of users on subcarrier m, and $p_{i,m,l}$ denotes the power allocated to equivalent parallel channel l.

Combining (18) and (19), we can observe that when $p_{i,m,l}$ decreases, transmission power

($P=\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} p_{i,m,l}$) will decrease, however, total system rate ($\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} r_{i,m,l}^{\partial_{i,m}}$) will also decrease given a certain $\partial_{i,m}$. It means the two objectives are contradictory and cannot reach optimum at the same time. Common way to resolve this kind of double-objective optimization model is to adopt a sub-optimal method which can transform the model to a single-objective optimization model.

For example, we can use utility function method to merge the two objectives into a single objective, which can be expressed as $\min_{p_{i,m,l},\partial_{i,m}} a\cdot P=\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} p_{i,m,l}-b\cdot\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} r_{i,m,l}^{\partial_{i,m}}$. However, the effectiveness factor a, b are difficult to decide, thus, we use another method, main-object method. We choose the transmission power minimization objective to be our primary objective and convert the total system rate maximization objective into a constraint, thus we have

$$\begin{aligned}&\min_{\{p_{i,m,l}\}}\ P=\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} p_{i,m,l}\\&subject\ \ to\ \max_{\{\partial_{i,m}\}}\Big(\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} r_{i,m,l}^{\partial_{i,m}}\Big)\geq R\end{aligned}\tag{20}$$

Generally, a convex optimization problem that can be directly solved by convex optimization method requires its function and inequality constraint functions to be convex and no nonlinear constraints exist. In (20), the optimization variable $r_{i,m,l}^{\partial_{i,m}}$ in the constraint is a logarithmic function that is nonlinear. Therefore, (20) is a non-convex optimization problem that cannot be solved directly by convex optimization method.

In order to solve the problem, Lagrange dual decomposition algorithm is used in this paper [17]-[20]. The basic idea of the method is to merge the two objectives into one goal based on weighted sum rules. According to this idea, the following Lagrange function is constructed

$$\ell(\{p_{i,m,l}\},\upsilon)=\left(\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} p_{i,m,l}-\upsilon\Big(\max_{\{\partial_{i,m}\}}\Big(\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} r_{i,m,l}^{\partial_{i,m}}\Big)-R\Big)\right)\tag{21}$$

where υ （$\upsilon\geq 0$）denotes the weight, also called dual variables. Let

$$h(\upsilon)=\min_{p_{i,m,l}\geq 0,\forall i,m,l}\ell(\{p_{i,m,l}\},\upsilon)=\min_{p_{i,m,l}\geq 0,\forall i,m,l}\left(\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} p_{i,m,l}-\upsilon\Big(\max_{\{\partial_{i,m}\}}\Big(\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} r_{i,m,l}^{\partial_{i,m}}\Big)-R\Big)\right)\tag{22}$$

We propose the dual problem of the original problem as

$$\begin{aligned}&\max_{\{\upsilon\}}\ \ h(\upsilon)\\&subject\ \ to\ \ \upsilon\geq 0\end{aligned}\tag{23}$$

Next, we first give two definitions, and then prove that the optimal solution of the original problem (20) can be obtained by solving its dual problem (23).

Definition 1: The lower bound of the optimal value of the original problem. Denoting the optimal value of the original problem as P^*, if q that satisfies $q\leq P^*$ exists, then q is a lower bound of P^*.

Definition 2: The infimum of the optimal value of the original problem. Denoting a set of real numbers as Q, if any element $q\in Q$ satisfies $q\leq P^*$, then all elements in Q are the lower bounds of P^* and the maximum element $q_{\max}$ in Q is the infimum of P^*.

According to duality theorem in [20], the dual function is the lower bound of the optimal value of its original problem. Thus, for any $\upsilon \geq 0$, we have

$$d^{\wedge} \leq P^{*} \tag{24}$$

where $d^{\wedge}$ denotes a feasible solution of dual problem (23). That is, the solution of dual problem is the lower bound of the optimal value of the original problem. The proof is shown in **Appendix I**.

Denoting the optimal solution of dual problem as υ^{*} and its corresponding optimal value as d^{*} which is the maximum value of $h(\upsilon)$. Then, d^{*} is the infimum of the optimal value of the original problem. The difference between P^{*} and d^{*} is defined as duality gap [20]. Generally, for a non-convex problem, the duality gap does not equal to 0. However, for the optimization problem in this paper, it is proved in **Appendix II** that the duality gap is 0, which means strong duality is established. Under this condition, we bring the dual optimal solution υ^{*} into (21) and have the following optimization objective

$$\min_{\{p_{i,m,l}\}} \ \ell(\{p_{i,m,l}\},\upsilon^{*}) \tag{25}$$

Accordingly, under the condition that the strong duality is established, the optimal solution of (25) is a feasible solution as well as the optimal solution of the original problem [20]. Therefore, by using this property, we can obtain the optimal solution of the original problem by solving the optimal solution of its dual problem.

In dual problem (23), $h(\upsilon)$ denotes Lagrange dual function, which is the minimum value of Lagrange function (21) of variable $p_{i,m,l}$. $h(\upsilon)$ is a cluster of pointwise infimums of an affine function concerning υ [20]. Therefore, $h(\upsilon)$ is a concave function with its inequality constraints being convex. Then, the dual problem is another form of convex optimization problem, that is, concave maximization problem [20] which can be solved by convex optimization method.

The concrete solution process is as follows. First, the minimum value of Lagrange function (21) of variable $p_{i,m,l}$ is calculated to obtain the dual function $h(\upsilon)$. Then, calculate υ^{*} that can maximize $h(\upsilon)$ among all feasible υ.

Therefore, first, we bring (16) into (22), and $h(\upsilon)$ can be written as follows:

$$h(\upsilon) = \sum_{m=1}^{N} h_m^{'}(\upsilon) + \upsilon R \tag{26}$$

where

$$h_m^{'}(\upsilon) = \min_{\{\partial_{i,m}\}} \min_{p_{i,m,l} \geq 0, \forall i,l} \left(\sum_{i=1}^{K_m} \sum_{l=1}^{\eta_{i,m}} p_{i,m,l} - \upsilon \sum_{i=1}^{K_m} \sum_{l=1}^{\eta_{i,m}} \log_2 \left(1 + \frac{p_{i,m,l} (s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0}\right) \right) \tag{27}$$

(26) shows that the original problem can be decomposed into N independent sub-problems that can be solved separately, thus obtaining $h(\upsilon)$.

The solution of (27) can be divided into two steps. First, supposing the selection result $\{\partial_{i,m}\}$ on subcarrier m is known, we calculate the optimal power and the corresponding rate expression by using convex optimization method. Then, we perform user selection and calculate υ^{*} that can maximize $h(\upsilon)$.

Following the above idea, the minimization problem (27) can be transformed to the following optimization problem:

$$\min \quad g_m(p_{i,m,l}) = \left(\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} p_{i,m,l} - \upsilon \sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \log_2\left(1 + \frac{p_{i,m,l}(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0}\right) \right) \tag{28}$$

$$subject \quad to \quad p_{i,m,l} \ge 0, \forall i,m,l$$

This is a standard convex optimization problem which satisfies the three essential conditions of convex optimization, thus it can be solved by standard convex optimization method [20]. To this end, the Lagrange multiplier vector $\boldsymbol{\theta}$ （$\theta_{i,m,l}$） is introduced, and (28) can be written as a Lagrange function in the following form:

$$g_m(p_{i,m,l}) + \sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \theta_{i,m,l}\, p_{i,m,l} \tag{29}$$

According to the optimality condition of convex optimization problem in [20], the point satisfying KKT conditions is the optimal solution. In optimization problem (28), in order to obtain the optimal solution, two KKT conditions that need to be satisfied are as follows:

$$\nabla g_m(p_{i,m,l}) + \sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \theta_{i,m,l} \nabla p_{i,m,l} = 0 \tag{30}$$

$$p_{i,m,l}\theta_{i,m,l} = 0 \tag{31}$$

where ∇ represents taking the derivative of $p_{i,m,l}$. The condition when (31) is always true is

$$\theta_{i,m,l} = 0 \quad \forall i,m,l \tag{32}$$

Bringing (32) into (30) yields

$$p_{i,m,l} = \max\left(\frac{\upsilon}{\ln 2} - \frac{\Gamma N_0}{(s_{i,m,l}^{\partial_{i,m}})^2}, 0\right) = \left(\frac{\upsilon}{\ln 2} - \frac{\Gamma N_0}{(s_{i,m,l}^{\partial_{i,m}})^2}\right)^+ \tag{33}$$

where $(a)^+$ represents taking the maximum one between a and 0. Binging (33) into (16), we can further have

$$r_{i,m,l} = \max\left(\log_2\left(\frac{\upsilon(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0 \ln 2}\right), 0\right) = \left(\log_2\left(\frac{\upsilon(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0 \ln 2}\right)\right)^+ \tag{34}$$

Bringing (33) and (34) into (27), we can get

$$h_m^{'}(\upsilon) = \min_{\{\partial_{i,m}\}} \left(\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \left(\frac{\upsilon}{\ln 2} - \frac{\Gamma N_0}{(s_{i,m,l}^{\partial_{i,m}})^2}\right)^+ - \upsilon\left(\log_2\left(\frac{\upsilon(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0 \ln 2}\right)\right)^+ \right) \tag{35}$$

(35) is a function of $\partial_{i,m}$ that can be obtained by selecting the optimal users. Finally, we utilize bisection method [21] to calculate υ^* that can maximize $h(\upsilon)$, during which the greedy algorithm is adopted to do user selection. The algorithm optimizes power and rate simultaneously, and the iterative precision of each step can be controlled. The iteration process is as follows:

Bisection method:

1. Initialize $\upsilon_{\min} = 0$, $\upsilon_{\max} = \upsilon_o$, where υ_o is the upper bound of the optimal value υ^* that can be derived from **Appendix III**;

2. Initialize $\upsilon = \dfrac{1}{2(\upsilon_{\min} + \upsilon_{\max})}$;

3. Implement greedy algorithm on subcarrier m （$1,2,\cdots,N$）to find $\partial_{i,m}$ that can minimize $h_m^{'}(\upsilon)$ in all possible user selection results. The algorithm will be specified later

4. Bring the obtained υ and $\partial_{i,m}$ into (33) and yield $p_{i,m,l}=(\frac{\upsilon}{\ln 2}-\frac{\Gamma N_0}{(s_{i,m,l}^{\partial_{i,m}})^2})^+$, $m=1,2,3,\cdots,N$, $i=1,2,\cdots,K_m$;

5. Bring $p_{i,m,l}$ and $\partial_{i,m}$ obtained in step 3 and step 4 into $\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}}\log_2(1+\frac{p_{i,m,l}(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_o})$, if $\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}}(\log_2(\frac{\upsilon(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0\ln 2}))^+\geq R$, set $\upsilon_{\max}=\upsilon$, otherwise set $\upsilon_{\min}=\upsilon$.

6. Repeat step 2 to step 5 until $\upsilon_{\max}-\upsilon_{\min}\leq\delta$, where δ denotes the constant that we set to control the accuracy of the algorithm.

User selection by greedy algorithm:

Bringing $\upsilon=\frac{1}{2(\upsilon_{\min}+\upsilon_{\max})}$ into (35) and then considering (35) as a function of $\partial_{i,m}$ on subcarrier m, we have

$$h^{'}(\partial_{i,m})=\min_{\{\partial_{i,m}\}}\left(\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}}((\frac{\upsilon}{\ln 2}-\frac{\Gamma N_0}{(s_{i,m,l}^{\partial_{i,m}})^2})^+-\upsilon(\log_2(\frac{\upsilon(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0\ln 2}))^+)\right) \tag{36}$$

1. Denote the total number of users in the system as K and the user selection set on subcarrier m as set_m.

2. On subcarrier m, set $set_m=\phi$. According to (13), the number of users on each subcarrier should be no more than N_T/n_r, where N_T denotes transmitting antennas and n_r denotes receiving antennas of each terminal. Among all K users in the system, select a user who can minimize $h^{'}(\partial_{i,m})$ and join it into set_m.

3. Randomly select a user from remaining $K-1$ users and join it into set_m. Calculate the value of $h^{'}(\partial_{i,m})$, if it is no more than the previous value of $h^{'}(\partial_{i,m})$, the user is selected, otherwise, the user is abandoned. Keep traversing all remaining users with the standard that $set_m\leq N_T/n_r$.

4. Repeat steps ii-iii on other subcarriers $(1,2,\cdots,m-1,m+1,\cdots,N)$, we can get the optimal user selection sets ($set_1,set_2,\cdots,set_N$).

In this way, the user selection process is completed. The user selection results are determined and the minimum $h^{'}(\partial_{i,m})$ is obtained.

5. Numerical Results and Discussions

We consider a downlink model for multi-user MIMO-OFDM systems. The number of transmitting antennas in base station is 4, and the number of receiving antennas of each terminal is 2. The system contains 10 users. The total bandwidth of the system is 1M, and be divided into 50 subcarriers. The required bit error rate is 10^{-3} and the required minimum total transmission rate is 10^{-7} (bit/s/Hz). The power of Gaussian white noise of the channel ranges from $0.5*10^{-12}$ to $3*10^{-12}$.

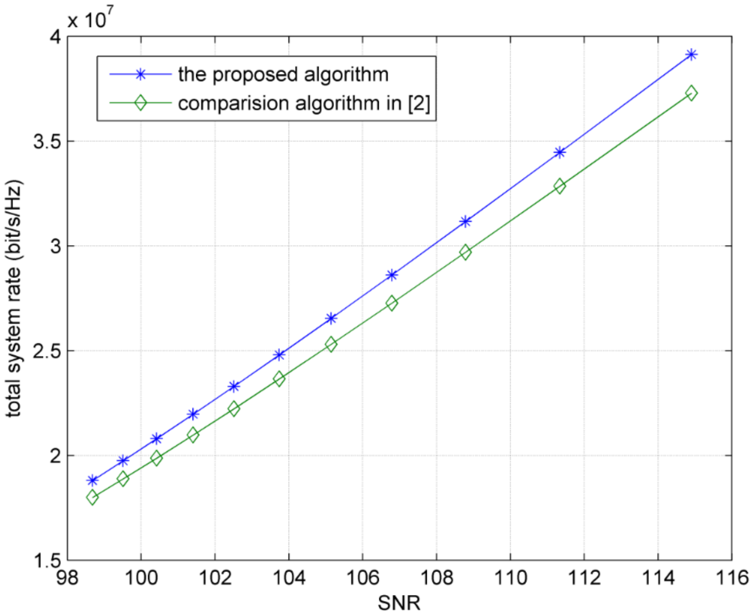

Fig. 1. The total system rate versus SNR

As can be seen from **Fig. 1**, with the increase of SNR, the total system rate obtained by the proposed algorithm is greater than that achieved by the algorithm in [2]. This is because authors in [2] only use greedy algorithm to optimize power. They select the subcarrier that can minimize the increment of power in each iteration until the total system rate R is satisfied. However, the proposed algorithm can optimize rate and power at the same time, so the total system rate is greater than that in [2].

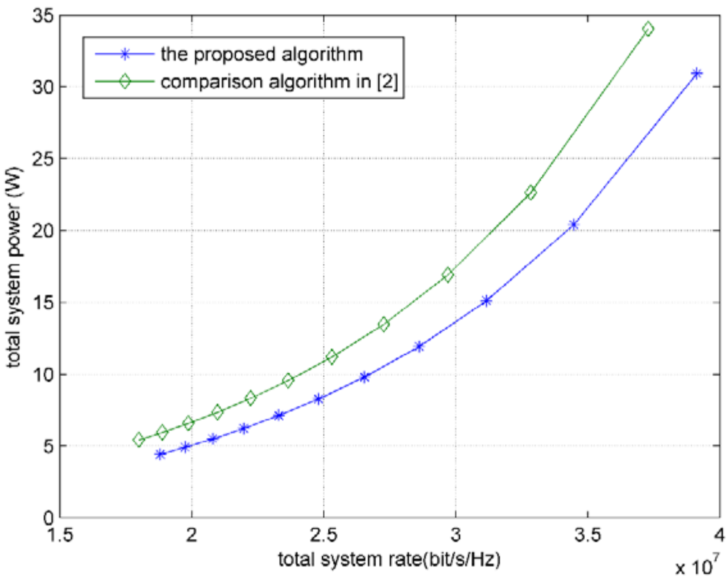

Fig. 2. The system power versus the total system rate

We can observe from **Fig. 2.** that the power consumed by the algorithm in [2] is more than the power consumed by the proposed algorithm when the total system rate increases. Moreover, greater total system rate results in greater gap between the power required by the two algorithms. The algorithm in [2] restricts that one subcarrier can only carry one user, while the subcarrier in the proposed algorithm can carry multiple users and the co-channel interference among users is eliminated by pre-coding technique. **Fig. 2.** reflects that SDMA can improve system throughput without increasing system power. This is because using appropriate number of multiplexing users can make the increment of multiplexing gain greater than the decrement of antenna gain in MIMO systems, and thereby improving system throughput without increasing power.

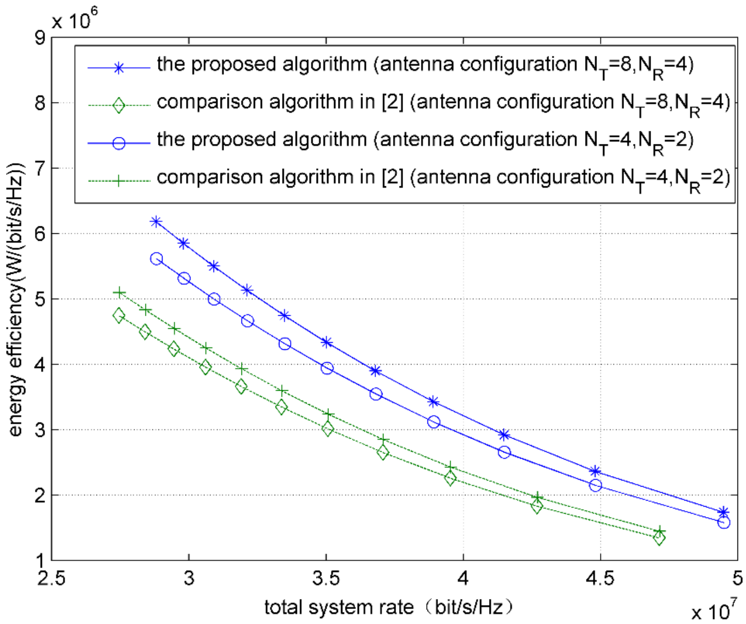

Fig. 3. The energy efficiency versus the total system rate under different antenna configurations

Fig.3. shows that with the increase of total system rate, the energy efficiency, i.e., the ratio of system power consumption to total system rate, of the proposed algorithm is always greater than that in [2] under two different antenna configurations. At the same time, with the increase of the number of antennas, the energy efficiency of the proposed algorithm increases accordingly, which comes from the fact that the increase of antennas brings an increase in the space division gain. On the contrary, the energy efficiency of algorithm in [2] decreases when the number of antennas increases. This is because increasing the number of antennas at both the transmitter and receiver will also increase system noise owing to the increase of the number of equivalent channels increases. It is difficult to resist the system performance degradation caused by noise when only utilizing antenna gain rather than spatial gain. Therefore, the energy efficiency in [2] does not increase when the number of antennas increases.

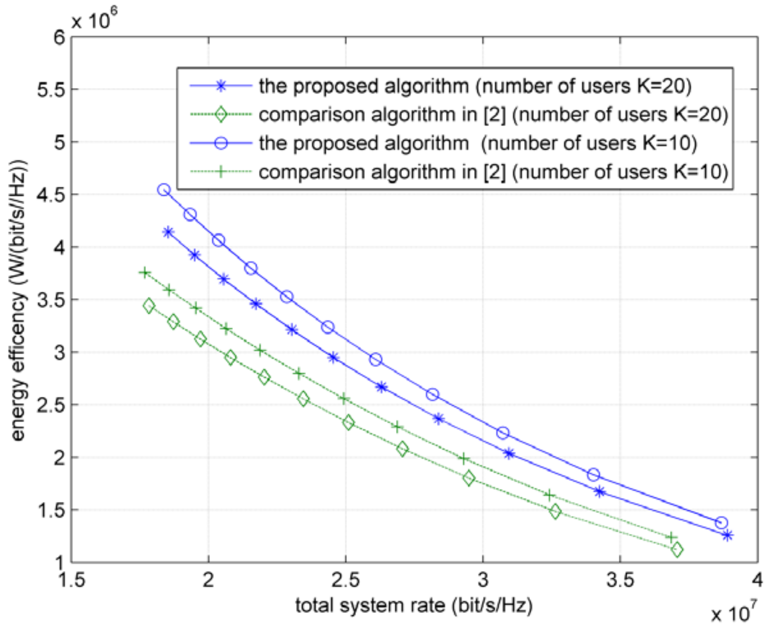

Fig. 4. The energy efficiency versus the total system rate under different system users

It can be seen from **Fig. 4.** that the energy efficiency of the proposed algorithm is always greater than that in [2] regardless of whether the number of system users is 10 or 20. The main reason is that the proposed algorithm makes full use of the spectrum gain and space division gain, and optimizes rate and power simultaneously; while algorithm in [2] does not use spatial gain and only optimizes power. Meanwhile, we can observe that the energy efficiency of the two algorithms both decrease when the number of users increases. This is because with the increase of the number of system users, the demand for resources will greatly increase and more power is needed to achieve high system throughput. Also, power grows proportionately more than the increase in rate, which will cause the decrease in the ratio of rate to power, that is, the degradation of system energy efficiency.

6. Conclusion

This paper proposes a resource allocation algorithm to simultaneously optimize two objectives for multi-user MIMO-OFDM systems. The algorithm can minimize power and maximize system throughput at the same time while ensuring the system throughput no lower than a certain requirement. To settle with the non-convexity of the original problem, its dual problem that can be solved by convex optimization method is introduced. The duality gap between the original problem and its dual problem is proved to be 0, which means the optimal solution of the original problem can be obtained by solving its dual problem. To solve the dual problem, an iterative method is proposed in this paper, which greatly reduces the complexity of the allocation. Moreover, the system energy consumption is minimized and the system throughput is maximized by controlling the accuracy of the iterative method. The numerical results demonstrate that the proposed algorithm is superior to traditional single-objective optimization method in both the system throughput and the system energy consumption.

APPENDIX I

According to duality theorem in [20], the dual function is the lower bound of the optimal value of original problem $P^* = \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} p^*_{i,m,l}$. Thus, for any $\upsilon \geq 0$, we have

$$h(\upsilon) \leq P^* \tag{37}$$

Proof:

Supposing $\tilde{p}_{i,m,l}(i=1,2,\cdots,K_m, m=1,2,\cdots N, l=1,2,\cdots,\eta_{i,m})$ is a feasible point of original problem (20) which satisfies

$$\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \log_2(1+\frac{\tilde{p}_{i,m,l}(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0}) - R \geq 0 \tag{38}$$

For any feasible υ ($\upsilon \geq 0$), combining (38), we have

$$\upsilon(\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \log_2(1+\frac{\tilde{p}_{i,m,l}(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0}) - R) \geq 0 \tag{39}$$

We can further obtain

$$\begin{aligned}\ell(\{\tilde{p}_{i,m,l}\},\upsilon) &= \left(\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \tilde{p}_{i,m,l} - \upsilon(\max_{\{\partial_{i,m}\}}(\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \log_2(1+\frac{\tilde{p}_{i,m,l}(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0})) - R)\right) \\ &\leq \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \tilde{p}_{i,m,l}\end{aligned} \tag{40}$$

Combining (40), we can get

$$h(\upsilon) = \min_{p_{i,m,l}\geq 0,\forall i,m,l} \ell(\{p_{i,m,l}\},\upsilon) \leq \ell(\{\tilde{p}_{i,m,l}\},\upsilon) \leq \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \tilde{p}_{i,m,l} \tag{41}$$

Since for any feasible υ or $p_{i,m,l}$, $h(\upsilon) \leq \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} p_{i,m,l}$, thus, $h(\upsilon) \leq P^*$ when $p_{i,m,l} = p^*_{i,m,l}$ and $h(\upsilon^*) \leq P^*$ when $\upsilon = \upsilon^*$.

APPENDIX Ⅱ

$h(\upsilon)$ is a concave function. According to (26) and (35), the optimal dual variable υ' should satisfy

$$\frac{\partial h(\upsilon)}{\partial \upsilon}\Big|_{\upsilon=\upsilon'} = 0 \tag{42}$$

Bring (16) into (22) and take a derivative of the obtained $h(\upsilon)$. Letting the derivative be 0, we have

$$\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}} \log_2(1+\frac{p_{i,m,l}(s_{i,m,l}^{\partial_{i,m}})^2}{\Gamma N_0}) = R \tag{43}$$

Letting $\partial'_{i,m}$ be the selection result that satisfies (43) and bringing $\partial'_{i,m}$ and υ' into (33), we can get

$$p^{'}_{i,m,l} = \left(\frac{\upsilon^{'}}{\ln 2} - \frac{\Gamma N_0}{(s^{\partial^{'}_{i,m}}_{i,m,l})^2}\right)^{+} \tag{44}$$

Bringing (44) into the constraint of (20), we can know that it is a feasible solution of the original problem, thus satisfying

$$\max_{\{\partial_{i,m}\}}\left(\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}}\log_2\left(\left(\frac{\upsilon^{'}(s^{\partial^{'}_{i,m}}_{i,m,l})^2}{\Gamma N_0 \ln 2}\right)^{+}\right)\right) \geq R \tag{45}$$

According to (42), (43) and (44), we can obtain the optimal value of the original problem, i.e., $h(\upsilon^{'}) = \sum_{m=1}^{N}\sum_{i=1}^{K}\sum_{l=1}^{\eta_{i,m}} p^{'}_{i,m,l}$. According to (45), $\{p^{'}_{i,m,l}\}$ is a feasible solution of original problem. Let $h(\upsilon^{'}) = d^{'}$, according to **Appendix Ⅰ**, the dual function is the lower bound of the optimal value of the original problem, thus, $d^{'} \leq P^{*}$. Therefore, $\{p^{'}_{i,m,l}\}$ is also optimal for original problem, that is, $P^{*} = d^{'}$. Accordingly, the duality gap is 0 and strong duality is established.

APPENDIX Ⅲ

Denote $\{\hat{\partial}_{i,m}\}$ and $\{\hat{p}_{i,m,l}\}$ as a user selection result and power allocation result that satisfy

$$\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}}\log_2\left(1+\frac{(s^{\hat{\partial}_{i,m}}_{i,m,l})^2\,\hat{p}_{i,m,l}}{\Gamma N_0}\right) = R+1 \tag{46}$$

Binging the optimal dual variable υ^{*} and (46) into the Lagrange function in (21) yields

$$\ell(\{\hat{p}_{i,m,l}\},\upsilon^{*}) = \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}}\hat{p}_{i,m,l} - \upsilon^{*} \tag{47}$$

Denote the optimal value of original problem as P^{*}. Since the duality gap is 0, the optimal value of the dual problem is $h(\upsilon^{*}) = P^{*}$. Since $\{\hat{p}_{i,m,l}\}$ is not the optimal power allocation result, thus,

$$\ell(\{\hat{p}_{i,m,l}\},\upsilon^{*}) \geq h(\upsilon^{*}) = P^{*} \tag{48}$$

Furthermore, since $P^{*} \geq 0$, combining (47) and (48), we have

$$0 \leq P^{*} = h(\upsilon^{*}) \leq \ell(\{\hat{p}_{i,m,l}\},\upsilon^{*}) = \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}}\hat{p}_{i,m,l} - \upsilon^{*} \tag{49}$$

According to the left-hand-side and the right-hand-side of (49),

$$\sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}}\hat{p}_{i,m,l} \geq \upsilon^{*} \tag{50}$$

Let $\upsilon_0 = \sum_{m=1}^{N}\sum_{i=1}^{K_m}\sum_{l=1}^{\eta_{i,m}}\hat{p}_{i,m,l}$, thus,

$$\upsilon_0 \geq \upsilon^{*} \tag{51}$$

Namely, the upper bound of υ^{*} is υ_0.

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