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Impact of Outdated CSI on the Performance of Incremental Amplify-and-Forward Opportunistic Relaying

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Abstract

This paper investigates the impact of outdated channel state information (CSI) on the performance of the incremental amplify-and-forward (AF) opportunistic relaying (OR) over dual-hop Rayleigh fading channels. According to the definition of distribution function, we obtain the cumulative distribution function (CDF) of the actual combined signal-to-noise ratio (SNR) received at the destination. Based on this CDF, the closed-form expressions of the average spectral efficiency and outage probability are derived for the incremental AF OR under outdated CSI. Numerical results show that in the low region of the average SNR of the direct link, outdated CSI deteriorates the system performance, whereas in the high region, outdated CSI has almost no impact on the system performance.

Keywords: Outdated CSI, incremental relaying, amplify-and-forward, opportunistic relaying, average spectral efficiency, outage probability

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1. Introduction

Cooperative diversity has been demonstrated as an efficient way to combat wireless channel impairments. In this cooperative system, multiple single-antenna-equipped nodes are utilized as relays to assist the source to forward data to the destination over independent wireless channels. Therefore, the system reliability and coverage can be increased significantly [1], [2]. However, cooperative diversity systems with fixed relaying (FR) lead to a certain loss in the spectral efficiency, because they require two phase periods for half-duplex transmissions. Incremental relaying (IR) has been introduced in [3] as an efficient relaying protocol to increase the spectral efficiency over FR as it makes efficient use of the degrees of freedom given by the channels.

In general, a cooperative system with multiple relays suffers from spectral efficiency loss caused by the use of time-division multiplexing. To overcome this loss, a simple cooperative diversity protocol, named as opportunistic relaying (OR), was proposed in [4]. The incremental OR scheme extends the IR scheme in conjunction with the best relay selection over multiple relay environments. It is shown that the incremental OR achieves an equivalent full diversity order as obtained by more complex schemes, where the coordination and distributed space-time coding for multiple relays is required [5].

The performance of incremental OR has been widely investigated in [6-10]. However, these studies have been conducted under the unrealistic assumption of perfect channel state information (CSI). That means the CSI remains constant from channel estimation to data reception over the flat fading channels. In practical scenarios, due to the time-varying nature of the fading channels, the instantaneous CSI at the instant of channel estimation may substantially differ from that at the instant of data reception. Commonly, the best relay is chosen based on the CSI at the instant of channel estimation, which implies that this CSI is outdated. As a result, the selected relay may not be the best relay at the instant of data reception, which will consequently deteriorate the system performance [11].

Recently, there have been some literatures [11-25] analyzing the impact of outdated CSI on the performance of the fixed OR systems, under different relay selection schemes. Relay selection schemes mainly include partial relay and dual-hop relay selections. A partial relay selection is analyzed based on the best instantaneous signal-to-noise ratio (SNR) of source - relay hop in [12], [13], [16], [17], or relay - destination hop in [14]. This scheme adapts to the single-hop relay scenarios, but the corresponding mathematical analysis is simpler. A dual-hop relay selection, which is suitable for the general scenarios, is investigated in [11], [15-25], yet its mathematical analysis is more complex. However, most of the above works have not considered the direct link from source to destination. Consequently, the degrees of freedom given by the channels have not been taken full advantage of.

The cooperative mechanism of the incremental amplify-and-forward (AF) OR is different from its fixed counterpart. Therefore, the impact of outdated CSI on the performance of the incremental AF OR is inevitably different from that of the fixed counterpart. From a practical point of view, it is thus necessary to evaluate the impact of outdated CSI on the performance of the incremental AF OR.

In this paper, we investigate the impact of outdated CSI on the average spectral efficiency and outage probability of the incremental AF OR over dual-hop Rayleigh fading channels. We first propose the cooperative mechanism of the incremental AF OR. Then, according to the definition of distribution function, we obtain the cumulative distribution function (CDF) of the actual combined SNR received at the destination. Based on this CDF, the closed-form expressions of the average spectral efficiency and outage probability are derived for the incremental AF OR under outdated CSI. Finally, using numerical evaluations, we analyze the impact of outdated CSI on the average spectral efficiency and outage probability in different cases.

The remainder of this paper is organized as follows. Section 2 introduces the system model and proposes the cooperative mechanism of the incremental AF OR. In Section 3, we derive the CDF of the actual combined SNR received at the destination, and the closed-form expressions of the average spectral efficiency and outage probability for the incremental AF OR under outdated CSI. Some numerical results are given and discussed in Section 4. Finally, some conclusions are drawn in Section 5.

2. System Model and Proposed Cooperative Mechanism

Consider an AF-based cooperative relaying system consisting of one direct channel from the source *S* to the destination *D*, and N dual-hop relay channels, denoted by $S-D$ and $S-R_i-D$ links (R_i is the relay node between *S* and *D*, $i=1,2,\ldots,N$), respectively. We assume that *S*, *D* and each relay employ only one antenna. The channel gain h_{AB} between any two nodes, *A* and *B*, is modeled as a zero mean complex Gaussian random variable, with variance σ_{AB}^2, denoted by $h_{AB} \sim \mathcal{CN}(0,\sigma_{AB}^2)$. Hence, $\left|h_{AB}\right|$ is a Rayleigh random variable, and $\left|h_{AB}\right|^2$ is a Chi-Square (χ^2) random variable with two degrees of freedom. The probability density function (PDF) of the latter is

$$f_{|h_{AB}|^2}(x)=\frac{1}{2\sigma_{AB}^2}e^{-\frac{x}{2\sigma_{AB}^2}}, \tag{1}$$

and its CDF can be obtained by taking the integral of (1) as follows

$$\begin{aligned} F_{|h_{AB}|^2}(x) &= \int_0^x \frac{1}{2\sigma_{AB}^2}e^{-\frac{t}{2\sigma_{AB}^2}}dx \\ &= 1-e^{-\frac{x}{2\sigma_{AB}^2}}. \end{aligned} \tag{2}$$

Fig. 1 illustrates the flow chart of the incremental AF OR in a centralized manner. The overall cooperative process can be divided into two phases as follows:

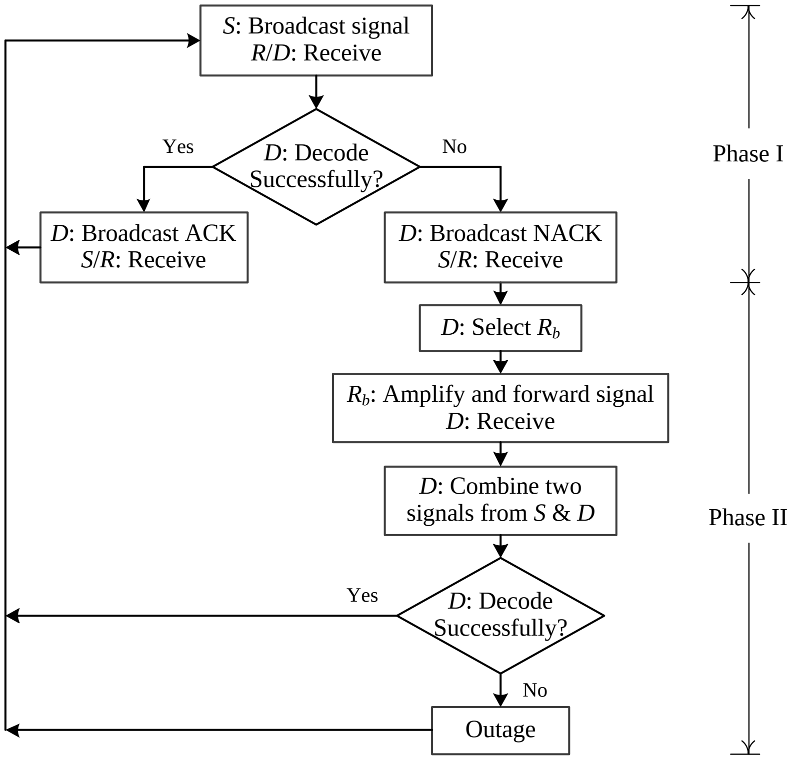

Fig. 1. Flow chart of incremental AF OR in a centralized manner (*R*: relay, R_b: best relay)

Phase I*: *S broadcasts signal, *D* decodes the received signal.

S broadcasts its signal with pilot symbols, denoted by *x*, to each relay and *D*. The signals received at the *i*-th relay (y_{SR_i}) and *D* (y_{SD}) can be written as

$$y_{SR_i} = \tilde{h}_{SR_i} x + n_{SR_i}, \tag{3}$$

and

$$y_{SD} = \tilde{h}_{SD} x + n_{SD}, \tag{4}$$

respectively. Where $\tilde{h}_{SR_i} \sim \mathcal{CN}(0, \sigma^2_{SR_i})$ is the channel gain between *S* and the *i*-th relay, $\tilde{h}_{SD} \sim \mathcal{CN}(0, \sigma^2_{SD})$ is the channel gain between *S* and *D*, the complex additive white Gaussian noise at the *i*-th relay and *D* is denoted by $n_{SR_i} \sim \mathcal{CN}(0, N_0)$ and $n_{SD} \sim \mathcal{CN}(0, N_0)$, respectively. Here, N_0 is the noise variance.

The *i*-th relay uses the received pilot symbol to estimate its $\tilde{h}_{SR_i}$, similarly, *D* uses the received pilot symbol to estimate its $\tilde{h}_{SD}$.

From the above signal transmission process, it can be seen that $\tilde{h}_{SR_i}$ and y_{SR_i} are obtained simultaneously at the *i*-th relay, and this is the same with $\tilde{h}_{SD}$ and y_{SD} at *D*. Therefore, a time delay does not exist between channel estimation and data reception. In other words, neither $\tilde{h}_{SD}$ nor $\tilde{h}_{SR_i}$ is outdated.

If *D* decodes the signal received from *S* successfully, it will broadcast an acknowledgement (ACK) signal to other nodes. After receiving the ACK, each relay remains idle, and *S* will transmit a new signal. There is an occasion that the signal needs to be sent only once, thus, time slots are saved, and the spectral efficiency is increased.

If *D* fails to decode the signal received from *S*, it will broadcast a negative acknowledgement (NACK) signal to other nodes. After receiving the NACK, *S* keeps silence, while the relay selection is activated in phase II.

Phase II**:** *D* selects a best relay to amplify and forward signal.

Upon receiving the NACK from *D*, according to the reciprocity of the $R_i - D$ and $D - R_i$ links, each relay estimates its $\tilde{h}_{R_iD}$. Then, in a centralized manner, each relay sends its $\tilde{h}_{SR_i}$ and $\tilde{h}_{R_iD}$ to *D*, respectively.

After receiving the $\tilde{h}_{SR_i}$ and $\tilde{h}_{R_iD}$ from each relay, *D* executes the OR selection algorithm to select the best relay

$$b = \arg\max_{i\in\mathcal{R}}\left[\min\left(\tilde{\gamma}_{SR_i}, \tilde{\gamma}_{R_iD}\right)\right], \tag{5}$$

where $\mathcal{R} = \{1, 2, \ldots N\}$, $\min\left(\tilde{\gamma}_{SR_i}, \tilde{\gamma}_{R_iD}\right)$ is an upper bound of the instantaneous end-to-end SNR of $S - R_i - D$ link for the *i*-th relay [3], $\tilde{\gamma}_{SR_i} = P_S\left|\tilde{h}_{SR_i}\right|^2 \big/ N_0$, and $\tilde{\gamma}_{R_iD} = P_S\left|\tilde{h}_{R_iD}\right|^2 \big/ N_0$ are the instantaneous SNRs of the $S - R_i$ and $R_i - D$ links at the instant of channel estimation, respectively. Here, P_S is the average power per symbol of *S*.

D then feeds back the index of the best relay in (5) to all potentially available relays.

The selected best relay amplifies and forwards its signal received from *S* to *D*. The signal received at *D* from the best relay is given by

$$y_{R_iD} = G_b h_{R_bD} y_{SR_b} + n_{R_bD}, \tag{6}$$

where $h_{R_bD} \sim \mathcal{CN}(0, \sigma^2_{R_bD})$ is the channel gain between the best relay and *D*, $n_{R_bD} \sim \mathcal{CN}\left(0, N_0\right)$ is the complex additive white Gaussian noise at *D*, and the gain of the best relay denoted by G_b is chosen as $G_b = \sqrt{P_S \Big/ \left(P_S\left|h_{SR_b}\right|^2 + N_0\right)}$.

Subsequently, *D* combines the two signals received from *S* and the best relay. The upper bound for the combined SNR received at *D* can be written as

$$\gamma_{ub} = \tilde{\gamma}_{SD} + \gamma_b = \tilde{\gamma}_{SD} + \max_{i\in R}\left[\min\left(\tilde{\gamma}_{SR_i}, \gamma_{R_iD}\right)\right], \tag{7}$$

where $\tilde{\gamma}_{SD} = P_S\left|\tilde{h}_{SD}\right|^2 \big/ N_0$ is the instantaneous SNR of the $S - D$ link at the instant of channel estimation.

There is still a probability that *D* fails to decode the combined signal, *D* is thus in outage. Further retransmissions would reduce the outage probability or eventually eliminate the outage at cost of an extra time delay. Therefore, we adopt one-transmission protocol in the proposed cooperative mechanism.

3. Performance Analysis

In this section, first, according to the definition of distribution function, we obtain the CDF of the actual combined SNR received at *D*. Then, based on this CDF, we derive the closed-form expressions of the average spectral efficiency and outage probability for the incremental AF OR under outdated CSI.

3.1 The CDF of the Actual Combined SNR Received at *D*

The end-to-end SNR of the best relay, selected by the OR selection algorithm from the outdated SNR values $\tilde{\gamma}_i$ of the relay links, at the instant of channel estimation, is given by $\tilde{\gamma}_b = \max_{i\in\mathcal{R}}\left[\min\left(\tilde{\gamma}_{SR_i}, \tilde{\gamma}_{R_iD}\right)\right]$. Assuming relay links are independent and identically distributed (i. i. d.), we have $\bar{\gamma}_{SR_i} = \bar{\gamma}_1$, $\bar{\gamma}_{R_iD} = \bar{\gamma}_2$, where $\bar{\gamma}_{SR_i} = E\left[\tilde{\gamma}_{SR_i}\right] = 2\frac{P_S}{N_0}\sigma^2_{SR_i}$ and $\bar{\gamma}_{R_iD} = E\left[\tilde{\gamma}_{R_iD}\right] = 2\frac{P_S}{N_0}\sigma^2_{R_iD}$ are the average SNRs of the $S-R_i$ and R_i-D links, respectively. Here, $E[\cdot]$ denotes the expectation operator. The PDF of $\tilde{\gamma}_b$ given in [26] is expressed as

$$f_{\tilde{\gamma}_b}(\tilde{\gamma}) = \frac{1}{\bar{\gamma}_C}\sum_{n=1}^{N}\binom{N}{n} n(-1)^{n-1} e^{\frac{-n\tilde{\gamma}}{\bar{\gamma}_C}}, \tag{8}$$

where $\bar{\gamma}_C = \frac{\bar{\gamma}_1 \cdot \bar{\gamma}_2}{\bar{\gamma}_1 + \bar{\gamma}_2}$.

When the OR selection algorithm is implemented in a real system, there may exist a time delay between channel estimation and data reception. Owing to the time-varying nature of the fading channels, the CSI corresponding to the selected relay is also time-varying. Consequently, the instantaneous CSI used in the relay selection can substantially differ from that at the instant of data reception, i.e., the CSI is outdated. Therefore, $\tilde{\gamma}_b$ at the channel estimation may be different from its actual value γ_b at the data reception. In other words, $\tilde{\gamma}_b$ is an outdated version of γ_b. As a result, the "best" relay selected according to the CSI at the instant of channel estimation, may not be the best relay at the instant of data reception, which may lead to the selection of the wrong relay, and thus deteriorate the system performance.

The power correlation coefficient ρ $(0 \le \rho \le 1)$ between $\tilde{\gamma}_b$ and γ_b is introduced to indicate the discrepancy for the CSI of the $S-R_i-D$ links. Under the assumption of the Jakes' model, the power correlation coefficient is given by [23]

$$\rho = J_0^2\left(2\pi f_D \tau\right), \tag{9}$$

where $J_0(\cdot)$ denotes the zero-order Bessel function of the first kind, $f_D = \frac{v f_c}{c}$ is the maximum Doppler frequency shift (here, v is the moving velocity of the node, f_c is the carrier frequency of the node, and $c = 3.0\times10^8\ m/s$ is the velocity of light in vacuum), and τ is the time delay between channel estimation and data reception. The small ρ denotes the great outdated degree of the CSI.

Let us consider the typical case of a mobile transceiver operating at the frequency of 1.8 GHz, which is assumed to be moving at a speed of 50 km/h. Then, maintaining the power correlation coefficient ρ at a value of 0.90 requires that the time interval τ between consecutive estimations is lesser than 857 μs.

The actual SNR γ_b, conditioned on its estimation $\tilde{\gamma}_b$, follows a non-central chi-square distribution with two degrees of freedom. Its PDF takes the following expression [22]

$$f_{\gamma_b|\tilde{\gamma}_b}\left(\gamma|\tilde{\gamma}\right)=\frac{1}{(1-\rho)\bar{\gamma}_C}e^{-\frac{\rho\tilde{\gamma}+\gamma}{(1-\rho)\bar{\gamma}_C}}I_0\left(\frac{2\sqrt{\rho\gamma\tilde{\gamma}}}{(1-\rho)\bar{\gamma}_C}\right), \tag{10}$$

where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind.

According to the total probability theorem, the PDF of the best relay SNR γ_b can be obtained as

$$f_{\gamma_b}\left(\gamma\right)=\int_0^{\infty} f_{\gamma_b|\tilde{\gamma}_b}\left(\gamma|\tilde{\gamma}\right)f_{\tilde{\gamma}_b}\left(\tilde{\gamma}\right)d\tilde{\gamma}. \tag{11}$$

Substituting (8) and (10) into (11), and using equations (6.614.3), (9.220.2) and (9.215.1) in [27], we derive the following expression for the PDF of γ_b under outdated CSI

$$f_{\gamma_b}\left(\gamma\right)=\sum_{n=1}^{N}\binom{N}{n}\frac{n(-1)^{n-1}}{\bar{\gamma}_C\left[n(1-\rho)+\rho\right]}e^{\frac{-n\gamma}{\bar{\gamma}_C\left[n(1-\rho)+\rho\right]}}. \tag{12}$$

The CDF of γ_b can be obtained by taking the integral of (12) as follows

$$\begin{aligned}F_{\gamma_b}\left(\gamma\right)&=\int_0^{\gamma}\sum_{n=1}^{N}\binom{N}{n}\frac{n(-1)^{n-1}}{\bar{\gamma}_C\left[n(1-\rho)+\rho\right]}e^{\frac{-nt}{\bar{\gamma}_C\left[n(1-\rho)+\rho\right]}}dt\\&=\sum_{n=1}^{N}\binom{N}{n}(-1)^{n-1}\left(1-e^{-\frac{n\gamma}{\bar{\gamma}_C\left[n(1-\rho)+\rho\right]}}\right)\\&=1-\sum_{n=1}^{N}\binom{N}{n}(-1)^{n-1}e^{-\frac{n\gamma}{\bar{\gamma}_C\left[n(1-\rho)+\rho\right]}}.\end{aligned} \tag{13}$$

According to the definition of distribution function, the CDF of the actual combined SNR $\gamma_{ub}=\tilde{\gamma}_{SD}+\gamma_b$ received at D can be expressed as

$$\begin{aligned}F_{\gamma_{ub}}\left(x\right)&=\Pr\left(\gamma_b+\tilde{\gamma}_{SD}<x\right)\\&=\int_0^x\left[\int_0^{x-v}f_{\gamma_b,\tilde{\gamma}_{SD}}(u,v)du\right]dv.\end{aligned} \tag{14}$$

Because $\tilde{\gamma}_{SD}$ and γ_b are independently distributed, (14) can be rewritten as

$$\begin{aligned}F_{\gamma_{ub}}\left(x\right)&=\int_0^x\left[\int_0^{x-v}f_{\gamma_b}(u)f_{\tilde{\gamma}_{SD}}(v)du\right]dv\\&=\int_0^x f_{\tilde{\gamma}_{SD}}(v)\left[\int_0^{x-v}f_{\gamma_b}(u)du\right]dv\\&=\int_0^x f_{\tilde{\gamma}_{SD}}(v)F_{\gamma_b}(x-v)dv.\end{aligned} \tag{15}$$

Substituting $f_{\tilde{\gamma}_{SD}}\left(v\right)=\frac{1}{\bar{\gamma}_{SD}}e^{\frac{v}{\bar{\gamma}_{SD}}}$ and (13) with $\gamma=x-v$ into (15), we have

$$\begin{aligned}F_{\gamma_{ub}}\left(x\right)&=\int_0^x \frac{1}{\overline{\gamma}_{SD}} e^{\frac{v}{\overline{\gamma}_{SD}}}\left(1-\sum_{n=1}^{N}\binom{N}{n}\left(-1\right)^{n-1} e^{-\frac{n(x-v)}{\overline{\gamma}_C\left[n(1-\rho)+\rho\right]}}\right)dv\\&=1-e^{-\frac{x}{\overline{\gamma}_{SD}}}-\sum_{n=1}^{N}\binom{N}{n}\frac{\left(-1\right)^{n-1}\overline{\gamma}_C\left[n\left(1-\rho\right)+\rho\right]}{n\overline{\gamma}_{SD}-\overline{\gamma}_C\left[n\left(1-\rho\right)+\rho\right]}\left(e^{-\frac{x}{\overline{\gamma}_{SD}}}-e^{-\frac{nx}{\overline{\gamma}_C\left[n(1-\rho)+\rho\right]}}\right).\end{aligned} \tag{16}$$

Finally, we can evaluate the CDF of the actual combined SNR received at D by (16).

3.2 Average Spectral Efficiency

In the incremental AF OR scheme, the average spectral efficiency can be calculated as

$$\overline{\eta}=\eta_T\left[d\left(\gamma_{th}\right)+\frac{1}{2}c\left(\gamma_{th}\right)\right], \tag{17}$$

where η_T is the target spectral efficiency (bps/Hz), γ_{th} is the threshold SNR corresponding to η_T, and $\gamma_{th}=2^{\eta_T}-1$. The factor 1/2 is due to the fact that in the incremental AF OR scheme, the relaying cooperative transmission process is performed over two time slots.

In expression (17), $d\left(\gamma_{th}\right)$ is the probability that $\tilde{\gamma}_{SD}\geq\gamma_{th}$, and it can be expressed as

$$\begin{aligned}d\left(\gamma_{th}\right)&=\Pr\left(\tilde{\gamma}_{SD}\geq\gamma_{th}\right)\\&=1-\Pr\left(\tilde{\gamma}_{SD}<\gamma_{th}\right)\\&=1-F_{\tilde{\gamma}_{SD}}\left(\gamma_{th}\right).\end{aligned} \tag{18}$$

$c\left(\gamma_{th}\right)$ is the probability that $\gamma_{ub}=\tilde{\gamma}_{SD}+\gamma_b\geq\gamma_{th}$ on the condition of $\tilde{\gamma}_{SD}<\gamma_{th}$, and it can be calculated as

$$\begin{aligned}c\left(\gamma_{th}\right)&=\Pr\left(\tilde{\gamma}_{SD}+\gamma_b\geq\gamma_{th}\middle|\tilde{\gamma}_{SD}<\gamma_{th}\right)\Pr\left(\tilde{\gamma}_{SD}<\gamma_{th}\right)\\&=\Pr\left(\tilde{\gamma}_{SD}+\gamma_b\geq\gamma_{th},\tilde{\gamma}_{SD}<\gamma_{th}\right)\\&=\Pr\left(\tilde{\gamma}_{SD}<\gamma_{th}\right)-\Pr\left(\tilde{\gamma}_{SD}+\gamma_b<\gamma_{th},\tilde{\gamma}_{SD}<\gamma_{th}\right)\\&=\Pr\left(\tilde{\gamma}_{SD}<\gamma_{th}\right)-\Pr\left(\tilde{\gamma}_{SD}+\gamma_b<\gamma_{th}\right)\\&=\Pr\left(\tilde{\gamma}_{SD}<\gamma_{th}\right)-\Pr\left(\gamma_{ub}<\gamma_{th}\right)\\&=F_{\tilde{\gamma}_{SD}}\left(\gamma_{th}\right)-F_{\gamma_{ub}}\left(\gamma_{th}\right).\end{aligned} \tag{19}$$

Using (18) and (19), (17) can be rewritten as

$$\overline{\eta}=\eta_T\left[1-\frac{1}{2}F_{\tilde{\gamma}_{SD}}\left(\gamma_{th}\right)-\frac{1}{2}F_{\gamma_{ub}}\left(\gamma_{th}\right)\right]. \tag{20}$$

Substituting $F_{\tilde{\gamma}_{SD}}\left(x\right)=1-e^{-\frac{x}{\overline{\gamma}_{SD}}}$ and (16) with $x=\gamma_{th}$ into (20), we obtain

$$\overline{\eta}=\frac{\eta_T}{2}\left[2e^{-\frac{\gamma_{th}}{\overline{\gamma}_{SD}}}+\sum_{n=1}^{N}\binom{N}{n}\frac{\left(-1\right)^{n-1}\overline{\gamma}_C\left[n\left(1-\rho\right)+\rho\right]}{n\overline{\gamma}_{SD}-\overline{\gamma}_C\left[n\left(1-\rho\right)+\rho\right]}\left(e^{-\frac{\gamma_{th}}{\overline{\gamma}_{SD}}}-e^{-\frac{n\gamma_{th}}{\overline{\gamma}_C\left[n(1-\rho)+\rho\right]}}\right)\right]. \tag{21}$$

Then, the average spectral efficiency can be estimated by (21).

3.3 Outage Probability

In the incremental AF OR cooperative scheme, if $\tilde{\gamma}_{SD} < \gamma_{th}$, D will need the help of the best relay to forward another copy of the signal received from S. In this case, there is still a probability that $\gamma_{ub} = \tilde{\gamma}_{SD} + \gamma_b < \gamma_{th}$, D is thus in outage. This outage probability can be derived as

$$\begin{aligned} P_{out} &= \Pr\left(\tilde{\gamma}_{SD} + \gamma_b < \gamma_{th} \middle| \tilde{\gamma}_{SD} < \gamma_{th}\right) \Pr\left(\tilde{\gamma}_{SD} < \gamma_{th}\right) \\ &= \Pr\left(\tilde{\gamma}_{SD} + \gamma_b < \gamma_{th}, \tilde{\gamma}_{SD} < \gamma_{th}\right) \\ &= \Pr\left(\tilde{\gamma}_{SD} + \gamma_b < \gamma_{th}\right) \\ &= \Pr\left(\gamma_{ub} < \gamma_{th}\right) \\ &= F_{\gamma_{ub}}\left(\gamma_{th}\right). \end{aligned} \tag{22}$$

Substituting (16) with $x = \gamma_{th}$ into (22), it can be rewritten as

$$P_{out} = 1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}_{SD}}} - \sum_{n=1}^{N} \binom{N}{n} \frac{(-1)^{n-1} \bar{\gamma}_C \left[n(1-\rho)+\rho\right]}{n\bar{\gamma}_{SD} - \bar{\gamma}_C \left[n(1-\rho)+\rho\right]} \left(e^{-\frac{\gamma_{th}}{\bar{\gamma}_{SD}}} - e^{-\frac{n\gamma_{th}}{\bar{\gamma}_C \left[n(1-\rho)+\rho\right]}} \right). \tag{23}$$

Comparing (23) with (16) in [22], we can see that the proposed incremental AF OR has the same expression as that obtained by the fixed counterpart. Nevertheless, from the signal transmission process of the $S - R_i$ link mentioned above in Section 2, it can be seen that a time delay does not exist between the channel estimation and the data reception. Whereas, according to the cooperative scheme of the fixed counterpart, D initially selects the best relay, based on the CSIs of the $S - R_i - D$ links at the instant of channel estimation, then S broadcasts its signal to the selected best relay. Therefore, there exists a time delay between channel estimation and data reception. Thus, the time delay of the incremental AF OR is less one data transmission duration from S to relay than that of the fixed counterpart. Hence, the outage performance of the incremental AF OR is still superior to that of the fixed counterpart.

4. Numerical Results

In this section, using numerical evaluations of the expressions in (21) and (23), we analyze the impact of outdated CSI on the average spectral efficiency and outage probability of the incremental AF OR over dual-hop Rayleigh fading channels in different cases. For all numerical evaluations, we assume $\bar{\gamma}_C = \bar{\gamma}_{SD}$.

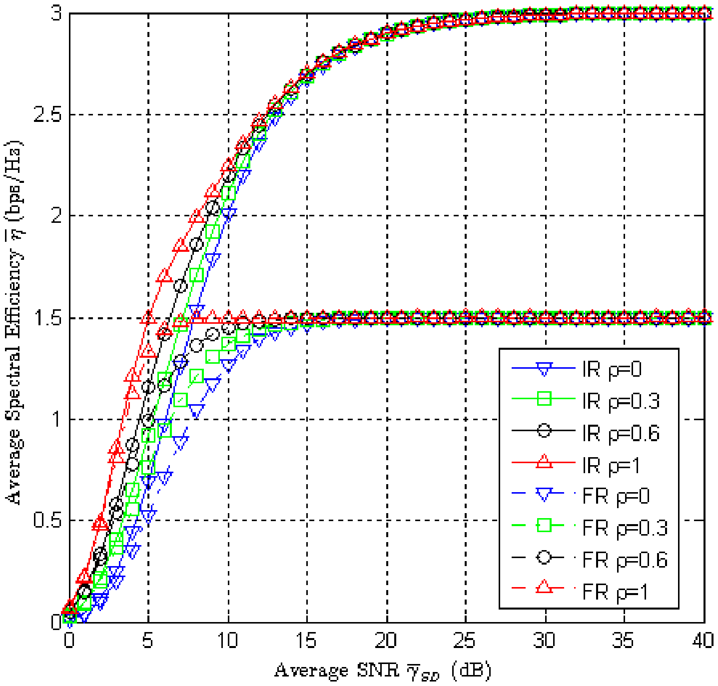

Fig. 2. Average spectral efficiency of incremental and fixed AF OR versus average SNR $\bar{\gamma}_{SD}$ of direct link for different values of power correlation coefficient ρ with number of relays $N=10$ and target spectral efficiency $\eta_T = 3bps/Hz$ (IR: incremental relaying, FR: fixed relaying)

Fig. 2 shows the average spectral efficiency of the incremental and fixed AF OR versus the average SNR $\bar{\gamma}_{SD}$ of the direct $S-D$ link for different values of $\rho \in \{0,\ 0.3,\ 0.6,\ 1\}$ with $N=10$ and $\eta_T = 3\,bps/Hz$. From the curves of the incremental AF OR, it can be observed that in the low SNR region ($\bar{\gamma}_{SD} < 15\,dB$), the average spectral efficiency of these four curves increases along with $\bar{\gamma}_{SD}$. For the same $\bar{\gamma}_{SD}$, as ρ increases, the average spectral efficiency also increases. Since the direct $S-D$ link failing to support the threshold SNR usually occurs in this low SNR region, the best relay is selected to forward data. Therefore, as ρ increases, i.e. outdated degree decreases, the degree of deviation from the best relay is reduced, leading to the increase in the average spectral efficiency. When $\bar{\gamma}_{SD} = 5\,dB$, calculated according to the numerical results, the decrease in the degree of the average spectral efficiency with respect to $\rho = 1$ (perfect CSI) for $\rho = 0, 0.3, 0.6$ (outdated CSI) is 53.9 %, 38.4 % and 22.8 %, respectively. Furthermore, in the high SNR region ($\bar{\gamma}_{SD} > 20\,dB$), it can be seen that the average spectral efficiency is hardly affected by the variation of ρ, and eventually converges to $\eta_T = 3bps/Hz$. Because the quality of the direct link is good in this high SNR region, the additional relay link is rarely used. Hence, the average spectral efficiency is nearly the same for $\rho = 0,\ 0.3,\ 0.6,\ 1$. Comparing the average spectral efficiency of the incremental AF OR with that of the fixed counterpart, we can see that for the same ρ, the greater the average SNR $\bar{\gamma}_{SD}$ is, the more obvious is the spectral efficiency improvement. While the average spectral efficiency of the fixed counterpart eventually converges to $\eta_T = 1.5\,bps/Hz$. The fixed AF OR scheme always demands the best relay to forward the signal, regardless of the direct $S-D$ link conditions, thus requiring two time slots, which results in the reduction by half in the spectral efficiency.

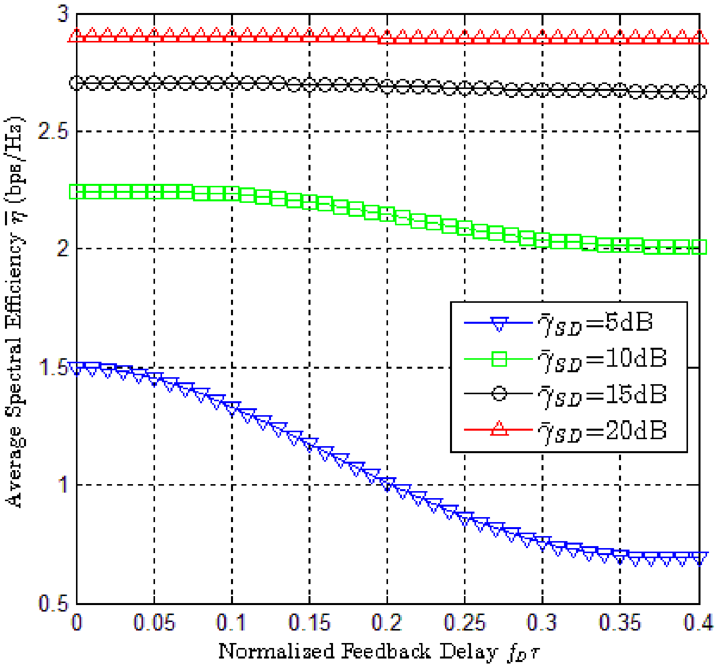

Fig. 3. Average spectral efficiency versus normalized feedback delay $f_D\tau$ for different average SNR $\overline{\gamma}_{SD}$ with $N = 10$ and $\eta_T = 3bps/Hz$

Fig. 3 shows the impact of the normalized feedback delay $f_D\tau$ on the average spectral efficiency, for different $\overline{\gamma}_{SD} \in \{5\,dB, 10\,dB, 15\,dB, 20\,dB\}$ with $N = 10$ and $\eta_T = 3bps/Hz$. It can be observed that as $f_D\tau$ increases, the average spectral efficiency decreases. This is because the outdated degree increases along with $f_D\tau$ according to (9). Furthermore, the smaller the average SNR $\overline{\gamma}_{SD}$ is, the more sensitive is the average spectral efficiency to the variation of $f_D\tau$. When $\overline{\gamma}_{SD}$ reaches up to 20dB, $f_D\tau$ has almost no effect on the average spectral efficiency. This is because the additional relay link is rarely used, due to the good quality of the direct link, in this high SNR region.

Fig. 4 shows the average spectral efficiency versus the threshold SNR γ_{th}, for different $\rho \in \{0,\ 0.3,\ 0.6,\ 1\}$ with $N = 10$ and $\overline{\gamma}_{SD} = 30\,dB$. It can be shown that as $\gamma_{th} < 20\,dB$, the variation of ρ has almost no effect on the average spectral efficiency. This is because the direct link is reliable in this case. Whereas, for $20\,dB < \gamma_{th} < 40\,dB$, as ρ increases, the average spectral efficiency increases. In this SNR region, the direct link fails to support the SNR threshold, and then the best relay is selected to forward data. Therefore, the variation of ρ may have an effect on the average spectral efficiency. Furthermore, when $\gamma_{th} > 40\,dB$, the average spectral efficiency eventually converges to zero, this is because both the direct and combined links are in outage.

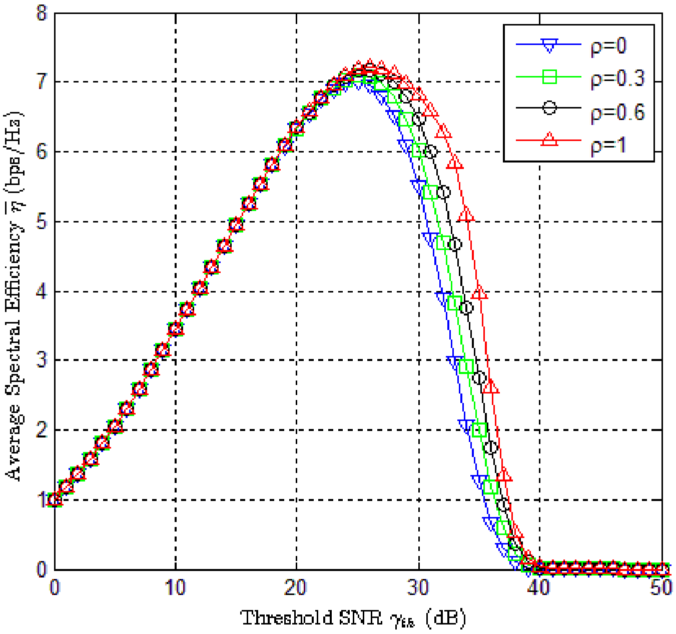

Fig. 4. Average spectral efficiency versus threshold SNR γ_{th} for different ρ with $N = 10$ and $\overline{\gamma}_{SD} = 30\ dB$

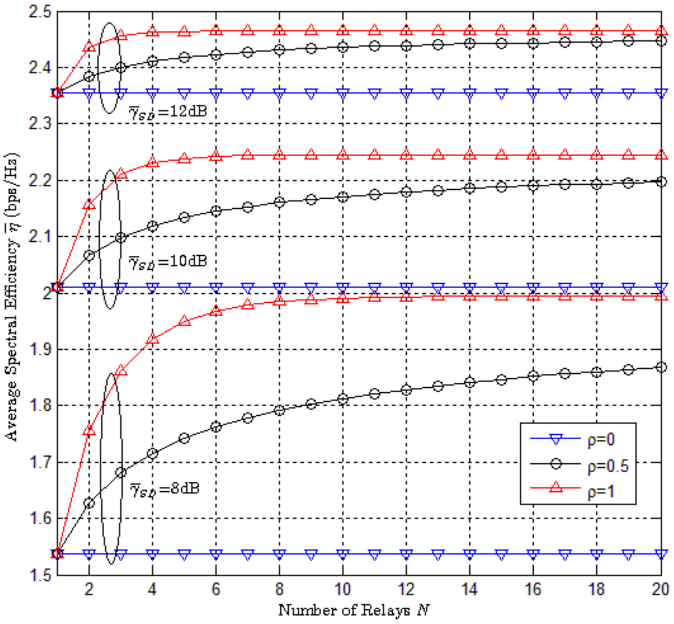

Fig. 5. Average spectral efficiency versus number of relays N for different ρ and $\overline{\gamma}_{SD}$ with $\eta_T = 3 bps/Hz$

Fig. 5 shows the average spectral efficiency versus the number of relays N for different $\rho \in \{0,\ 0.5,\ 1\}$ and $\overline{\gamma}_{SD} \in \{8\,dB, 10\,dB, 12\,dB\}$ with $\eta_T = 3bps/Hz$. It is shown that for $\rho = 0.5$, as the average SNR $\overline{\gamma}_{SD}$ increases, the average spectral efficiency improvement decreases along with N. As N increases from 1 to 20, calculated according to the numerical results for $\rho = 0.5$, the spectral efficiency improvement for $\overline{\gamma}_{SD} = 8, 10, 12\,dB$ is 21.4 %, 9.2 % and 3.9 %, respectively. Furthermore, it can be seen that for $\rho = 0.5, 1$, the average spectral efficiency increases along with N. This effect is motivated by the fact that the reliability of the $S - R_i - D$ link improves along with N, as there exist more candidates for the selected relay. Whereas, when $\rho = 1$, and $N \geq 10$, increasing the number of relays N has almost no effect on the average spectral efficiency. Finally, for $\rho = 0$, when N varies from 1 to 20, the average spectral efficiency remains the same as that of the system employing one relay ($N = 1$) and the direct $S - D$ link.

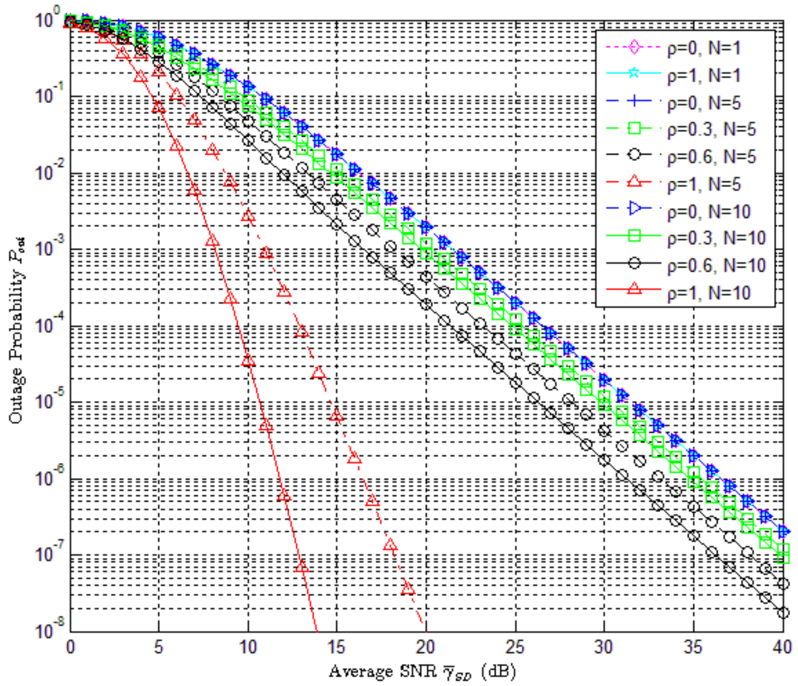

Fig. 6. Outage probability versus average SNR $\overline{\gamma}_{SD}$ of direct link for different values of power correlation coefficient ρ and number of relays N with $\gamma_{th} = 8\,dB$

Fig. 6 shows the outage probability (P_{out}) versus the average SNR $\overline{\gamma}_{SD}$ for different $\rho \in \{0,\ 0.3,\ 0.6,\ 1\}$ and $N \in \{5, 10\}$ with $\gamma_{th} = 8\,dB$. It can be seen that as ρ decreases, P_{out} increases. For $\rho \in \{0.3,\ 0.6\}$ (outdated CSI), a slight improvement in the outage performance can be obtained with the increase in the number of relay nodes N. Whereas for $\rho = 0$, there is no improvement of the outage performance as N increases, and P_{out} is equal to that of the system employing one relay ($N = 1$) and the direct $S - D$ link. From Fig.6, it can be seen that the four curves corresponding to $\rho = 0, N = 1$, $\rho = 0, N = 5$, $\rho = 0, N = 10$, and $\rho = 1, N = 1$

are overlapped together. Furthermore, for $\rho \in \{0.3, 0.6\}$, in the low SNR region, as N increases, the slope of P_{out} decreases, while it is still larger than that of $N=1$. Nevertheless, in the high SNR region, the slope eventually converges to the same value as for that of $N=1$. This indicates that the incorrect relay selection, because of the outdated CSI, reduces the diversity order of the incremental AF OR system to 2, i.e. the diversity order of the system with one relay and the direct $S-D$ link. As expected, for $\rho=1$ (perfect CSI), the diversity order is $(N+1)$ (the maximum achievable diversity order for the AF OR system). From Corollary 1 and the relevant proofs given by references [11] and [16], it can be shown that if $\rho<1$ (outdated CSI), the diversity order is 2, else if $\rho=1$ (perfect CSI), the diversity order is $(N+1)$ (the diversity order of the system with N relays and the direct $S-D$ link).

5. Conclusion

In this paper, we analyze the impact of outdated CSI on the average spectral efficiency and outage probability of the incremental AF OR over dual-hop i.i.d. Rayleigh fading channels. From the process of S broadcasting its signal to each relay and D, it is evident that there does not exist a time delay between the channel estimation and the data reception. In other words, neither the CSI of the first-hop relay link $S-R_i$, nor that of the direct $S-D$ link is outdated. Therefore, the time delay of the incremental AF OR is less the data transmission duration from S to relay than that of the fixed counterpart. When the average SNR $\bar{\gamma}_{SD}$ of the direct link is in the low region, the direct link cannot transmit data reliably. In this case, the relay is required to cooperatively forward data to D, and thus the system performance will be affected by outdated CSI. Numerical results show that the smaller the $\bar{\gamma}_{SD}$ is, the more sensitive is the spectral efficiency to outdated CSI, and the more obvious is the spectral efficiency improvement with the increase of the number of relays. Furthermore, when $\bar{\gamma}_{SD}$ is in the low region, the performance of the incremental AF OR is heavily dependent upon the level of correlation between the actual channel conditions and their corresponding outdated estimations. This result has an impact on the design of pilot symbols in practical applications, implying that the smaller pilot symbol spacing may be required to control the degree of outdated CSI. Moreover, when $\bar{\gamma}_{SD}$ is in the high region, the direct link is reliable, and the additional relay link is rarely used. Hence, the system performance is hardly influenced by outdated CSI.

This paper studies the single-source, single-destination, and multi-channel cooperative relaying under outdated CSI. In the future, we will consider the impact of outdated CSI on the performance of the multi-source, multi-destination, and multi-subchannel cooperative communications [28]. It is an interesting research topic since we will face some new challenging issues that are needed to be resolved.

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