SINR Pricing in Non Cooperative Power Control Game for Wireless Ad Hoc Networks

Sanjay Kumar Suman[1], Dhananjay Kumar [2] and L. Bhagyalakshmi [3]
[123]Dept. of Information Technology, Anna University, MIT campus
Chennai 600044 - INDIA
[e-mail: [1]suman.sanjaykumar@gmail.com, [2]dhananjay@annauniv.edu
[3] Dept. Of RCC, CEG, Anna University
Chennai 600025 - INDIA
[e-mail: eec.ecetimes@gmail.com]
*Corresponding author: Sanjay Kumar Suman

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Abstract

In wireless ad hoc networks the nodes focus on achieving the maximum SINR for efficient data transmission. In order to achieve maximum SINR the nodes culminate in exhausting the battery power for successful transmissions. This in turn affects the successful transmission of the other nodes as the maximum transmission power opted by each node serves as a source of interference for the other nodes in the network. This paper models the choice of power for each node as a non cooperative game where the throughput of the network with respect to the consumption of power is formulated as a utility function. We propose an adaptive pricing scheme that encourages the nodes to use minimum transmission power to achieve target SINR at the Nash equilibrium and improve their net utility in multiuser scenario.

Keywords: Adaptive Pricing, Nash equilibrium, Non cooperative power control game, SINR, Transmission Power, Wireless ad hoc networks.

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1. Introduction

The most widespread notion of a wireless ad hoc network is a network formed dynamically by autonomous systems of heterogeneous wireless devices interfaced through wireless links without an existing network infrastructure and without any central coordinator. The decentralized nature and quick deployment properties of these networks make them suitable for emergency and military application as well as any other communication related application. In future, it is possible to operate these networks in the unlicensed band without strict rules and regulation. The increasing demand for CDMA based wireless ad hoc networks has generated a number of issues. One such important issue is transmission power control that not only saves the battery power but also increases the system capacity by influencing the proper utilization of radio frequency. Since radio frequency is a scarce resource, it is advised to reuse the same frequency band at some distance to increase the system capacity. In order to reduce the reuse distance, it is needed to reduce the interference which, in turn, requires implementing transmission power control in a system [1].

The wireless devices, called nodes, take part in these networks are battery power operated and having an intention of using maximum transmission power for gaining higher signal to interference plus noise ratio (SINR) for better services. This tendency of a node makes its neighboring nodes to face higher interference. As a result, neighboring nodes also raise their transmission power to combat the posed interferences. All nodes, thereby, drain their battery power soon. There may be a solution to reduce the power level, but on the cost of compromising QoS [2]. Therefore, there is a need of minimizing interference by optimizing transmission power to increase channel capacity as well as to extend battery life. We found an optimum SINR in game theoretical framework and investigate how power management reduce the interference and increase energy efficiency, in turn, QoS in wireless ad hoc networks.

Many different approaches have been suggested on the power control issue in wireless networks, in general, and wireless ad hoc networks, in particular [3]. Recently an alternative approach in game theoretic framework drags the attention of researchers to study and analyze the power control problem in wireless ad hoc and sensor networks. Game theory is an art of decision making tool that helps several players, each having its own individual objectives, to increase their profit in a competitive situation. In some cases, like a wireless network, there may be a common goal, such as network performance, that has to be maximized by all players. In this case, some players may compromise with their personal objective and contend with each other under a set of rules. In a non cooperative situation the game finds an equilibrium solution at which each player is at a local maximum and it cannot do better by a unilateral deviation [4].

In this approach, the network operation is considered as a non cooperative game where the nodes of the networks are players who play the game to maximize their profit by applying some action plan in response to other nodes' actions. In ad hoc networks scenario, the action plan could be the utilization of system resource, such as increasing transmission power, to get higher SINR or higher throughputs as their utility function. Since all the nodes want to maximize their own utility, the total utility of networks is typically difficult to achieve due to the rational behavior of the nodes. The purpose of applying game theory is to maximize total networks' utility rather than individual node's utility. To mitigate this problem, therefore, the game enforces price that causes the marginal loss in utility due to a marginal increase in interference. Now nodes change their behavior by knowing their profit and loss thus

eventually the game reaches an equilibrium solution referred to as Nash Equilibrium, where all players have to use optimum transmission power to obtain networks' goal.

Energy efficiency issue is well addressed in cellular networks and various power control algorithms are developed for cellular networks. It is of its first kind, an embodied energy is used in modeling the energy efficient cellular networks in [5]. The fast growth of data communication in cellular networks leads to a huge operating cost which can be reduced by designing energy efficient cellular networks. In [6] distribution of traffic load and power utilization is considered to model a Poisson-Voronoi tessellation (PVT) cellular networks to reduce the load of working outlay. The effect of interference on spectrum and energy efficiency is a critical issue in all form of wireless networks such as cellular networks and femtocell networks [7]. A Markov chain is used to determine the channel access in two tier femtocell networks and energy efficiency is analysed [8]. Wireless body area network is a new wireless network with constrained energy. A new energy efficient relay mac protocol is proposed in [9] to save energy using dynamic power control mac methodology.

A Substantial amount of research on the power control issue in wireless networks has been done using game theoretic approach. Chao Liang and Kapil R. Dandekar [10] considered the power control problem in stationary MIMO ad hoc networks as a game theoretic problem and they devised a link shut down mechanism, if an inefficient link consuming more power. The link shut-down mechanism obviously increases the total networks' performance, but it is a contentious approach as many people don't want to get disconnected even if they have extremely low data rates. Joint beam forming and power allocation problem in MIMO cognetive radio is studied in [11] using game theory to avoide the excessive interference for primerary user.

The wireless sensor networks, a part of an ad hoc network, also consists of battery operated powered device, are used for sensing the environmental changes, data processing and communicating wirelessly [12]. The data processing in sensor networks consumes more power, therefore, transmission power control is equally important in wireless sensor networks too. To solve the distributed power control problem in wireless sensor networks, Shamik Sengupta, et al. framed a utility function as a non cooperative game and suggested 'not to transmit' if the channel condition is not suitable for transmission [13]. They advised to fix a threshold power level with price mechanism to punish the nodes that transmit above threshold limit of power level. Chengnian Long et al. applied SFP theory, instead of pricing mechanism, in non cooperative power control algorithm to gain requisite quality of services by scheduling and controlling the power level of users in ad hoc network [14]. The authors recommended a self incentive scheme which makes the user decide based on its own belief without falling in a selfish behavior. Nash Equilibrium is an important criterion to get the optimum solution in a game. But sometimes it is possible to increase users' payoff beyond Nash Equilibrium. In this case the Pareto optimal theory is being used to measure the efficiency of an outcome which is defined as "An outcome is a Pareto optimal if and only if, there exists, no other outcome that makes every player at least as well off while making at least one player better off". Using the Pareto optimality theory, Mehdi Rasti et al. provided a trade-off solution between fairness and aggregate throughput using a SIR based linear pricing scheme in wireless networks [15]. Recently, Mehdi Monemi et al. proposed a Pareto efficient distributed multiple target SINRs tracking power control algorithms. According to this algorithm, if a user cannot achieve target SINR, it selects some lower target SINR to continue transmission instead of get disconnected [16]. Game theory has been successfully applied to solve power control, rate control and spectral efficiency issues in cellular networks. Madhusudhan et al. used non cooperative game to provide pliable transmission rate by utilizing radio resource efficiently using the power and

rate control in uplink single cell CDMA system [17]. Eirini et al. further accosted a convex pricing to balance the throughput and power consumption to maximize utility of each user in wireless networks [18]. The power control problem is again investigated by Yuan hang Xiao et al. using the concept of intervention device to monitor and accordingly, adjust the power level of selfish users in single hop ad hoc networks to get the desired result [19].

There exist several literatures on power control issues in wireless ad hoc networks. Most of the proposed works are framed in non cooperative strategic form game with incomplete information. This work also comes under the non cooperative strategic form game with pricing since the characteristics of ad hoc networks are very much similar to a non cooperative game. But the pricing scheme proposed in this work is different from many previously proposed pricing schemes. Maximum proposed pricing schemes are linear pricing scheme which charges a common price to all the nodes irrespective of the channel condition and their distances from receivers whereas this adaptive pricing scheme can be tuned according to transmission feasibility.

The rest of the paper is organized as follows. The next section provides the system model that comprises network topology and SINR model. The utility function is developed in section 3 using non cooperative power control game and validated analytically by proving the existence of Nash equilibrium in section 4. A new proposed pricing scheme is introduced in section 5 to obtain the net utility. Finally, the proposed NCPCG is simulated and evaluated in section 6 followed by the conclusion in section 7.

2. System Model

For easier reference the remaining works begin with the major notations, used in this paper, are listed in **Table 1.**

Table 1. Major Notation

Notation	Description	Notation	Description
$\mathcal{N}$	Set of all nodes	$\mathbf{p}_{-i}$	Set of power level selected by nodes other than i
N	Number of nodes		
$h_{i,j}$	Path gain	u_i	Utility function of node i
$d_{i,j}$	Distance between two nodes	q	Packet success probability
$\propto$	Path loss exponent	f	Efficiency function
P_i	Transmit power level of node	λ_i	Price function
p_i	Power level of node i; $p_i \in P_i$	L	Number of information
$\mathbf{p}$	Power profile of nodes	M	Size of packet
$\gamma_{i,j}$	SINR	W	Bandwidth in hertz
η_j	Back ground noise at receiver j	R	Transmission rate in bits/second
σ^2	Additive white Gaussian noise	G	Non cooperative power control game

A single hop wireless ad hoc network is considered, shown in **Fig. 1 (a)**, with several nodes located in a neighborhood of a node. The SINR model shown in **Fig. 1 (b)** reveals that the transmission from T1 interferes R2 and R3.

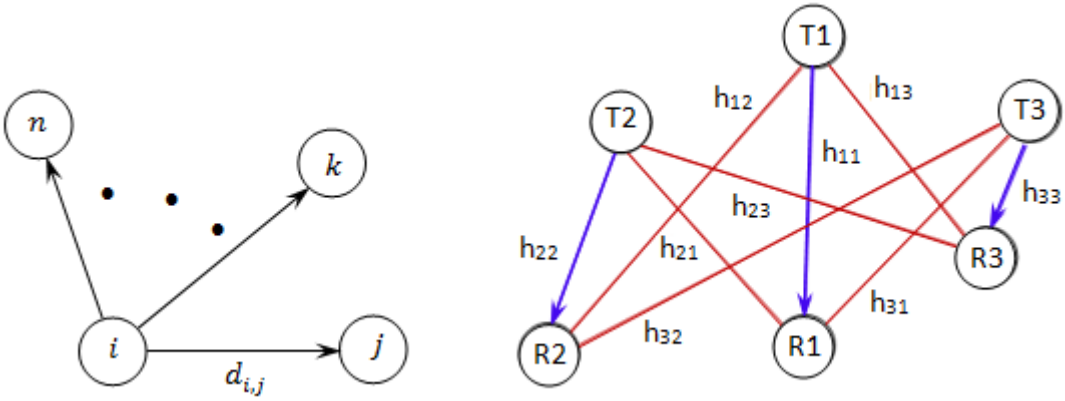

Fig. 1. Network Topology: (a) A node is surrounded by $N-1$ nodes, (b) SINR model where transmitter 1 interfere receiver 2 and receiver 3.

The framework presented here is based on SINR calculation. To obtain SINR, many sets of power vectors $P_1, P_2, \ldots, P_N$ for nodes $1, 2, \ldots, N$ are assumed where each set consists of power levels ranging from p_{min} to p_{max}. Let the node 1 selects power level p_1 from set P_1, node 2 selects power level p_2 from set P_2, and so on, then the power profile can be define as $\mathbf{p} = \{p_1, p_2, \ldots, p_N\} \in \mathcal{P}$, where $\mathcal{P} \triangleq \prod_{i=1}^{N} P_i$ known as space of power profile and P_i is the set of power level of node i. If all the nodes are considered to be identical, then the power profile of all nodes will also be identical i.e., $\mathcal{P} = \prod_{N\ times} P$. Suppose that nodes transmit packet at the rate of R bits/second over a bandwidth of W Hz, then the signal to interference plus noise ratio of link (i, j) is defined as:

$$\gamma_{i,\mathrm{j}} = \frac{W}{R}\left(\frac{h_{\mathrm{i,j}}\, p_{\mathrm{i}}}{I_{\mathrm{i,j}} + \eta_{\mathrm{j}}}\right) \tag{1}$$

Where

$$I_{i,\mathrm{j}} = \sum_{k=1, k\neq i, k\neq j}^{N} h_{\mathrm{k,j}}\, p_{\mathrm{k}}$$

and η_{j} is the background noise at the receiver. Assuming the background noise is additive white Gaussian noise (AWGN) then $\eta_{\mathrm{j}} = \sigma^2$. Thus, the SINR at receiver is given by:

$$\gamma_{i,j} = \frac{W}{R}\frac{h_{\mathrm{i,j}}\, p_{\mathrm{i}}}{\sum_{\mathrm{k=1,k\neq i,k\neq j}}^{\mathrm{N}} h_{\mathrm{k,j}}\, p_{\mathrm{k}} + \sigma^2}$$

In dB, SINR can be obtained as:

$$\gamma_{i,j} = 10\log_{10}\left(\frac{W}{R}\frac{h_{\mathrm{i,j}}\, p_{\mathrm{i}}}{\sum_{\mathrm{k=1,k\neq i,k\neq j}}^{\mathrm{N}} h_{\mathrm{k,j}}\, p_{\mathrm{k}} + \sigma^2}\right)\mathrm{dB} \tag{2}$$

The term $h_{\mathrm{i,j}}$ is the path gain between the transmitter (say node i) and the receiver (say node j) and can be calculated using simple path loss model $h_{\mathrm{i,j}} = \mathrm{K}/\mathrm{d}_{\mathrm{i,j}}^{\propto}$ where K is a constant and $\propto$

is the path loss exponent [20]. The distance, $d_{i,j}$, between node i and node j can be found as:

$$d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \tag{3}$$

Where x and y indicate coordinate positions of the nodes. The numerator term in equation (2) indicates the received power at the jth node, whereas denominator is the total interferences caused by the other nodes' received power at the jth receiver plus noise power at the receiver.

4. Utility Function

Utility function in the proposed model is adopted from the data transmission model of D.J Goodman and N.B. Mandayam [21]. Data are transmitted in packets, each containing L bits of information plus overhead bits, at the rate of R bits/sec. A strong error detection code is assumed to be used for detecting errors and retransmitting packets, if error found, until the receiver receives error free data. Including error detecting code, the size of packet becomes M bits, where $M > L$ and $(M - L)$ bits are used for error detection code. The number of retransmission required to receive a packet correctly is a function of geometric random variable X. Let all retransmissions are statistically independent then the mean value of X is equal to $E[X] = 1/q(\gamma_{i,j})$ where $\gamma_{i,j}$ is the SINR at node j (explain in previous section) and $q(\gamma_{i,j})$ is the probability of correct reception of data transmitted by node i. If the duration of each transmission is calculated as M/R seconds then the expected total transmission time for correct reception is $E[X]\,M/R$ seconds. With transmitted power p_i watts the expected energy consumed during transmission is $E[X]\,p_i M/R$ joules. But the actual data transmitted is only L bits; therefore, the utility function is modeled as:

$$u_i(p_i, \mathbf{p}_{-i}) = \frac{L}{E[X]\,p_i M/R}\ bits/joule$$

$$u_i = \frac{LR}{Mp_i}\, q(\gamma_{i,\mathrm{j}})\ \frac{bits}{joule} \tag{4}$$

Where

$$q(\gamma_{i,\mathrm{j}}) = (1 - P_e)^M \tag{5}$$

and P_e is the bit error rate that depends on many factors such as channel condition, interference from other nodes and various modulation schemes.

Modulation schemes used in wireless networks are either coherent or non-coherent. Coherent techniques, such as, FSK, BPSK, QPSK and MSK, exhibit excellent performance, but they require prior knowledge of the phase information of the carrier signal which is impractical in a wireless environment. On the other hand, the performance of non-coherent technique, DPSK and FSK, are almost equal to coherent PSK and coherent FSK [22]. Moreover, they are robust with respect to amplitude variation hence they are widely accepted schemes in practices. Both the modulation schemes (DPSK and NCFSK) are suitable for the proposed utility model, albeit, NCFSK is more suitable for the pricing scheme proposed in this paper.

Hence, by selecting non-coherent FSK modulation scheme and substituting its BER $P_e = 0.5e^{-\gamma_{i,j}/2}$ in equation (5), the packet success probability is expressed as:

$$q(\gamma_{i,j}) = (1 - 0.5\, e^{-\gamma_{i,j}/2})^M \tag{6}$$

The utility function defined in equation (4) has a mathematical anomaly. If $P_i = 0$, the utility becomes infinity. To avoid this problem, $q(\gamma_{i,j})$ can be modified in such a way that it could give a close approximation of the packet success rate given in equation (6) and also it should yield $f(0) = 0$ and $f(\infty) = 1$. Thus, the efficiency function is now defined as:

$$f(\gamma_{i,j}) = (1 - e^{-\gamma_{i,j}/2})^M \tag{7}$$

This function gives the desirable properties, e.g., for $\gamma_{i,j} = 0$; $f(0) = 0$ and for $\gamma_{i,j} = \perp\!\!\!\perp$; $f(\perp\!\!\!\perp) = 1$. Also $f(\gamma_{i,j})/P_i = 0$ for $P_i = 0$. The close behavior of PSR and efficiency function is depicted in **Fig. 2**. Finally the modified utility function is formulated as [23]:

$$u_i = \frac{LR}{Mp_i} f(\gamma_{i,j}) \frac{bits}{joule} \tag{8}$$

Equation (8) represents the total number of information bits that are transmitted to the receiver without an error per joule of energy consumed and well balances between throughput and battery life. The similar shapes of $q(\gamma_{i,j})$ and $f(\gamma_{i,j})$ ensures that a power profile that maximizes utility given in equation (8) will be close to the power profile that maximizes the utility given in equation (4). Also, it is observed that the utility function obtained in equation (8) by each node is a function of its own power level plus the power level selected by other nodes.

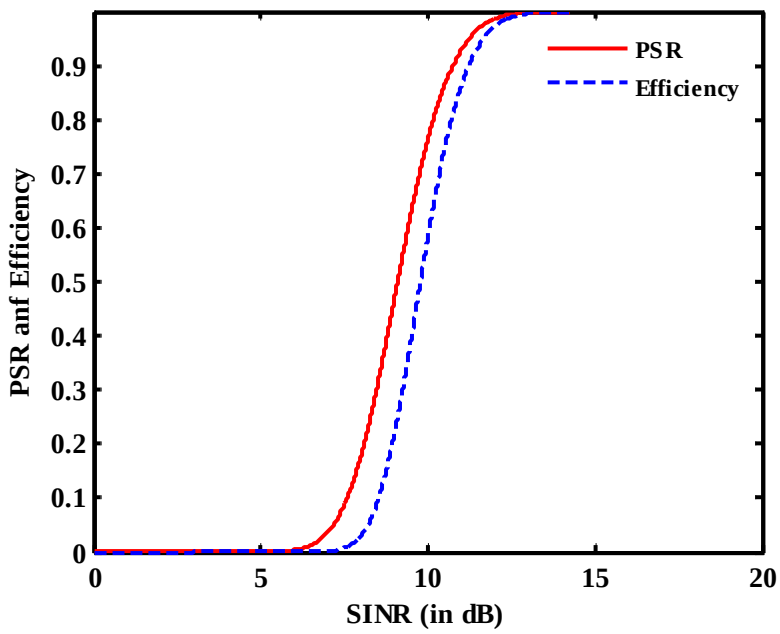

Fig. 2. Close approximation of packet success rate and efficiency function with respect to SINR.

5. Existence of Nash Equilibrium

Non-cooperative Game tries to predict stable outcomes of a competition that is defined by Nash equilibrium. It is defined based on the best action plan chosen by a player to maximize its utility for a given action profile of other players [24][25]. Specifically, in wireless ad hoc network, $\bar{p}$ is the best response by node i to $\mathbf{p_{-i}}$ if

$$\bar{p} = \{\text{argmax}\, u_i\,(p_i^*, \mathbf{p}_{-\mathbf{i}}^*)\} \tag{9}$$

Definition: A power profile $p^* = (p_1^*, p_2^*, p_3^*, \ldots, p_N^*)$ is a Nash Equilibrium for non cooperative power control game $G = \langle \mathcal{N}, \{P_i\}, \{u_i\}\rangle$ if, for every $i \in \mathcal{N}$.

$$u_i\,(p_i^*, \mathbf{p}_{-\mathbf{i}}^*) \geq \; u_i\,(p_i, \mathbf{p}_{-\mathbf{i}}^*) \;\; \text{for } \forall\, p_i \in P_i \tag{10}$$

At Nash equilibrium, for the given power levels of other nodes, no node can improve its utility by changing the power level of its own. However, such a point is not guaranteed to exist in a game. The sufficient conditions at which a unique Nash equilibrium is guaranteed to exist are delineated in following theorem.

Theorem: A Nash equilibrium exists in game $G = \langle \mathcal{N}, \{P_i\}, \{u_i\}\rangle$ for $\forall i \in \mathcal{N}$ if:

(i) The power profile P_i is a nonempty, convex and compact subset of some Euclidean space $\mathcal{R}^n$.

(ii) $u_i(\mathbf{P})$ is continuous in $\mathbf{P}$ and quasi-concave in p_i.

Proof: To satisfy the above conditions it is required to calculate the power that maximizes the utility of node i by equating the first order derivative of u_i to zero [25]. The first order derivative of u_i from equation (8) is as follows:

$$\frac{\partial u_i}{\partial p_i} = \frac{LR}{M}\left[\frac{p_i\,\dfrac{\partial f(\gamma_{i,j})}{\partial p_i} - f(\gamma_{i,j})}{p_i^2}\right]$$

$$= \frac{LR}{M}\left[\frac{p_i\,\dfrac{df(\gamma_{i,j})}{d\gamma_{i,j}}\dfrac{\partial \gamma_{i,j}}{\partial p_i} - f(\gamma_{i,j})}{p_i^2}\right]$$

$$= \frac{LR}{M}\left[\frac{p_i\, f'(\gamma_{i,j})\dfrac{\partial \gamma_{i,j}}{\partial p_i} - f(\gamma_{i,j})}{p_i^2}\right]$$

$$= \frac{LR}{M}\left[\frac{p_i\, f'(\gamma_{i,j})\dfrac{W}{R}\dfrac{h_{\mathrm{i,j}}}{\sum_{\mathrm{k=1,k\neq i,k\neq j}}^{\mathrm{N}} h_{\mathrm{k,j}}\, p_{\mathrm{k}} + \sigma^2} - f(\gamma_{i,j})}{p_i^2}\right]$$

$$\frac{\partial u_i}{\partial p_i} = \frac{LR}{Mp_i^2}\left(f'(\gamma_{i,j})\gamma_{i,j} - f(\gamma_{i,j})\right) \tag{11}$$

Where

$$f'(\gamma_{i,j}) = \frac{M}{2}\left(1 - e^{-\gamma_{i,j}/2}\right)^{M-1}\left(e^{-\gamma_{i,j}/2}\right) \tag{12}$$

Equation (11) yields $(\partial u_i)/(\partial p_i) = 0$ for $p_i = 0$. Therefore, $p_i = 0$ is a stationary point at which the utility becomes zero. With some small positive value of p_i, say ε, the utility becomes positive, that means, the utility is increasing at $p_i = 0$. But zero cannot be a local maximum. To find a local maximum for a non-zero value of power, further study is required to examine the value of SINR. Equating second part of the equation (11) to zero:

$$f'(\gamma_{i,j})\gamma_{i,j} - f(\gamma_{i,j}) = 0 \tag{13}$$

and substituting (12) and (7) in (13) it gives:

$$\frac{M}{2}\gamma_{i,j}\left(1 - e^{-\gamma_{i,j}/2}\right)^{M-1}\left(e^{-\gamma_{i,j}/2}\right) - \left(1 - e^{-\gamma_{i,j}/2}\right)^{M} = 0 \tag{14}$$

$$\frac{M}{2}\gamma_{i,j}\frac{\left(1 - e^{-\gamma_{i,j}/2}\right)^{M}}{\left(1 - e^{-\gamma_{i,j}/2}\right)}\left(e^{-\gamma_{i,j}/2}\right) - \left(1 - e^{-\gamma_{i,j}/2}\right)^{M} = 0$$

$$\frac{M}{2}\gamma_{i,j}\frac{f(\gamma_{i,j})\left(e^{-\gamma_{i,j}/2}\right)}{\left(1 - e^{-\gamma_{i,j}/2}\right)} - f(\gamma_{i,j}) = 0$$

$$\frac{M}{2}\gamma_{i,j}\frac{\left(e^{-\gamma_{i,j}/2}\right)}{\left(1 - e^{-\gamma_{i,j}/2}\right)} - 1 = 0$$

$$\frac{M}{2}\gamma_{i,j}e^{-\gamma_{i,j}/2} - \left(1 - e^{-\gamma_{i,j}/2}\right) = 0$$

$$\frac{M}{2}\gamma_{i,j} + 1 = e^{\gamma_{i,j}/2} \tag{15}$$

The LHS of the equation (15) is monotonically increasing in $\gamma_{i,j}$, and the RHS is convex in $\gamma_{i,j}$. Moreover, it is satisfied at $\gamma_{i,j} = 0$. That leads to the existence of a single value γ_T which satisfies the equation (15) for $\gamma_{i,j} > 0$, where γ_T (12.4205 = 10.94 dB for $M = 80$) is numerically derived from the above equation. Substituting γ_T in equation (11) and taking its second order partial derivative, it reveals that $\partial^2 u_i/\partial p_i^2 < 0$ and this point is a local maximum and therefore the function is global maximum. Hence, the utility is quasi-concave in p_i for $\forall i$ and P_i is nonempty. This completes the proof of existence of a unique Nash equilibrium [25].

6. Net Utility

In the absence of any punishment, the selfish nodes may try to maximize their throughput by consuming high power and cause interferences to other nodes. Thereby, it may result a cascading effect where all nodes lose their energy. In order to restrict a node using a high power level to achieve maximum throughput, the proposed algorithm NCPCG inflicts a pricing function defined as:

$$\lambda_i(p_i) = \beta\left(1 - e^{-\gamma_{i,j}/\tau}\right) \tag{16}$$

where the parameters β and τ are used as control parameters in order that SINR does not deviate from threshold and maintained greater than or equal to 1. Now the net utility of node i can be redefined as:

$$u_i^{net} = \begin{cases} u_i - \lambda_i(p_i), & \text{if transmitting,} \\ 0, & \text{if not transmitting.} \end{cases} \tag{17}$$

With this pricing scheme, the node that produces more interference to other nodes gets punished in terms of poor SINR and thereby forced to use less transmission power.

Based on the discussion presented in section 2 through section 6, algorithm is listed in **Table 2**.

Table 2. Algorithm

Step 1.	Setup network with N nodes
Step 2.	Initialize: $n = 0$
	Power Profile $p_i \in P_i$; where $i = 1, 2, \ldots, N$
	Power level p_{min} and p_{max}
Step 3.	Estimate path gain between transmitting and receiving nodes
	using $h_{i,j} = K/d_{i,j}^{\propto}$
	where $d_{i,j}^{\propto}$ is as per equation (3)
Step 4.	For $n = n + 1$
	Select power level $p_i = \{P_i\}$, for node i where $i = 1, 2, 3, \ldots N$
	and $p_{min} < p_i < p_{max}$
	Calculate SINR using equation (2)
	Calculate utility function using equation (8) and equation (17)
Step 5.	Optimize: Nash Equilibrium
	Find best response power level $\bar{p} = \{\text{argmax}\, u_i\,(p_i^*, \mathrm{p}_{-\mathrm{i}}^*)\}$

if for every $i \in N,\ u_i\,(p_i^*, \mathrm{p}_{-\mathrm{i}}^*) \geq\ u_i\,(p_i, \mathrm{p}_{-\mathrm{i}}^*)$ for $\forall\, p_i \in P_i$

Step 6. The nodes are checked for utilizing more power so as to maximize utility function and are imposed pricing in terms of SINR reduction.

Step 7. Repeat steps 4, 5, 6 and 7 until $P(n+1) - P(n) = \varepsilon$ where ε is small positive value where $P(n)$ and $P(n+1)$ are allocated power in previous and next iteration respectively.

7. Numerical Analysis

Existence of Nash Equilibrium for the utility function is proven in section 5. In order to show the effect of the proposed adaptive pricing scheme on net utility, an ad hoc network is set up with 10 nodes (for avoiding the run time delay), deployed randomly in 100 x 100 meters of area. The values of control parameters β and τ and other parameters are summarized in **Table 3**. For emphasizing on the pricing effect of the proposed work, hidden nodes and exposed nodes are ignored.

Table 3. Major Notation

Parameters	Value
$\propto$	4
$p_{\min}$	0.001 watts
$p_{\max}$	2 watts
σ^2	5 x 10^{-15} Watts
L	64
M	80
W	1 MHz
R	10^4 bits/second
k	0.097
β	100, 500, 1000 and 5000
τ	1, 5, 10, 50 and 100

7.1 Effect of β while keeping τ constant ($\tau = 10$)

With the above network setup, the resulting utility and net utility curves with respect to SINR are shown in **Fig. 3** to **Fig. 5**. It is observed that both the utility and net utility of node i increase as SINR increases.

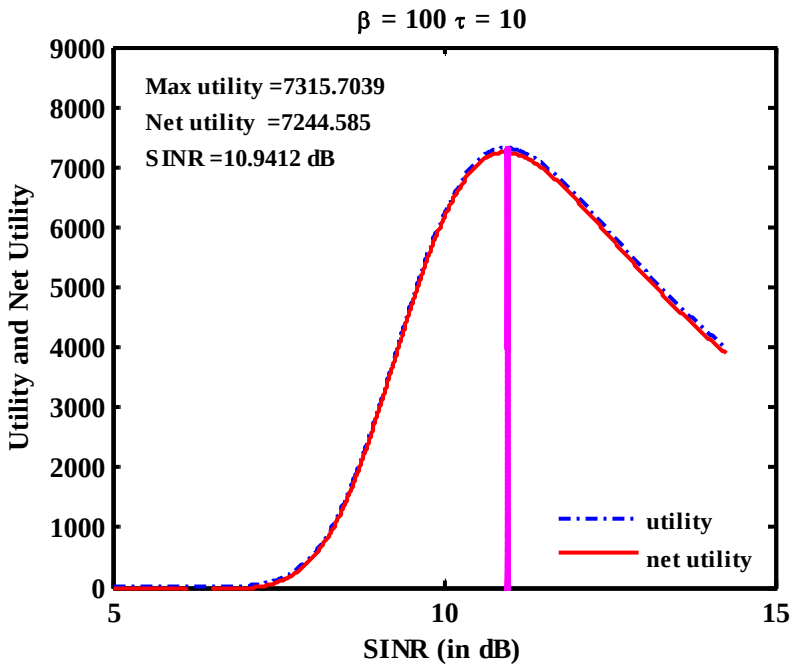

Fig. 3. Less value of β has negligible effect on utility and net utility.

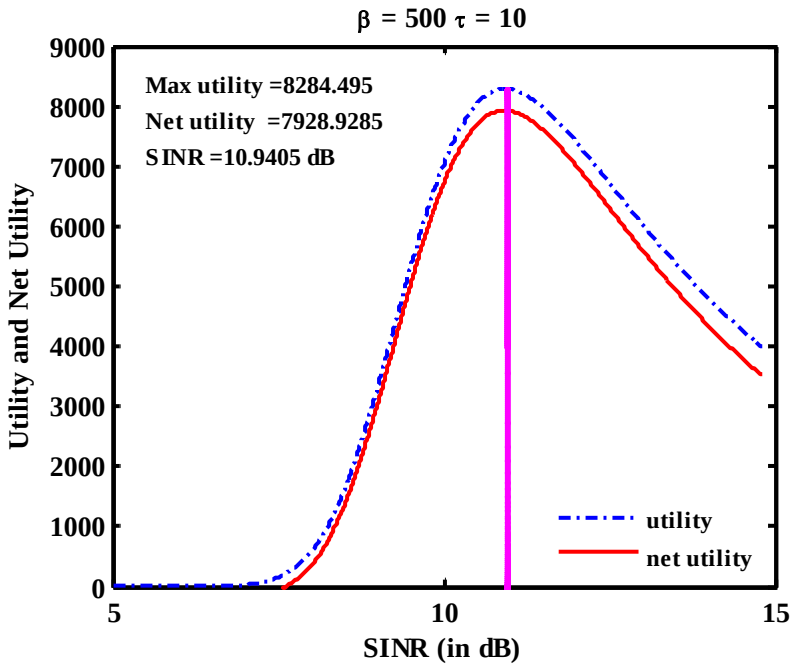

Fig. 4. The effect of β is visible when increased. Difference between utility and net utility is also increased.

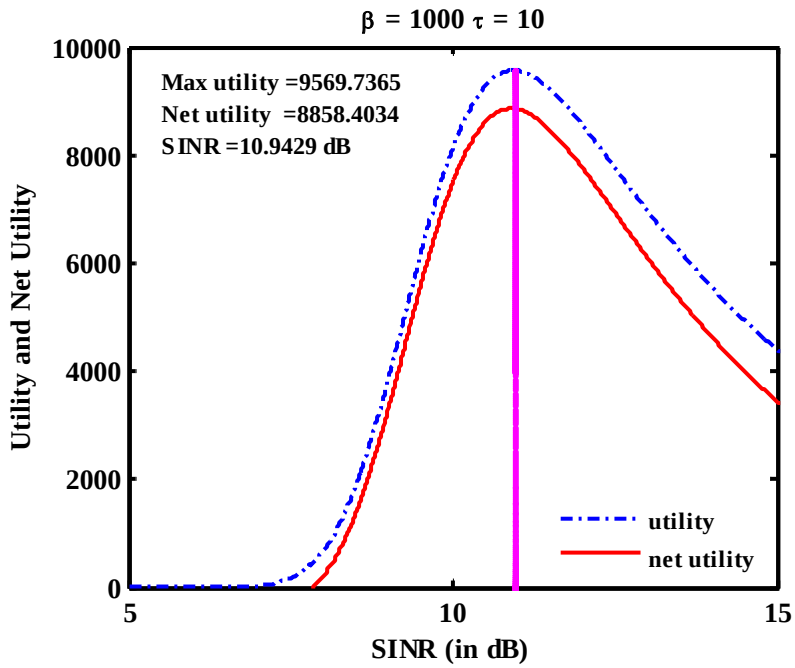

Fig. 5. Both utility and net utility are improved at the higher value of β, however, the difference between these two curves also increased (709.93) when weight is factor increased.

Once SINR obtains its equilibrium state, the utility starts decreasing as expected from NCPCG. Also, it can be noticed that all the upper set of utility curve are convex that illustrates the property of quasi-concavity of utility function. Furthermore the effect of weight factor in pricing scheme is evident from these figures.

The main importance of introducing β is to improve the net utility even if the transmitting node is far away from the receiving node but inside the transmission range. A node far away from the receiving node normally has a small path gain, so it is more likely to get a lower SINR. In this case either transmission may fail or delayed. So by increasing β, it is possible to improve the utility, consequently, net utility, as depicted in Figures for β = 100, 500 and 1000.

7.2 Effect of τ while keeping β constant ($\beta = 500$ *and* $\beta = 5000$)

Fig. 6 to **Fig. 8** depicts the effect of τ on utility as well net utility function with respect to transmission power. The effect of pricing is evident when τ is set to minimum value ($\tau = 1$) but there could be a further improvement in utility as τ is increased. It is apparent that when τ is nearly set equal to γ_T at the Nash equilibrium, the utility and hence net utility is raised to the highest value while taking lowest value transmission power. It is not necessary that the net utility will increase always with increasing τ. Even though τ is increased up to 100 (**Fig. 8**), the utility (also net utility) does not increase while consuming more power compared to $\tau = 10$.

Fig. 9 to **Fig. 10** further demonstrate the effect of τ on net utility with respect to both SINR and transmission power. As shown in the figures the net utility is maximum when the value of τ is nearer to optimum SINR (γ_T = 10.94 dB). When a node tries to deviate from optimum SINR unilaterally, it has to pay in terms of reducing its net utility. As a result, the nodes who increase power to get high SINR in order to maximize their net utility, are charged at high prices, in turn, the nodes will rationally reduce their transmit power.

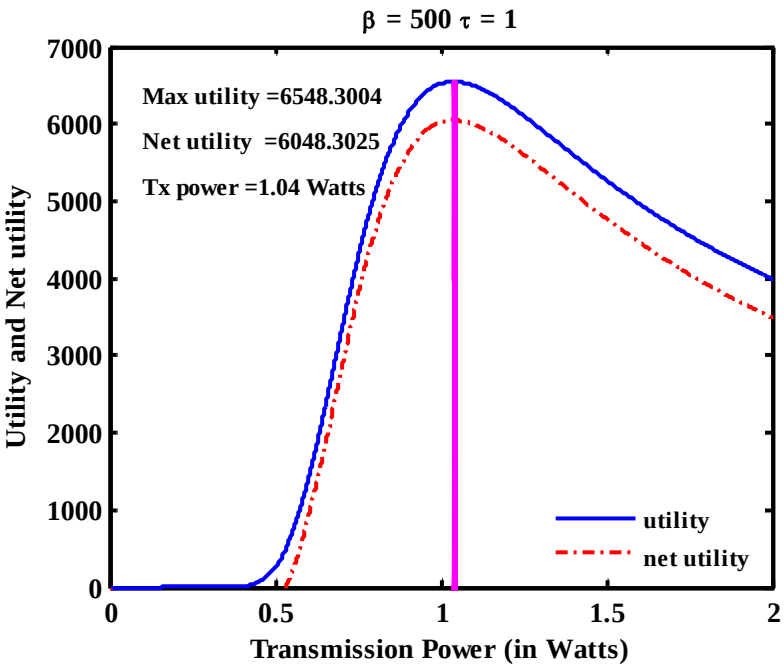

Fig. 6. Utility and net utility with respect to transmission power.

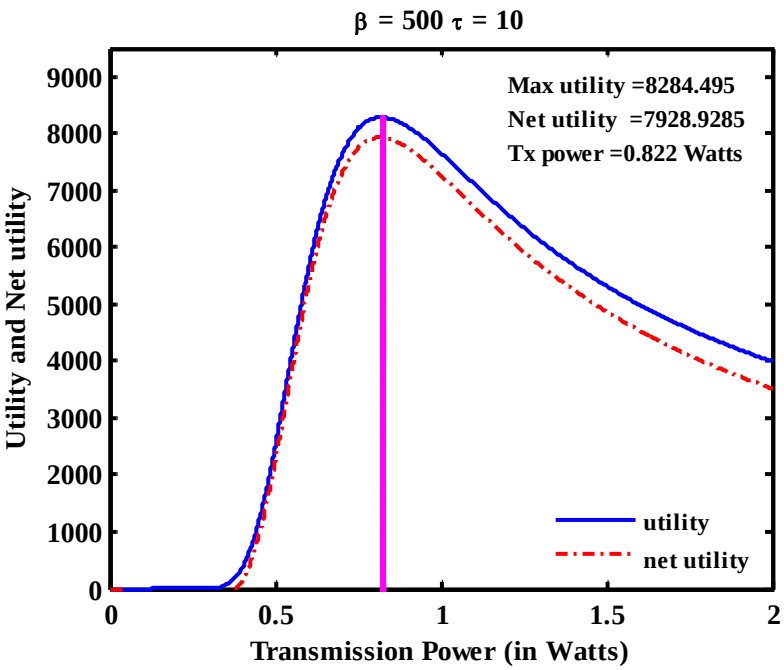

Fig. 7. At τ = 10, the nodes achieve the highest level of maximum utility and net utility at the lowest transmission power.

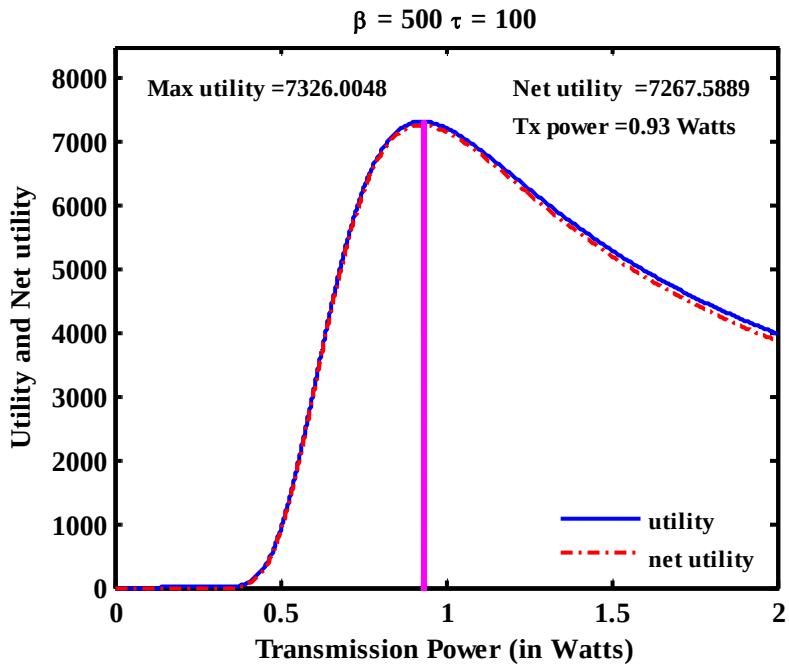

Fig. 8. When τ is deviated from Nash equilibrium, both utility and net utility decrease and transmission power increases.

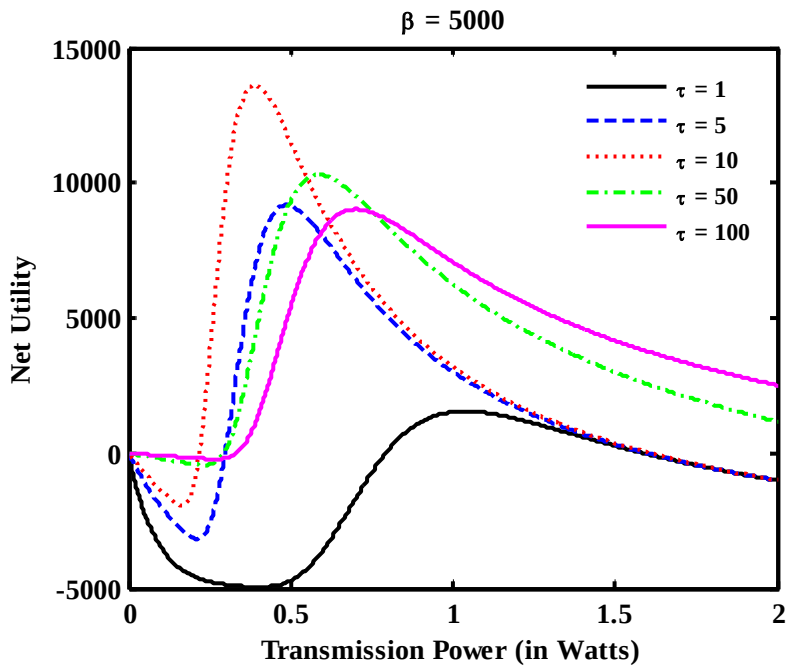

Fig. 9. Net utilities of nodes with respect to transmission power at different level of τ.

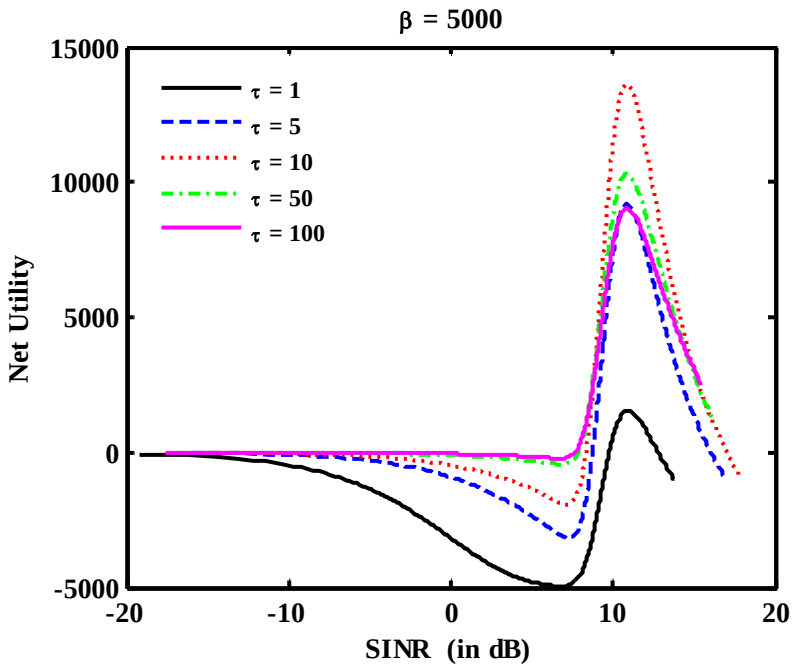

Fig. 10. Net utilities of nodes with respect to SINR at different level of τ.

7.3 Comparison with Existing Pricing Schemes

The other pricing schemes, found in various literatures [13], are given in equation (18) along with their respective curves in **Fig. 11**.

$$\lambda_i(p_i) = \begin{cases} \lambda p_i & \text{Linear} \\ \lambda p_i^2 & \text{Quadratic} \\ \lambda e^{p_i} & \text{Exponential} \end{cases} \tag{18}$$

Where λ is a positive scalar quantity having the same unit as bits/joule. **Fig. 11** exemplified that all these pricing schemes except the proposed pricing scheme (for $\beta = 100$ and $\tau = 10$) are increasing function of SINR. The existing pricing schemes shown in equation (18) charges a common price to all nodes, whereas the proposed pricing scheme can be tuned by using adaptive parameter β and τ depend on their attainable SINR. The nodes that use more power to gain higher SINR have to compromise with net utility. As a result, they reduce power, in turn, reduce interference and the purpose of this scheme is fulfilled.

Plot of net utilities at various pricing functions are shown in **Fig. 12.** The utility curve obtained from the proposed pricing scheme shows nearly 10 % improvement in net utility and nearly 17 % reduction in transmission power as compared to a nearest exponential pricing scheme.

Finally the net utility verses τ and transmission power verses τ is shown in **Fig. 13** and **Fig. 14** respectively. It is seen in these figures that at the equilibrium point ($\tau = 10$ or SINR $= 0.94$ dB) nodes achieves maximum net utility while using minimum transmission power as the objective of this paper.

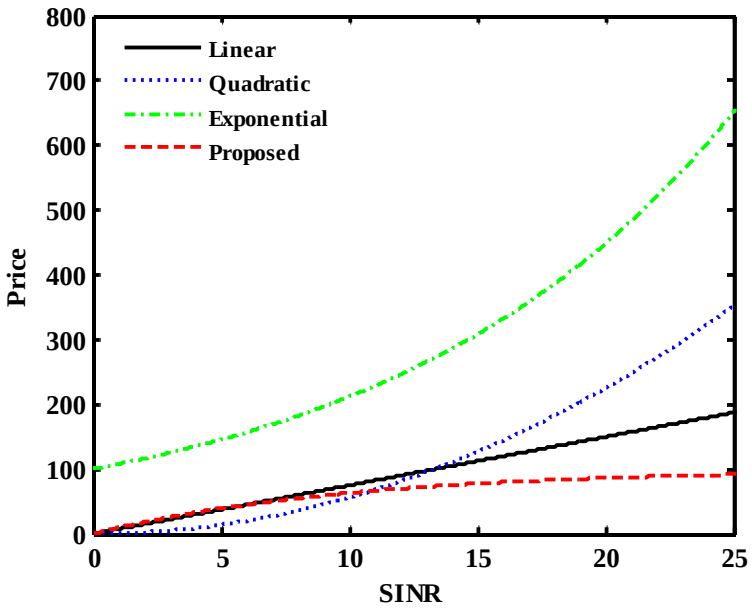

Fig. 11. Curves of various pricing schemes.

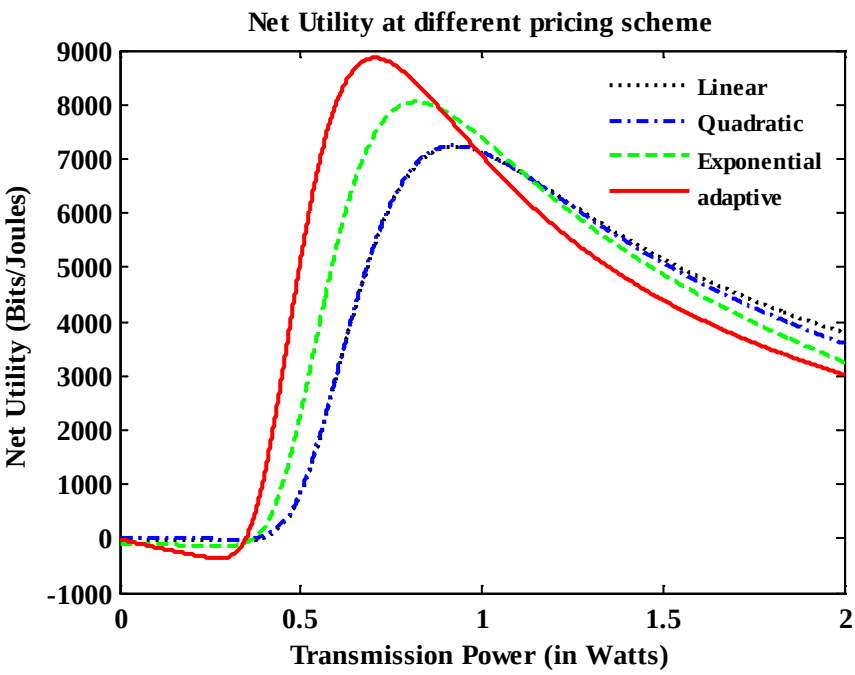

Fig. 12. Comparision curves of net utilities at various pricing schemes.

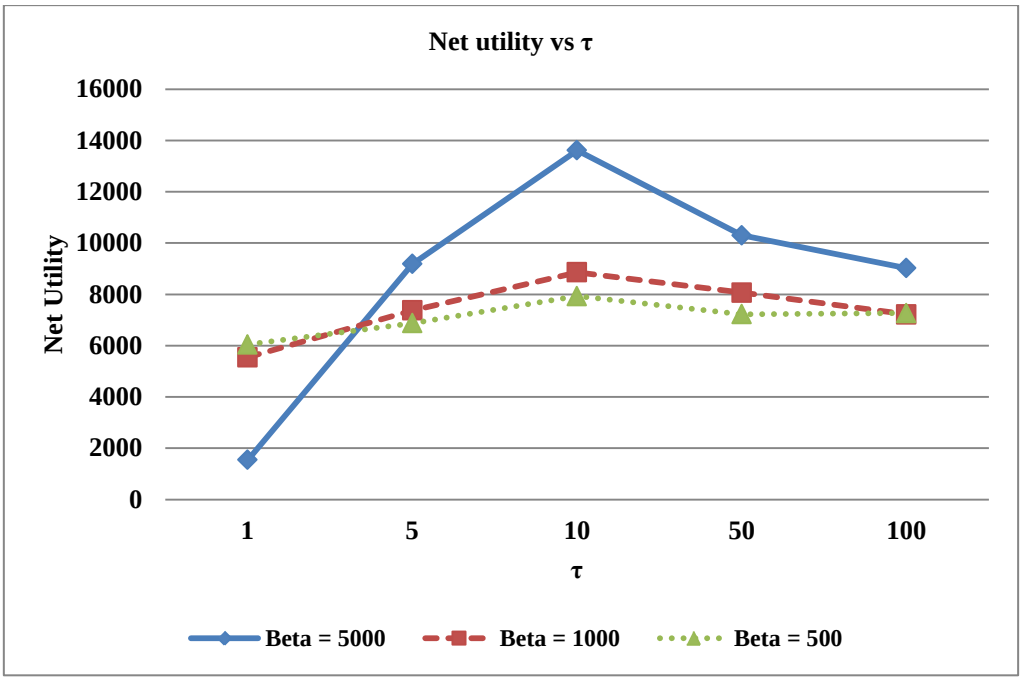

Fig. 13. Plot for net utility with respect to τ at different level of β.

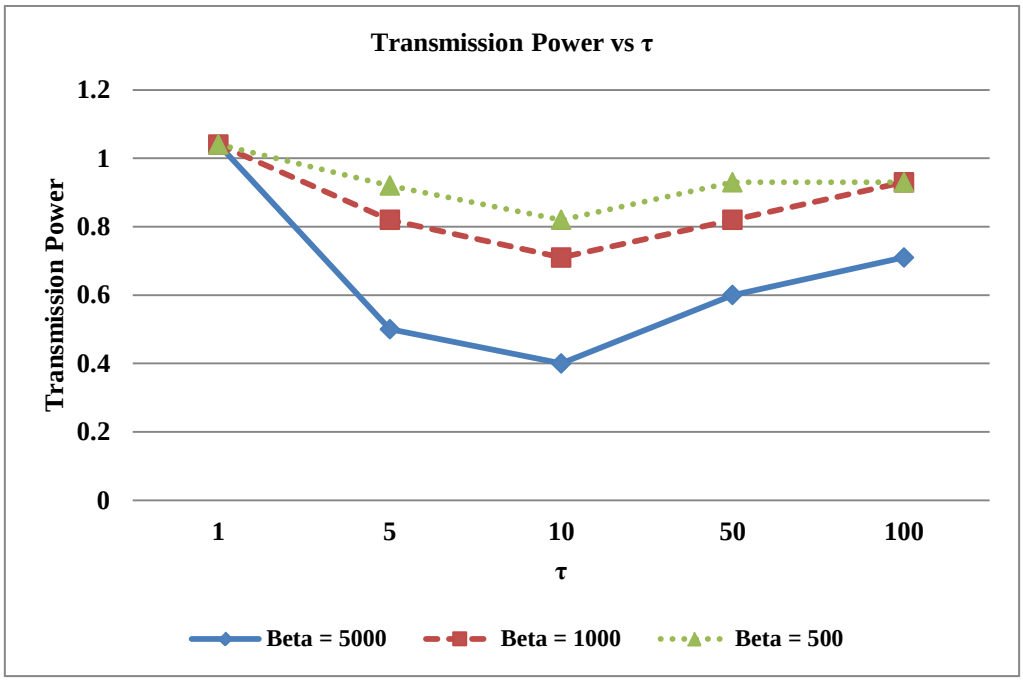

Fig. 14. Plot for transmission power with respect to τ at different level of β.

8. Conclusion

The control of transmitting power by controlling SINR using game theory offers a synergistic way of reducing interference while improving connectivity. In this approach, the power control problem in wireless ad hoc networks is framed as a non cooperative power control game. A new SINR based pricing scheme with two control parameters is proposed for non cooperative power control game which is validated with the proof of the existence of Nash equilibrium. The improvements of nodes' utility at minimum transmission power by using control parameters of the proposed pricing scheme are demonstrated using simulation while SINR was kept fixed. It is investigated that with this pricing scheme the nodes are encouraged to use a minimum power level to save their battery power as well as improving channel capacity, in turn, net utility without compromising QoS.

References

[1] Stefano Basagni, Marko Conti, Silvia Giordano and Ivan Stojmenovic, *Mobile ad hoc networking*, IEEE Press, Piscataway, New Jersey John Wiley & Sons, Inc., Publication, 2004. Article (CrossRef Link)

[2] Sanjay Kumar Suman, L. Bhagyalakshmi, and K. Murugan, "Non cooperative power control game for wireless ad hoc networks," in *Proc. of IEEE International conf. on Advance Computing* (ICoAC-2012), December 13-15, 2012. Article (CrossRef Link)

[3] Vikas Kawadia and P.R. Kumar, "Principal and protocols for power control in wireless ad hoc networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 1, pp. 76-88, January 2005. Article (CrossRef Link)

[4] Martin J. Osborne and Ariel Rubinstein, *A Course in Game Theory*, MIT Press, 2011. Article (CrossRef Link)

[5] L. Xiang, X. Ge, C.-X. Wang, Frank Y. Li and Frank Reichert, "Energy Efficiency Evaluation of Cellular Networks Based on Spatial Distributions of Traffic Load and Power Consumption," *IEEE Transaction on Wireless Communications*, vol. 12, no. 3, pp. 961-973, March. 2013. Article (CrossRef Link)

[6] I. Humar, X. Ge, L. Xiang, J. Ho, M. Chen, "Rethinking Energy-Efficiency Models of Cellular Networks with Embodied Energy," *IEEE Network Magazine*, vol. 25, no. 3, pp. 40-49, March 2011. Article (CrossRef Link)

[7] S. Porselvi and V. Bhaskar, "Novel Finite Summation Expressions for Total Interference Power due to Carrier Frequency Offset in Uplink Multiuser OFDM Systems," *Wireless personal Communication*, vol. 72, issue 4, pp. 2587-2604, October 2013. Article (CrossRef Link)

[8] X. Ge, T. Han, Y. Zhang, G. Mao, C-X. Wang, J. Zhang, B. Yang and S. Pan, "Spectrum and Energy Efficiency Evaluation of Two-Tier Femtocell Networks with Partially Open Channels," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 3, pp. 1306-1319, March 2014. Article (CrossRef Link)

[9] Xuelian Cai, Jingjing Yuan, Wu Zhu, Jiandong Li, Changle Li and Sana Ullah, "Energy- efficient relay MAC with dynamic power control in wireless body area networks," *KSII Transactions on Internet and Information Systems*, vol. 7, no. 7, pp. 1547-1568, July 2013. Article (CrossRef Link)

[10] Chao Liang and Kapil R. Dandekar, "Power management in mimo ad-hoc networks: a game-theoretic approach," *IEEE Transaction on Wireless Communication*, vol. 6, no. 4, pp. 1164-1170, April 2007. Article (CrossRef Link)

[11] Feng Zhao, Jiayi zhang and Hongbin Chen, "Joint beam forming and power allocation for multiple primary users and secondary users in cognitive mimo systems via game theory," *KSII Transaction on Internet and Information Systems*, vol. 7, no. 6, pp. 1379-1397, June 2013.

Article (CrossRef Link)
[12] L. Bhagyalakshmi, Sanjay Kumar Suman, and K. Murugan, "Corona based clustering with mixed routing and data aggregation to avoid energy hole problem in wireless sensor network," in *Proc. of IEEE Int. conference on Advance Computing* (ICoAC-2012), December 13-15, 2012. Article (CrossRef Link)
[13] Shamik Sengupta, Mainak Chaterjee, and Kevin A. Kwiat, "A game theoretic framework for power control in wireless sensor networks," *IEEE Transactions on Computers*, vol. 59, no. 2, pp. 231–242, February 2010. Article (CrossRef Link)
[14] Chengnian Long, Qian Zhang, Bo Li, Huilong Yang and Xinping Guan, "Non-cooperative power control for wireless ad hoc networks with repeated game," *IEEE Journal on Selected Area in Communication*, vol. 25, no. 6, pp. 1101-1111, August 2007. Article (CrossRef Link)
[15] Mehdi Rasti, Ahmad R. Sharafat, and Babak Seyfe, "Pareto-efficient and goal-driven power control in wireless networks: a game-theoretic approach with a novel pricing scheme," *IEEE/ACM Transaction on Networking*, vol. 17, no. 2, pp. 556-569, April 2009. Article (CrossRef Link)
[16] Mehdi Monemi, Ali Reza Zolghadr Asli, Shapoor Golbahar Haghighi, and Mehdi Rasti, "Distributed multiple target-SINRs tracking power control in wireless multirate data networks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 4, pp. 1850-1859, April 2013. Article (CrossRef Link)
[17] Madhusudhan R. Musku, Anthony T. Chronopoulos, Dimitrie C. Popescu, and Anton Stefanescu, "A game - theoretic approach to joint rate and power control for uplink CDMA communications," *IEEE Transactions on Communication*, vol. 58, no. 3, pp. 923-932 March 2010. Article (CrossRef Link)
[18] Eirini Eleni Tsiropoulou, Georgios K. Katsinis, and Symeon Papavassiliou, "Distributed uplink power control in multiservice wireless networks via a game theoretic approach with convex pricing," *IEEE Transactions on Parallel and Distributed Systems*, vol. 23, no. 1, pp. 61-68, January 2012. Article (CrossRef Link)
[19] Yuanzhang Xiao, Jaeok Park, and Mihaela van der Schaar, "Intervention in power control games with selfish users," *IEEE Journal of Selected topic in Signal Processing*, vol. 6, no. 2, pp. 165-179 April 2012. Article (CrossRef Link)
[20] Theodore S. Rappaport, "*Wireless Communications*, Prentice Hall of India Pvt. Ltd, 2006. Article (CrossRef Link)
[21] D. J. Goodman and N. B. Mandayam, "Power control for wireless data," *IEEE Personal Communication Magazine*, vol. 7, no. 2, pp. 48-54, April 2000. Article (CrossRef Link)
[22] Haykin Simon. (1994). *Communication Systems*. John Willy & Sons, Inc. Article (CrossRef Link)
[23] Vivek Srivastava, *et al.*, "Using game theory to analyze wireless ad hoc networks," *IEEE communications surveys and tutorials*, vol. 7, no. 4, pp. 46-56, fourth quarter 2005. Article (CrossRef Link)
[24] Nash J. F. (1951). *Non-cooperative games*. Ann. Math., vol. 54, pp. 289-295. Article (CrossRef Link)
[25] Cem U. Saraydar, Narayan B. Mandayam. and David J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Transactions Communication*, vol. 50, no. 2, pp. 291-303, February 2002. Article (CrossRef Link)

Sanjay Kumar Suman received his M.E degree in Electronics Systems and Communication from National Institute of Technology (Formerly REC), Rourkela, Orissa. He is currently pursuing research in the area of wireless ad hoc network in Dept. of IT, Anna University, MIT campus, Chennai, India. His area of interest includes wireless networks, ad hoc and sensor networks, digital signal processing and wireless communication.

Dhananjay Kumar received his Ph. D. degree under the Faculty of Information and Communication Engineering at Anna University, Chennai. He did his M. E. in Industrial Electronics Engineering, at Maharaja Sayajirao University of Baroda and M. Tech. in Communication Engineering at Pondicherry Engineering College, Pondicherry. He is currently working as Associate professor in Dept. of Information Technology, Anna University, MIT Campus, Chennai, India. His technical interest includes mobile computing & communication, multimedia systems, and signal processing. Currently he is developing a system to support medical video streaming over 3G wireless networks which is sponsored by the University Grant Commission of Government of India, New Delhi.

L. Bhagyalakshmi received her M.E degree in Electronics from Anna University, MIT campus, Chennai. She is currently pursuing research in the area of wireless sensor network in the Dept. of RCC, Anna University, Chennai, India. Her area of interest includes wireless ad hoc and sensor networks, signal processing and communication engineering.