# Stability of Slotted Aloha with Selfish Users under Delay Constraint

**Chang-Ho Chin[1], Jeong Geun Kim[2] and Deokjoo Lee[1]**

[1] Department of Industrial & Management Systems Engineering, Kyung Hee University
Yongin-si, Gyeonggi-do 446-701 - ROK
[e-mail: {chin, ldj}@khu.ac.kr]
[2] Department of Electrical Engineering, Kyung Hee University
Yongin-si, Gyeonggi-do 446-701 - ROK
[e-mail: jg_kim@khu.ac.kr]
*Corresponding author: Chang-Ho Chin

## *Abstract*

Most game-theoretic works of Aloha have emphasized investigating Nash equilibria according to the system state represented by the number of network users and their decisions. In contrast, we focus on the possible change of nodes' utility state represented by delay constraint and decreasing utility over time. These foregone changes of nodes' state are more likely to instigate selfish behaviors in networking environments. For such environment, in this paper, we propose a repeated Bayesian slotted Aloha game model to analyze the selfish behavior of impatient users. We prove the existence of Nash equilibrium mathematically and empirically. The proposed model enables any type of transmission probability sequence to achieve Nash equilibrium without degrading its optimal throughput. Those Nash equilibria can be used as a solution concept to thwart the selfish behaviors of nodes and ensure the system stability.

*Keywords:* Slotted Aloha, stability, game theory, Bayesian game, Nash equilibrium, delay constraint

## 1. Introduction

$A$loha protocol has attracted remarkable attention in random access communications due to its simplicity and robustness. In slotted Aloha, time is discretized into slots and nodes synchronized to the timeslot transmit a packet over it. Successful transmission is achieved only when one packet is transmitted in a given slot. Otherwise, collision occurs, causing unsuccessful reception. Packets backlogged due to collision are probabilistically retransmitted in the next timeslot.

Since the advent of Aloha in [1], most of works on Aloha and slotted Aloha have been done on the implicit assumption that all nodes in the network undoubtedly obey the standard specified by the system designer. However, the stability and performance of systems designed on this assumption could be easily compromised by "selfish nodes" capable of deviating from the standard [2]. In this paper, "selfish" nodes refer to the ones that are free to manipulate transmission probabilities for unilaterally augmenting their own gains without considering others' loss over timeslots of slotted Aloha networks. For example, a selfish node possibly increases its transmission probability higher than specified by the standard in order to improve the individual performance, which provokes others' deviating. This vicious cycle leads to consistent packet collision and then, finally system failure.

The presence and discretion of "selfish nodes" make network environment intractable with conventional analytical methods which require complete control over all the determinants of systems. In a recent decade, there have been some pioneering attempts to address this difficulty with game theory [2][3][4]. Game theory is a collection of mathematical tools useful in analyzing the interaction among multiple decision makers competing for conflicting interests. As a critical solution concept to competition, Nash equilibrium represents a combination of all players' decisions in which no player unilaterally augments its interest by changing its own decision specified in the combination.

There are two major reasons for modeling slotted Aloha as a game. Firstly, slotted Aloha is distributed in nature. Each network node makes its own access decision. Furthermore, these decisions are interactive and one's decision affects the others'. Each node has a similar role to a player in a game. Secondly, as nodes become more and more flexible and intelligent [5], it becomes more appropriate to analyze the system behavior of slotted Aloha on the assumption that nodes are rational and selfish. Game theory is a suitable tool to model such selfish behaviors and conjecture the outcomes of their decisions.

Among the aforementioned pioneering attempts to apply game theory to analyzing system behavior in presence of selfish users, MacKenzie and Wicker [2] established a Aloha game model for the first time and provided Nash equilibrium transmission probabilities for different numbers of users in the system. In [3], they extended this work to a multipacket slotted Aloha game. Relaxing the perfect information assumption of [2] and [3] about the number of users, [4] established a Markov chain based slotted Aloha game of partial information.

In contrast that past works [2][3][4][7][8][9][10] focused on selecting Nash equilibrium transmission probabilities according to the system state represented by the number of users and their decisions, we investigate the selfish behavior of users experiencing the change of their individual state represented by delay constraint and decreasing utility. These foregone changes of individual state are more likely to instigate selfish behaviors in network environments.

This paper is mainly intended to analyze the system behavior of Aloha-based random access networks with selfish users and design a game-theoretic framework facilitating convergence to Nash equilibria. The proposed framework enables any type of transmission probability sequence to achieve Nash equilibrium without degrading its throughput. Those Nash equilibria can be used as a solution concept to thwart the selfish behaviors of nodes and ensure the system stability. To the best of our knowledge, this paper is the first to formulate a repeated Bayesian slotted Aloha game focusing on the change of user's individual states. Our main contributions can be summarized as follows. In Section 3, we formulate a repeated Bayesian slotted Aloha game with partial information, in which each user may determine its transmission probability every timeslot over the limited lifetime and has a decreasing utility of successful transmission. We reveal the mathematical existence of the Bayesian Nash equilibrium for the game and also explain it empirically with Monte Carlo simulations. In Section 4, we show that the proposed Bayesian game model enables any type of transmission probability sequence strategy (TPSS) to be a Nash equilibrium without degrading its throughput. Also, we compare the performance of the time-invariant TPSS of traditional Aloha system and a time-variant TPSS in terms of throughput and packet loss rate. Interestingly, time-variant TPSSs proposed in this paper outperform time-invariant TPSSs. We provide a lemma and theorems about the characteristics of the proposed game model.

## 2. Related Work

There has been a rich literature using game theory to study medium access. In addition to early works of slotted Aloha [1][2][3], the work in [11] provides another model, in which users announce their transmission probabilities to the others while their desired throughput are kept secret. By knowing other users' actions, a user adjusts its transmission probability in an attempt to attain the desired throughput and then make known its new action. It is proved that the dynamic game with such behavior of users converges to a stable equilibrium point. In [12], a one-shot random access game for wireless networks is presented to study the behavior of the selfish nodes. Necessary and sufficient conditions on selfish nodes are given for the purpose of thoroughly characterizing the Nash equilibria of the game. Furthermore, the authors also provide the asymptotic analysis of the game as the number of selfish transmitters goes to infinity. In [13], an interference-aware MAC protocol is proposed via a game-theoretic approach. A channel access game in this work considers nodes concurrently transmitting in nearby areas. Under such interference-present environment, the authors derive a decentralized transmission strategy, which achieves a Bayesian Nash equilibrium (BNE).

Some previous works [6][14][15] propose game-theoretic approaches for backoff schemes. In [16], the backoff attacks in ad hoc networks with anonymous stations are analyzed in two different non-cooperative game models: one-shot and repeated CSMA/CA games. Furthermore, the authors developed a strategy for stations, which provides a fair Pareto efficient and sub-game perfect Nash equilibrium of repeated CSMA/CA games. A reverse-engineering of backoff-based random access MAC protocols using a game-theoretic approach is presented in [17]. As shown in this paper, the exponential backoff protocol is reversed-engineered through a non-cooperative game in which the objective of each link is maximizing a selfish local utility function. Additionally, the authors prove the existence of the Nash equilibrium and provide sufficient conditions for its uniqueness and stability for the game. In [18], the authors investigate the stability of CSMA/CA based wireless networks with selfish users engaging in a non-cooperative CSMA/CA game. In this game, each user's price is different and is dynamically determined by the network's congestion and power consumption

status, not by a network. In addition, a proposed iterative method guarantees convergence to the unique Nash equilibrium. In [6][19], a novel concept of incompletely cooperative game theory and its implementation are proposed to improve the performance of CSMA/CA in mobile ad hoc networks. In this game model, nodes' equilibrium strategies are tuned based on the estimated game state to achieve the optimal performance. The extension of this work is shown in [20]. In this work, the authors present a method for estimating the conditional collision probability based on the Virtual-CSMA technique and propose a simplified game-theoretic MAC protocol that can be implemented in wireless mesh networks.

Nash equilibria of threshold transmission policy are discussed [5][21]. [21] formulates the problem of finding a channel state information (CSI) based transmission policy for each node in slotted Aloha as a non-cooperative game, in which each node attempts to maximize its individual utility. The condition for the existence of a Nash equilibrium threshold transmission policy is given and a stochastic gradient based algorithm is employed to handle the best response dynamic adjustment process for the transmission game. The authors in [5] present a game theoretic approach to design robust random access control protocols for wireless networks with fading channels. Specifically, the opportunistic transmissions in slotted Aloha and CSMA adapted to channel information states are modeled as Bayesian games in which each transmission threshold is a Bayesian Nash equilibrium of the game.

The focus of [7][8][9] is on applying game theory to generalized General cases or models. In [7], the authors consider a generalized case for transmission strategy in Aloha networks. In this model, the user can change transmission probabilities of both fresh and backlogged packets. Their concerns are throughput and throughput fairness among users. They study the users' behavior in cooperative, competitive and adversarial environments. In [8], the authors generalize the game access control for the case where each node can observe multiple contention signals to guide them to the Nash equilibrium and provide the conditions for the unique existence of this equilibrium. In [9], the authors present a general game-theoretic model to study the interactions among the nodes contending for the common wireless channel. Additionally, they investigate the Nash equilibrium of this game and present a method for achieving it in a distributed manner.

## 3. Slotted Aloha Game

### 3.1 The Model

A game is basically composed of a set of players $\boldsymbol{K} = \{1, \ldots, n\}$, an action space $A_k$ for each player $k \in \boldsymbol{K}$, the space $\mathscr{A}_k$ of probability distributions over $A_k$, and utility functions $g_k : \Omega \to \mathbf{R}$, where $\Omega = \times_{k \in \boldsymbol{K}} A_k$. In a competitive situation referred to as a *game*, players rationally choose one of their available actions to maximize their individual utilities which are determined by the other players' actions as well as their own. The utilities from all possible combinations of chosen actions, called as action profile, are mathematically expressed by utility function such that $g_k(a)$ representing player $k$'s von Neumann- Morgenstern utility for an action profile $a \in A$ [23].

We consider a slotted Aloha protocol with $N$ homogenous nodes transmtting their own packets competitively. In this protocol, collision occurs when two or more nodes transmit packets simultaneously in a given slot, and the backlogged packets are retransmitted with a specified probability in the next timeslot. That is, the transmission of only one packet over all

nodes in a given slot leads to successful reception. The homogeneous network nodes behave selfishly as they are contending for successful reception which is a conflicting goal. This can be viewed as a typical game-theoretic situation, where a stage game $G = \left\{ \boldsymbol{K}, \left( A_k \right)_{k \in K}, \left( g_k \right)_{k \in K} \right\}$ is repeated every timeslot. The homogeneous nodes can be regarded as the players of game. Thus, the player set coincides with the set of nodes, denoted by $\boldsymbol{K} = \left\{ 1, ..., N \right\}$. The actions available to each node at every timeslot are *Transmit* (*T*) and *Wait* (*W*): $A_k = \left\{ T, W \right\}$. Without the loss of generality, we can omit subscript $k$ from $A_k$ and $g_k(a)$ in case all players are homogeneous like our model.

We assume that a backlogged packet is expired when a specified amount of time has elapsed since its arrival, and its gross utility gradually deceases in every timeslot in which the packet is backlogged. These assumptions correspond to the growing tendency of real-time delay sensitive services such as a car navigation, stock trading, video conference, and voice over IP (VoIP) for mobile devices. Those services convey importance to timely reception as well as successful reception. The time-point of a packet's successful reception determines its utility and also influences the following transmissions. In order to reflect this, we add the aforementioned two time-related conditions (packet expiration and decreasing utility) to conventional slotted Aloha protocols.

The gross utility given to a node by the successful transmission of its packet depends on the number of times the packet has been backlogged. The gross utility is greatest when a packet is successfully transmitted in the first timeslot following its arrival and subsequently decreases every time the packet is backlogged. A backlogged packet loses its worth for transmission after a certain time-point, and expires. These time-related conditions motivate nodes' deviating from the standard. Unlike conventional slotted Aloha, thus, we assume that each node may choose different transition probabilities according to the number of backlogged timeslots.

To describe time-dependent strategies and utility functions, we define the age of a packet, denoted by $\tau$, as the number of elapsed timeslots since its arrival. A packet at the first timeslot following its arrival has an age of one.

We also assume that all nodes have the same delay constraint, such that that a backlogged packet is discarded at the age of $L+1$. Thus, $L$ is the packet lifetime limit. Let $u_\tau$ denote a gross utility assigned to a node when its packet is successfully transmitted at the age of $\tau$. The net utility of successful transmission is $u_\tau - c$, where $c$ is a transmission cost. The decreasing gross utility over lifetime is calculated by:

$$u_\tau = U - \sum_{i=1}^{\tau-1} d_i \, ,$$

where $U$ is the gross utility assigned to the node when its packet is successfully received at the age of one and $d_\tau$ is the loss of utility per timeslot as packet age increases from $\tau$ to $\tau+1$. Thus,

$$u_\tau = u_{\tau-1} - d_{\tau-1} \, .$$

For each competitive timeslot, as shown in **Fig. 1**, a node takes action *T* or *W* with transmission probability $r_\tau \in \mathscr{A}^\tau$ at the age of $\tau$, where $\mathscr{A}^\tau$ is the space of probability distributions over $A$ at the age of $\tau$. This is referred to as a mixed strategy, while taking an action deterministically is referred to as a pure strategy. Note that $0 \leq r_\tau \leq 1$ for $1 \leq \tau \leq L$

and otherwise, $r_\tau = 0$. We refer to $\mathbf{r}_k = (r_1, r_2, ..., r_L)$ as transmission probability sequence strategy (TPSS) for node $k$. We assume that each node is free to determine its transmission probability at the age of $\tau$ in order to maximize the expectation of the discounted payoff over the *remnant lifetime*. We define the remnant lifetime of a packet as the time from the current timeslot to the timeslot at which the packet is successfully transmitted. All nodes are assumed to possess a packet for transmission at all times. All systems under analysis in this paper are assumed to be in a steady state.

In order to achieve an equilibrium TPSS, we add the concept of compensation to the slotted Aloha game model. Compensation is assigned to a node when it takes action *W*. This concept of compensation can be understood with pricing, as in [4][15], in terms of the relationship between cost and performance. Let $\alpha_{\mathbf{r},L,\tau}$ denote the compensation assigned to a node of age $\tau$ when its packet is determined to wait, all nodes select a TPSS $\mathbf{r}$, and their packet lifetime limit is $L$.

The utility function of the aforementioned stage game, $g_k$, is defined for each timeslot at age of $\tau$ as follows: a successful transmission accruing net utility of $u_\tau - c$, a collision net utility of $-c$, and a compensation of $\alpha_{\mathbf{r},L,\tau}$ for waiting.
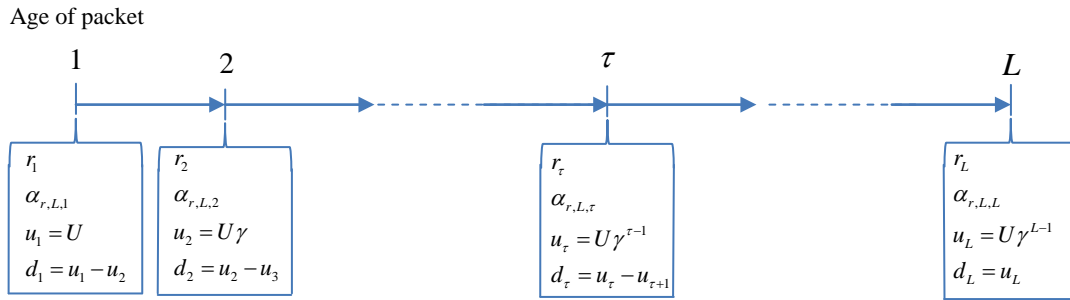
Age of packet



Fig. 1. Description of notations along the age of packet.

In this paper, we assume that no information about the age of one node is given to the others. The packet age $\tau$ is considered as the *type* element of Bayesian game model. Hence, the slotted Aloha protocol considered in this paper can be modeled as a repeated Bayesian game that the aforementioned stage game is repeated every timeslot. TPSS $\mathbf{r} = (r_1, r_2, ..., r_L)$ can be regared as a strategy taken by a player over packet lifetime $L$ and the strategy space is $H = \times_{\tau \in \Psi} r_\tau$, where $\Psi = \{1, 2, ...., L\}$ is the type space.

The packet age $\tau$ is a regular Markov chain. When all nodes select TPSS $\mathbf{r}$ with delay constraint $L$, the transition probabilities of the Markov Chain are given by

$$P_{i,j}(\mathbf{r}) = \begin{cases} r_i S_{\mathbf{r},L}, & 1 \le i \le L, j = 1 \\ 1 - r_i S_{\mathbf{r},L}, & 1 \le i \le L, j = i+1 \\ 1, & i = (L+1), j = 1 \\ 0, & otherwise \end{cases},$$

where $S_{\mathbf{r},L}$ is the probability that a packet is successfully received when it is transmitted in the

steady-state and can be calculated with using the steady-state probability distribution $\pi$ by:

$$S_{\mathbf{r},L} = \left( \sum\nolimits_{\tau=1}^{L} \left( \pi_\tau \left( 1 - r_\tau \right) \right) \right)^{N-1} \tag{1}$$

The steady state throughput can be calculated using

$$thpt = \sum\nolimits_{\tau=1}^{L} N \pi_\tau r_\tau S_{\mathbf{r},L} \tag{2}$$

See [24] for details in calculating $S_{\mathbf{r},L}$ and $\pi_\tau$.

When all nodes play TPSS $\mathbf{r}$, each node's expected discounted payoff over the remnant packet lifetime from taking action $A$ at the age of $\tau$ is denoted by $v_\tau(A)$ and is expressed as follows, depending on which action, $T$ or $W$, is taken :

$$v_{\mathbf{r},\tau}(T) = S_{\mathbf{r},L} u_\tau + \left( 1 - S_{\mathbf{r},L} \right) \left( \delta E\left[ v_{\tau+1}(A) \right] \right) - c \tag{3}$$

$$v_{\mathbf{r},\tau}(W) = \delta E\left[ v_{\tau+1}(A) \right] + \alpha_{\mathbf{r},L,\tau} \tag{4}$$

where $\delta$ is a per-slot discount factor and is mostly close to 1 for wireless network applications, and $v_\tau(A) = 0$ for $\tau > L$.

With the above utility functions, we formally define the Bayesian game model for this paper as follows.

Definition 1: The repeated Bayesian game for slotted Aloha protocol with $N$ homogenous nodes is defined as $\mathsf{G} := \{ \boldsymbol{K}, A, \Psi, \mathrm{H}, \mathrm{B}, v \}$, where $\boldsymbol{K} = \{1, 2, \dots N\}$ is the set of players, $A = \{T, W\}$ is the action space for all players, $\Psi = \{1, 2, \dots, L\}$ is the type space, $r_\tau$ is the probability that a player of type $\tau$ takes action $T$ and $\mathrm{H} = \times_{\tau \in \Psi} r_\tau$ is a strategy space, $\mathrm{B} = \{ \pi_1, \pi_2, \dots, \pi_L \}$ is the belief space, and $v_{\mathbf{r},\tau}$ is the utility function when all players select TPSS $\mathbf{r} \in \mathrm{H}$ and the player's own type is $\tau \in \Psi$.

## 3.2 Nash Equilibrium

Individual manipulations of system configuration have been facilitated more and more by easier accessibility to relevant information and tools over internet. Users' attempting to attain privileged controls within smartphone operating systems is one of typical examples. It allows users to customize products according to their preferences, beyond the restrictions of manufacturers. These attempts are overlooked or even encouraged by manufacturers due to rapidly growing diversity in user preferences. However, selfish manipulations neglecting others' loss in strategic situations with conflicting interests should be restrained to ensure stability and functionality of the system.

The Nash equilibrium concept of game theory is applicable to dealing with the fears associated with the slotted Aloha system in terms of performance deterioration attributed to the selfish behaviors of nodes. In game theory, a mixed strategy Nash equilibrium is defined as a combination of all players' mixed strategies where no player unilaterally augments its utility by changing its own strategy specified in the combination [25]. For slotted Aloha protocols susceptible to users' selfish manipulations of transmission probabilities, a mixed strategy Nash equilibrium TPSS $\mathbf{r}^*$ can be used to thwart such manipulations by hindering selfish users from achieving their intention. In other words, slotted Aloha protocols based on a mixed strategy Nash equilibrium TPSS $\mathbf{r}^*$ do not allow each node to unilaterally increase its utility by taking another sequence strategy rather than that specified at the Nash equilibrium. Hence,

no player has an incentive to deviate from the standard. For more information of game-theoretic applications in communications and networking, see [26].

Let 2-tuple $\left(\mathbf{r}_k, \mathbf{r}_{-k}^*\right)$ represent that node $k$ and the other nodes select $\mathbf{r}$ and $\mathbf{r}^*$ as their mixed sequence strategies over packet lifetime, respectively. Node $k$'s expected discounted payoff for sequence strategy profile $\left(\mathbf{r}_k, \mathbf{r}_{-k}^*\right)$ at the age of $\tau$ is denoted by function $v_\tau\left(\mathbf{r}_k, \mathbf{r}_{-k}^*\right)$. $\mathbf{r}^*$ is a Nash equilibrium if $v_\tau\left(\mathbf{r}_k^*, \mathbf{r}_{-k}^*\right) \geq v_\tau\left(\mathbf{r}_k, \mathbf{r}_{-k}^*\right)$ for $\forall k \in \mathbf{K}$ and $\forall \mathbf{r}_k \in \mathrm{H}$. $\mathbf{r}^*$ is a best response to the other nodes and no node improves its payoff by unilaterally deviating from Nash equilibrium $\mathbf{r}^*$.

In the repeated Bayesian game model proposed in this paper, TPSS $\mathbf{r}$ achieves a Nash equilibrium at $v_\tau(T) = v_\tau(W)$, $\tau \in \{1, 2, ... L\}$, which yields the following equations: from (4),

$$E\left[v_\tau(A)\right] = \delta E\left[v_{\tau+1}(A)\right] + \alpha_{\mathbf{r}, L, \tau}, \ \tau \in \{1, 2, ..., L-1\} \tag{5}$$

from (3) and (5),

$$E\left[v_{\tau+1}(A)\right] = \frac{1}{\delta}\left(u_\tau - \frac{c + \alpha_{\mathbf{r}, L, \tau}}{S_{\mathbf{r}, L}}\right), \ \tau \in \{2, 3, ..., L-1\} \tag{6}$$

from (5) and (6),

$$\alpha_{\mathbf{r}, L, \tau-1} = \delta\left(\left(1 - S_{\mathbf{r}, L}\right)\alpha_{\mathbf{r}, L, \tau} + S_{\mathbf{r}, L}d_{\tau-1}\right) + \left(1 - \delta\right)\left(S_{\mathbf{r}, L}u_{\tau-1} - c\right), \ \tau \in \{2, 3, ..., L\} \tag{7}$$

$$\alpha_{\mathbf{r}, L, L} = S_{\mathbf{r}, L}u_L - c \tag{8}$$

Note that $v_L(T) = v_L(W)$ makes nodes at the age of $L$ compensated in a manner set to be equal to the payoff from taking action $T$, as shown in (8). $\alpha_{\mathbf{r}, L, \tau}$ can be recursively calculated backward in time via (7) for $\tau = \{2, 3, ..., L-1\}$. The expectation of aggregate compensation given to a packet over packet lifetime, $Q$, and the packet loss rate (PLR) can be respectively computed by:

$$E[Q] = \sum_{\tau=1}^{L-1} \alpha_{\mathbf{r}, L, \tau} \prod_{i=1}^{\tau}\left(1 - r_i S_{\mathbf{r}, L}\right)r_{\tau+1} S_{\mathbf{r}, L} + \alpha_{\mathbf{r}, L, L} \prod_{i=1}^{L}\left(1 - r_i S_{\mathbf{r}, L}\right) \tag{9}$$

$$PLR = \Pi_{\tau=1}^{L}(1 - r_\tau S_{\mathbf{r}, L}) \tag{10}$$

## 4. Numerical Analysis

### 4.1 Time-invariant Transmission Probability Sequence Strategy

In this section, we consider a slotted Aloha system with a time-invariant TPSS in contrast to the time-variant TPSS considered in Section 4.2. Just as in traditional slotted Aloha systems, a time-invariant TPSS is $\mathbf{r}$ of $r_t = r$ for $\forall t$ as shown in **Fig. 2-(a)**. The transmission probability $r$ is determined to maximize the throughput for this system and its optimal value is $1/N$, which is easily derived based on the fact that the number of successful receptions has a binomial distribution. In the slotted Aloha system, the number of nodes is $N = 5$, the discount factor is $\delta = 0.999$, and the transmission cost is $c = 0.2$.

The slotted Aloha system considered in this section has a packet expiration and decreasing gross utility over the packet lifetime. We assume that the packet lifetime limit is $L = 50$, the maximum gross utility of successful transmission is normalized to 1 (i.e., $U = 1$) [2][3], the decreasing gross utility is $u_\tau = U\gamma^{\tau-1}$, and $\gamma = 0.995$. As shown in **Fig. 2-(b)**, the gross utility exponentially decreases over the packet lifetime. The loss of utility per timeslot is computed by $d_\tau = U\gamma^{\tau-1}(1-\gamma)$ for $\tau < L$ and $d_L = u_L = U\gamma^{L-1}$. Compensation $\alpha_{\mathbf{r},L,\tau}$ is determined by (7) and (8) so that TPSS $\mathbf{r}$ is a Nash equilibrium.

**Fig. 2** shows the graphs of $r_\tau$, $u_\tau$, $d_\tau$, and $\alpha_{\mathbf{r},L,\tau}$ for the slotted Aloha protocol. $d_\tau$ linearly decreases up to $\tau = (L-1)$ and then abruptly shifts to a large value at $\tau = L$, due to the loss of the whole potential utility caused by packet expiration. The compensation is large at age $L$, because it is set to be equal to the utility from action $T$ at the age of $L$ in order to achieve Nash equilibrium, as shown in (8).



**(a)** Transmission probability

**(b)** Gross utility

**(c)** Loss of utility per timeslot

**(d)** Compensation
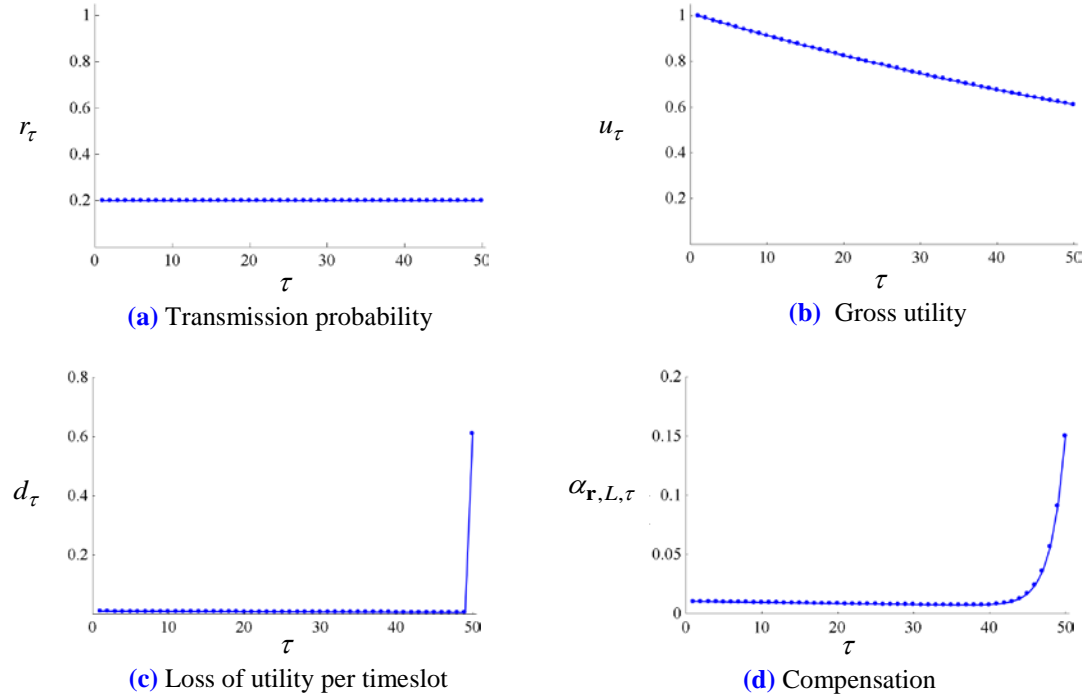
**Fig. 2**. Graphs of slotted Aloha transmission with a time-invariant TPSS.

We conducted Monte Carlo simulations using 100 replicates to empirically verify the existence of the Nash equilibrium for the aforementioned slotted Aloha game model. The length of each replicate was $5 \times 10^6$ timeslots and all metrics were computed with numerical values from the steady state. We considered two scenarios. In the first scenario, all nodes used the same Nash Equilibrium transmission probability of $r = 1/5$ over the entire packet lifetime. In the second scenario, only selfish Node 1 increased its Nash equilibrium transmission probability of $r^*$ up to $r = 2r^*$ in order to improve its utility, while the others behave as in the first scenario.

The simulation results in **Table 1** show that selfish Node 1 cannot benefit from deviation.

The same is true for any other value of $r \in [0,1]$, consistent with the Nash equilibrium concept that no node is able to improve the expectation of the aggregate discount payoff over the remnant lifetime through unilateral deviating. Hence, the rational behavior of all nodes is to stay at $r = 1/5$, the Nash Equilibrium transmission probability for this game model. Note that the values of all metrics in Table 1 are consistent with the values obtained through (1), (2), (7), and (8), validating the game modeling approach based on the steady-state probability distribution of packet age.

**Table 1**. Numerical results of Monte Carlo simulations.

| | Scenario 1 | | Scenario 2 | |
|---|---|---|---|---|
| | Transmission probability | Expectation of discounted payoff over the entire lifetime at $\tau = 1$ | Transmission probability | Expectation of discounted payoff over the entire lifetime at $\tau = 1$ |
| Node 1 | **0.2** | **0.5038** ($7.0 \times 10^{-4}$) | **0.4** | **0.5038** ($4.5 \times 10^{-4}$) |
| Node 2 | 0.2 | 0.5039 ($6.4 \times 10^{-4}$) | 0.2 | 0.3454 ($1.0 \times 10^{-3}$) |
| Node 3 | 0.2 | 0.5039 ($6.6 \times 10^{-4}$) | 0.2 | 0.3453 ($1.0 \times 10^{-3}$) |
| Node 4 | 0.2 | 0.5038 ($6.6 \times 10^{-4}$) | 0.2 | 0.3455 ($1.1 \times 10^{-3}$) |
| Node 5 | 0.2 | 0.5039 ($6.3 \times 10^{-4}$) | 0.2 | 0.3453 ($1.3 \times 10^{-3}$) |
| | Aggregate throughput: 0.4096 ($3.3 \times 10^{-4}$) | | Aggregate throughput: 0.4096 ($3.4 \times 10^{-4}$) | |

\* The standard errors are shown in parentheses.

## 4.2 Time-variant Transmission Probability Sequence Strategy

In slotted Aloha systems with limited packet lifetimes, the packet loss possibility increases as the packet age approaches the expiration time. Thus, the concern about packet loss possibly instigate nodes to gradually increase their retransmission probabilities, as the packet age increases. We introduce an interesting Nash equilibrium TPSS in which such concern and desire are reflected. It is graphically shown in **Fig. 3-(a)**. Approximately up to the first half of the maximum packet lifetime of 50, the transmission probability constantly stays at a certain level just as in the time-invariant TPSS of Section 4.1. Over the remnant packet lifetime, the transmission probability exponentially increases and finally reaches 1 at $\tau = 50$. We refer to this type of time-variant TPSSs as a hill-shaped TPSS [24]. We assume that all other parameters are as same as those for the slotted Aloha system based on the time-invariant TPSS in the previous section.

**Fig. 3-(b)** shows the compensation $\alpha_{\mathbf{r}, L, \tau}$ for the Nash equilibrium. The hill-shaped TPSS and the time-invariant TPSS have the almost same value of successful reception probability $S$, which leads to nearly the same compensation policy, as expected in (7) and (8). The hill-shaped TPSS yields a substantially better PLR of $4.1 \times 10^{-2}$% than 1.4% of the time-invariant TPSS in the previous section, while both systems have the same throughput of 0.4096. Thus, hill-shaped TPSSs are worth considering in the light of providing equal throughput and lower PLR and reflecting the desires of impatient nodes.

The lower PLR of the hill-shaped TPSS can be attributed to its characteristic in favor of later transmission. The hill-shaped TPSS transmits with a much higher probability after the age of 25. It substantially reduces the loss probability of backlogged packets. Instead, the hill-shaped TPSS has a larger expected delay for backlogged packets [24], resulting in receipt of a smaller

number of newly arrived packets compared to that of the time-invariant TPSS. Despite the lower PLR, hence, the hill-shaped TPSS has the same throughput as the time-variant TPSS.
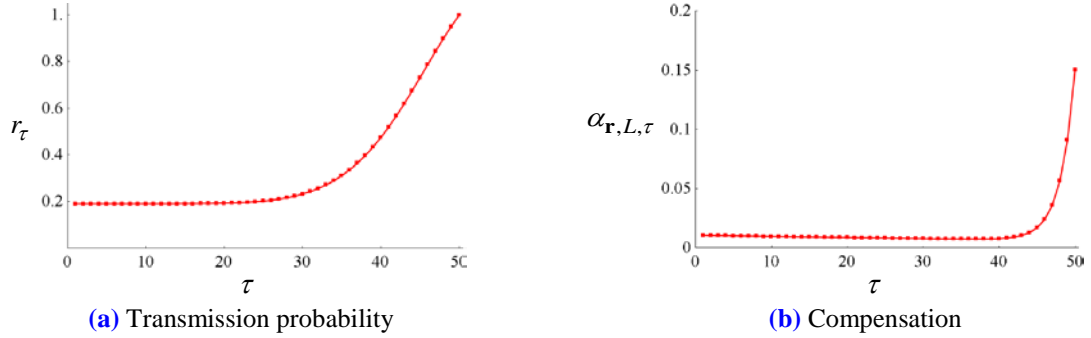


**(a)** Transmission probability                                        **(b)** Compensation

**Fig. 3**. Graphs of the slotted Aloha protocol with a time-variant transmission probability sequence.

## 4.3 Comparison of Two Transmission Probability Sequence Strategies: Time-invariant TPSS versus Hill-shaped TPSS

The number of nodes and the packet lifetime limit are major factors associated with the performance of the slotted Aloha system under delay constraints. In order to analyze how these factors influence the system performance, we compare time-invariant TPSSs to hill-shaped TPSSs for ten combinations of the packet lifetime limit $L$ and the number of nodes $N$. The first comparison considers the change of $N \in \{5,6,7,8,9,10\}$ with $L$=50 and the second considers the change of $L \in \{50,60,70,80,90,100\}$ with $N$=10. All slotted Aloha systems in this analysis are assumed to have the same decreasing utility function, where $U = 1$, $u_\tau = U\gamma^{\tau-1}$, $\gamma = 0.995$, $c = 0.2$, and $\delta = 0.999$. For each combination of $N$ and $L$, both TPSSs are determined to maximize the throughput while maintaining the typical shape, using the iterative search method proposed in [24].

In the first comparison scenario, as shown in **Fig. 4**, the $r$ value of the optimal time-invariant TPSSs is determined to be $1/N$ and the optimal hill-shaped TPSSs are shown to have different initial transmission probabilities according to the number of nodes. The initial transmission probability is lower with a larger number of nodes, which helps to mitigate collision during the first half. The compensation policy for each combination is determined by (7) and (8), enabling the optimal TPSS to be a Nash equilibrium.

**Fig. 4** also shows the graphs of the throughput, probability of successful reception, expected aggregate compensation over lifetime, and PLR for the first comparison scenario. For each combination, the two TPSSs have the same throughput and show a negligible difference with regard to the probability of successful reception. Hill-shaped TPSSs require the smaller expected aggregate compensations to achieve a Nash equilibrium compared to those of its corresponding time-invariant TPSS, and the deviation increases as $N$ increases. The time-invariant TPSS's PLR rapidly increases as $N$ increases, while the hill-shaped TPSS's remains satisfactorily low. These trends in the expected compensation and the PRL are consistent for $N$, for which both TPSSs have the same throughput. The preference of one TPSS over the other depends on which metric is more critical.
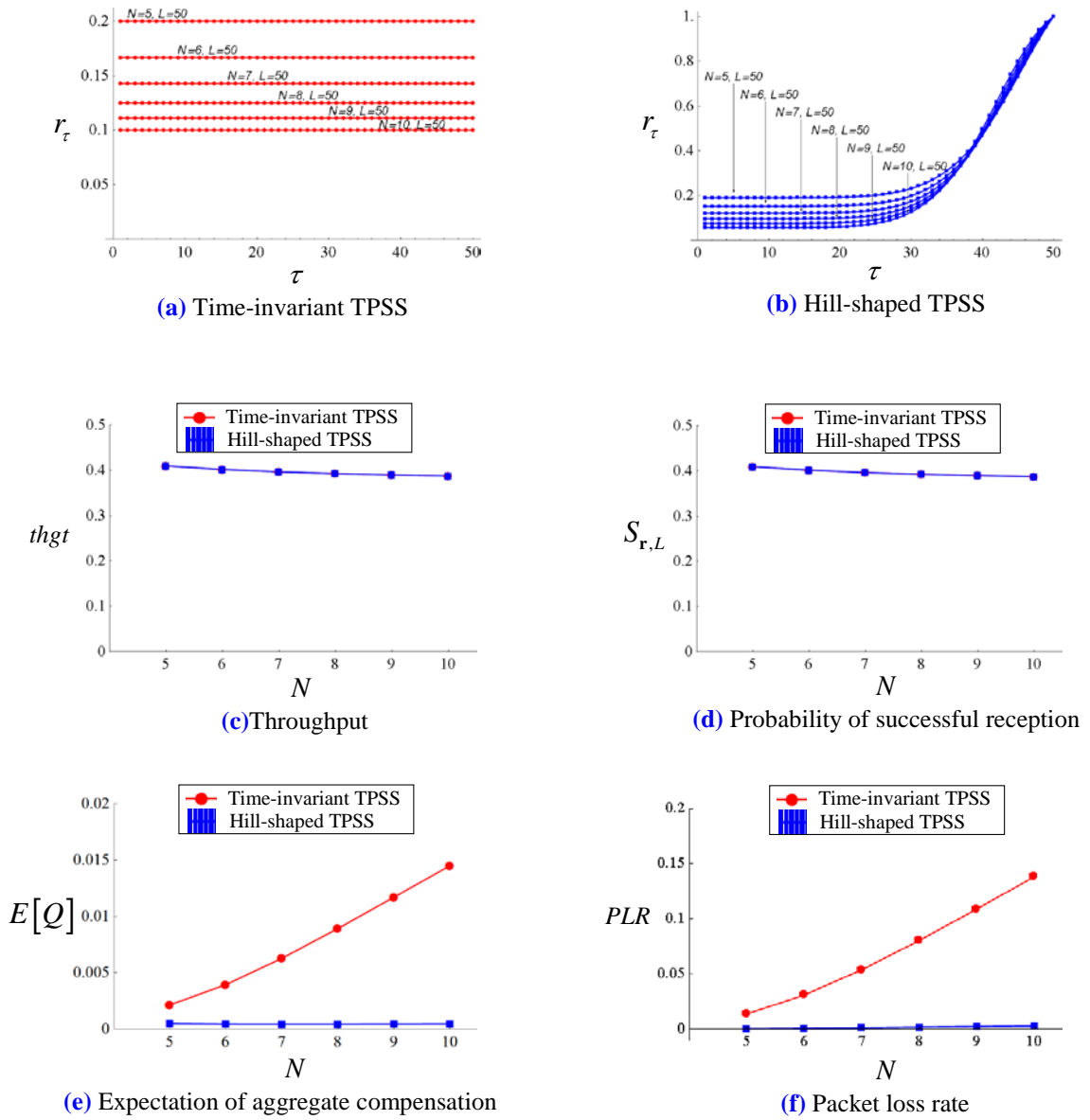
**(a)** Time-invariant TPSS



**(b)** Hill-shaped TPSS



**(c)** Throughput



**(d)** Probability of successful reception



**(e)** Expectation of aggregate compensation



**(f)** Packet loss rate

**Fig. 4**. Comparison of node number changes from 5 to 10 with a fixed packet lifetime limit of $L$=50.

In the second comparison scenario, shown in **Fig. 5**, the $r$ value of the optimal time-invariant TPSSs is $1/10$ regardless of the packet lifetime limit. Hill-shaped TPSSs have different initial transmission probabilities and different increasing rates in order to maximize throughput according to the packet lifetime limit. The initial transmission probability is higher at the larger packet lifetime limit, implying that the larger packet lifetime limit allows for more aggressive transmission during the first half. Both TPSSs for each combination have the same throughput and nearly the same probability of successful reception. These two metrics are both constant over all of the packet lifetime limits. **Fig. 5** also shows that hill-shaped TPSSs requires the smaller expected aggregate compensations over the lifetime in order to achieve Nash equilibrium compared to those of their corresponding time-invariant TPSSs, while time-variant TPSSs have substantially lower PLRs. The deviations decrease as $N$ increases.

In short, both perform more compatible as $L$ increases.



**(a)** Time-invariant TPSS



**(b)** Time-variant TPSS



**(c)** Throughput



**(d)** Probability of successful reception



**(e)** Expectation of aggregate compensation



**(f)** Packet loss rate
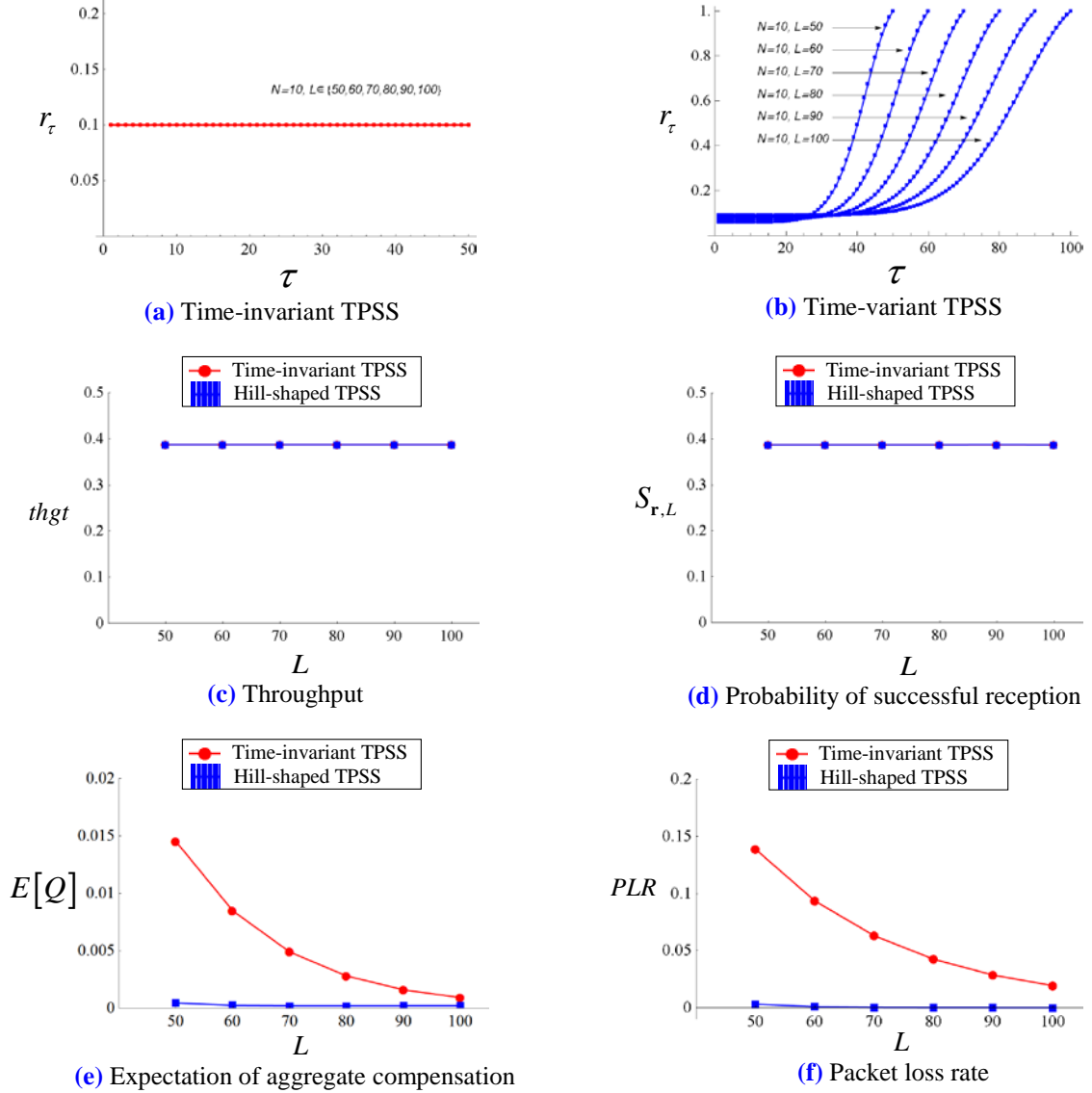
**Fig. 5**. Comparison when the packet lifetime changes from 50 to 100 with a fixed number of node $N$=5.

Based on the numerical examples, we observe convergence in the expected aggregate compensation and the expected discounted payoff over the remnant packet lifetime. The theoretical bases for these convergences are given in Theorems 2 and 3, respectively.

Lemma 1. Consider a slotted Aloha system in which all nodes are homogenous, transmit with a Nash equilibrium time-invariant transmission probability, have a payoff function of decreasing utility, and are compensated according to (7) and (8) for taking action $W$. The compensation at the age of $\tau$, denoted by $\alpha_{\mathbf{r},L,\tau}$, converges as $L$ increases and becomes sufficiently close to its converged value for $L \geq (\tau + \omega^*)$ where $\omega^*$ is a positive integer large

enough to have $\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^{\omega^*} \approx 0$ .

*proof*: $(\forall \tau)\ r_\tau = r$ . Since all nodes are homogenous, $S_{\mathbf{r},L} = S_{\mathbf{r}}$ . Let $\omega^*$ be a positive integer large enough to have $\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^{\omega^*} \approx 0$ . For $L \geq \left(\tau + \omega^*\right)$ ,

$$
\begin{aligned}
\alpha_{\mathbf{r},L,\tau} &= \delta\left(1-S_{\mathbf{r}}\right)\alpha_{\mathbf{r},L,\tau+1} + \delta S_{\mathbf{r}}d_\tau + \delta(1-\delta)\left(S_{\mathbf{r}}u_\tau - c\right) \\
&= \delta\left(1-S_{\mathbf{r}}\right)\left(\delta\left(1-S_{\mathbf{r}}\right)\alpha_{\mathbf{r},L,\tau+2} + \delta S_{\mathbf{r}}d_{\tau+1} + \delta(1-\delta)\left(S_{\mathbf{r}}u_{\tau+1} - c\right)\right) \\
&\quad + \delta S_{\mathbf{r}}d_\tau + \delta(1-\delta)\left(S_{\mathbf{r}}u_\tau - c\right) \\
&= \left(\delta\left(1-S_{\mathbf{r}}\right)\right)^2 \alpha_{\mathbf{r},L,\tau+2} + \delta S_{\mathbf{r}}d_\tau + \delta S_{\mathbf{r}}\left(\delta\left(1-S_{\mathbf{r}}\right)\right)d_{\tau+1} \\
&\quad + \delta(1-\delta)\left(S_{\mathbf{r}}u_\tau - c\right) + \delta(1-\delta)\left(\delta\left(1-S_{\mathbf{r}}\right)\right)\left(S_{\mathbf{r}}u_{\tau+1} - c\right) \\
&= \left(\delta\left(1-S_{\mathbf{r}}\right)\right)^2 \left(\delta\left(1-S_{\mathbf{r}}\right)\alpha_{\mathbf{r},L,\tau+3} + \delta S_{\mathbf{r}}d_{\tau+2} + \delta(1-\delta)\left(S_{\mathbf{r}}u_{\tau+2} - c\right)\right) + \delta S_{\mathbf{r}}d_\tau \\
&\quad + \delta S_{\mathbf{r}}\left(\delta\left(1-S_{\mathbf{r}}\right)\right)d_{\tau+1} + \delta(1-\delta)\left(S_{\mathbf{r}}u_\tau - c\right) + \delta(1-\delta)\left(\delta\left(1-S_{\mathbf{r}}\right)\right)\left(S_{\mathbf{r}}u_{\tau+1} - c\right) \\
&= \left(\delta\left(1-S_{\mathbf{r}}\right)\right)^3 \alpha_{\mathbf{r},L,\tau+3} + \delta S_{\mathbf{r}}d_\tau + \delta S_{\mathbf{r}}\left(\delta\left(1-S_{\mathbf{r}}\right)\right)d_{\tau+1} + \delta S_{\mathbf{r}}\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^2 d_{\tau+2} \\
&\quad + \delta(1-\delta)\left(S_{\mathbf{r}}u_\tau - c\right) + \delta(1-\delta)\left(\delta\left(1-S_{\mathbf{r}}\right)\right)\left(S_{\mathbf{r}}u_{\tau+1} - c\right) \\
&\quad + \delta(1-\delta)\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^2\left(S_{\mathbf{r}}u_{\tau+2} - c\right) \\
&= \left(\delta\left(1-S_{\mathbf{r}}\right)\right)^\omega \alpha_{\mathbf{r},L,\tau+\omega} + \delta S_{\mathbf{r}}\sum_{i=0}^{\omega-1}\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^i d_{\tau+i} \\
&\quad + \delta(1-\delta)\sum_{i=0}^{\omega-1}\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^i \left(S_{\mathbf{r}}u_{\tau+i} - c\right) \\
&\approx \delta S_{\mathbf{r}}\sum_{i=0}^{\omega^*-1}\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^i d_{\tau+i} + \delta(1-\delta)\sum_{i=0}^{\omega^*-1}\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^i \left(S_{\mathbf{r}}u_{\tau+i} - c\right) \\
&\qquad\qquad\qquad\qquad\qquad \left\langle \because \left(\delta\left(1-S_{\mathbf{r}}\right)\right)^{\omega^*} \alpha_{\mathbf{r},L,\tau+\omega^*} \approx 0 \right\rangle \quad \blacksquare
\end{aligned}
$$

**Theorem 1.** Consider a slotted Aloha system in which all nodes are homogenous, transmit with a Nash equilibrium time-invariant transmission probability, have a payoff function of decreasing utility, and are compensated according to (7) and (8) for taking action *W*. The expectation of aggregate compensation $Q$ converges as $L$ increases and becomes sufficiently close to its converged value for $L \geq \left(\zeta^* + \omega^*\right)$ where $\zeta^*$ and $\omega^*$ are positive integers large enough to have $\left(1-rS_{\mathbf{r}}\right)^{\zeta^*} \approx 0$ and $\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^{\omega^*} \approx 0$ , respectively.

*proof*: $(\forall \tau)\ r_\tau = r$ . Since all nodes are homogenous, $S_{\mathbf{r},L} = S_{\mathbf{r}}$ . Let $\zeta^*$ and $\omega^*$ be positive integers large enough to have $\prod_{i=1}^{\zeta^*}\left(1-r_i S_{\mathbf{r}}\right) \approx 0$ and $\left(\delta\left(1-S_{\mathbf{r}}\right)\right)^{\omega^*} \approx 0$ , respectively. For $L \geq \left(\zeta^* + \omega^*\right)$ ,

$$E[Q] = \sum_{\tau=1}^{L} \alpha_{r,L,\tau} \prod_{i=1}^{\tau}(1-r_i S_\mathbf{r}) \approx \sum_{\tau=1}^{\zeta^*} \alpha_{r,L,\tau} \prod_{i=1}^{\tau}(1-r_i S_\mathbf{r})$$

$$\left\langle \because \prod_{i=1}^{\tau}(1-r_i S_\mathbf{r}) \approx 0 \text{ for } \tau \geq \zeta^* \right\rangle$$

From Lemma 1, $\alpha_{\mathbf{r},L,\tau}$ for $\tau = \{1,2,...,\zeta^*\}$ converges as $L$ increases and becomes sufficiently close to its converged value for $L \geq \zeta^* + \omega^*$ . For $L \geq \zeta^* + \omega^*$ ,

$$E[Q] \approx \sum_{\tau=1}^{\zeta^*}\left[\left(\delta S_\mathbf{r}\sum_{i=0}^{\omega^*-1}\left(\delta(1-S_\mathbf{r})\right)^i d_{\tau+i} + \delta(1-\delta)\sum_{i=0}^{\omega^*-1}\left(\delta(1-S_\mathbf{r})\right)^i (S_\mathbf{r}u_{\tau+i}-c)\right)\right.$$
$$\left.\prod_{i=1}^{\tau}(1-r_i S_\mathbf{r})\right] \qquad\qquad \blacksquare$$

   Theorem 2. Consider a slotted Aloha system in which all nodes are homogenous, transmit with a Nash equilibrium time-invariant transmission probability, have a payoff function of decreasing utility, and are compensated according to (7) and (8) for taking action *W*. The expectation of discounted payoff over the entire lifetime at a Nash equilibrium, represented as the present value at $\tau = 1$, converges as $L$ increases and becomes sufficiently close to its converged value for $L \geq (1+\omega^*)$ where $\omega^*$ is an positive integer large enough to have

$$\left(\delta(1-S_\mathbf{r})\right)^{\omega^*} \approx 0 .$$

*proof*: $(\forall \tau)$ $r_\tau = r$ . Since all nodes are homogenous, $S_{\mathbf{r},L} = S_\mathbf{r}$. Let $\omega^*$ be a positive integer large enough to have $\left(\delta(1-S_\mathbf{r})\right)^{\omega^*} \approx 0$. From (5) and (6), the expectation of discounted payoff over the whole lifetime at a Nash equilibrium, represented as the present value at $\tau = 1$, is $E\left[v_1(A)\right] = \left(u_1 - \dfrac{c+\alpha_{\mathbf{r},L,1}}{S_\mathbf{r}}\right) + \alpha_{\mathbf{r},L,1}$. The proof reduces to show that $\alpha_{r,L,1}$ converges as

$L$ increases and becomes sufficiently close to its converged value for $L \geq (1+\omega^*)$, which follows directly from Lemma 1.                                                                                        ■

## 4.4 Overall Remark on Numerical Results

The throughput, the probability of successful reception and the expectation of the discounted payoff decrease as the number of nodes increases in the Nash equilibrium slotted Aloha systems aforementioned. In contrast, the expectation of aggregate compensation and the PRL both increase. The additional extension of the packet lifetime limit does not improve the probability of successful reception and the expectation of discounted payoff, if the original limit is sufficiently large enough to yield the optimal throughput. However, it consistently reduces the expectation of aggregate compensation and the PRL.
   In comparing time-invariant TPSSs and hill-shaped TPSSs with the same optimal throughput, hill-shaped TPSSs are superior to time-invariant TPSSs in the PRL, which becomes more prominent as the number of nodes increases. Hill-shaped TPSSs require a smaller expectation of aggregate compensation in order to achieve Nash equilibrium. The difference in the expectation of aggregate compensation increases as the number of nodes increases. Conversely, it decreases as the packet lifetime limit increases.
   For slotted Aloha protocols with delay constraints, a time-variant TPSS in favor of later

retransmission is more desirable with respect to performance as long as its aggregate compensation is affordable. The larger packet lifetime limit is also more favorable for performance; however, it is not controllable and is determined by the packet's inherent properties. The type of TPSS has less effect on performance as the packet lifetime limit increases. With respect to Nash equilibrium and performance, it is obviously advisable to limit the number of nodes simultaneously accessing a shared medium (e.g., by improving the capture capability such as multiple packet reception (MPR)).

## 5. Conclusions

We analyzed the system behavior of Aloha-based random access networks formed by selfish nodes. Our proposed framework enables any type of transmission probability sequence to achieve Nash equilibrium without degrading its throughput. It was verfied that no node can unilaterally augment its payoff by deviating from the Nash equilibrium and each node rationally decides to stay at the equilibrium even in network environments which instigate selfish behaviors. Hence, those Nash equilibria can be used as a solution concept to thwart the selfish behaviors of nodes and ensure the system stability. We anticipate that such findings can be used immediately and provide insight to the behavior of random access networks in which absolutely distributed nature of random access networks allows any node to change system parameters, including transmission probability, in an adaptive and autonomous way.

In the slotted Aloha protocol considered in this paper, nodes experience packet expiration and decreasing utility. Each node is assumed to be free to determine its transmission probability at every timeslot in order to maximize the utility over its remnant lifetime. For the slotted Aloha protocol, we proposed a repeated Bayesian game model and investigated the Nash equilibria for the various combinations of node number and packet lifetime limit. We demonstrated the existence of the Nash equilibrium mathematically and empirically. Additionally, we added the concept of compensation to the game model in order to achieve Nash equilibrium without degrading the optimal throughput. A larger expected aggregate compensation is required to achieve the Nash equilibrium as the number of nodes increases. We showed that it converges as the packet lifetime limit increases.

With respect to performance, we revealed that hill-shaped TPSSs outperformed time-invariant TPSSs with the same throughput and lower PLRs. The two types of TPSSs perform more similarly as the packet lifetime limit increases. We demonstrated that the throughput and the probability of successful reception are constants regardless of the packet lifetime limit

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**Chang-Ho Chin** received the B.S. degree in Industrial Engineering from Korea University, Seoul and the M.S. and Ph. D. degrees in Industrial Engineering from Texas A&M University, College Station. He was a post-doctoral research associate at Texas A&M University from 2004 to 2005. He is currently an assistant professor of Industrial & Management Systems Engineering at Kyung Hee University, Republic of Korea. His research interests are in applied game theory, statistical analysis, data mining, and activity monitoring.

**Jeong Geun Kim** received the B.S. and M.S. degrees, both in electrical engineering, from Yonsei University, Seoul, Korea, in 1990 and 1992, respectively, and the Ph.D. degree in electrical and computer engineering from the University of Arizona, Tucson, in March 2000. He was a Senior Systems Engineer at Qualcomm Inc., Santa Clara, CA until 2003. He is currently Associate Professor in Department of Electrical Engineering, Kyung Hee University, Korea. His research interests include wireless networks and performance evaluation.

**Deokjoo Lee** received the Ph.D., M.S., and B.S. degrees in Industrial Engineering from Seoul National University in 1998, 1990 and 1995 respectively. He is currently the chair and associate professor in the Industrial & Management Systems Engineering Department at Kyung Hee University, Korea. His research interests include economic analysis of telecommunication systems, game theory application, management of technology, performance evaluation of R&D, etc.