

# Performance Analysis of Space-time Coded MIMO System with Discrete-rate Adaptive Modulation in Ricean Fading Channels

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## Abstract

The performance of a MIMO system with adaptive modulation (AM) and space-time coding over Ricean fading channels for perfect and imperfect channel state information (CSI) is presented. The fading gain value is partitioned into a number of regions by which the modulation is adapted according to the region the fading gain falls in. Under a target bit error rate (BER) constraint, the switching thresholds for AM are given. Based on these results, we derive the calculation formulae of the theoretical spectrum efficiency (SE) and average BER. As a result, closed-form SE expression and accurate BER expression are respectively obtained. Besides, using the approximation of complementary error function, a tightly closed-form approximation of average BER is also derived to simplify the calculation of accurate theoretical BER. Computer simulation shows that the theoretical SE and BER are in good agreement with the corresponding simulation, and the approximate BER is also close to the accurate one. The results show that the AM scheme in Ricean fading channel provides better SE than that in Rayleigh fading channel due to the direct-path propagation, and has performance degradation in SE and BER for imperfect CSI.

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**Keywords:** *Adaptive modulation, space-time coding, Ricean fading, MIMO, CSI*

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## 1. Introduction

The increasing demand of high data rate services always looks for spectrally efficient communication systems. Adaptive modulation (AM), as a powerful technique for improving spectrum efficiency (SE), has received much attention recently. It can take advantages of the time-varying nature of wireless channels to transmit data at higher rates under favorable channel conditions and to maintain reliability by varying transmit power, symbol rate, and/or code rate under poor channel condition [1][2][3][4]. Multiple antennas approach is another well known SE technique with diversity and/or coding gain [5][6][7]. Especially, multi-antenna systems with space-time block code (STBC) provide effective transmit diversity for combating fading effects [8][9][10][11][12]. Therefore, effective combination of adaptive modulation and space-time coding techniques has received much attention in the literatures [13][14][15][16][17][18][19][20][21].

Discrete-rate adaptive  $M$ -ary Quadrature Amplitude Modulation (MQAM) schemes with space-time block coding over flat Rayleigh and Nakagami fading channel have been studied in [13] and [14], respectively. A general performance analysis of STBC with fixed and variable rate MQAM over Rayleigh fading channel is provided in [15]. The impact of feedback delay on AM for multi-antenna systems in Rayleigh fading channel can be found in [16][17]. The performance of single-and multicarrier AM with perfect channel information is analyzed over Nakagami fading channel in [18]. The performance of adaptive modulation and adaptive coded modulation with STBC over Rayleigh fading channel for imperfect channel state information (CSI) are analyzed in [19] and [4], respectively. The performance analysis of an AM scheme with STBC in spatially correlated Rayleigh channels is presented in [20]. Based on the time-varying channel conditions and perfect CSI, adaptive modulation is used in ad hoc DS/CDMA networks to obtain high capacity [21], where the Ricean fading and lognormal shadowing are considered.

According to the above analysis, the MIMO system performance with AM in Rayleigh fading channel is studied well. Moreover, the above AM schemes basically assume that perfect channel information can be available at the receiver, and thus the expected performance is obtained. In practice, however, the system may experience the Ricean fading due to the direct-path propagation, and the channel state information will be imperfect due to estimation errors. For this reason, in this paper, we will present an AM scheme for space-time coded MIMO system over Ricean fading channel, where both perfect and imperfect CSI are considered. The SE and average bit error rate (BER) performance will be investigated. Based on the performance analysis and imperfect CSI, the effective signal-to-noise ratio (SNR) and its probability density function are derived. With these results, the fading gain switching thresholds for AM are obtained under the target BER constraint. Using the obtained switching thresholds and the generalized Marcum  $Q$ -function, we derive the accurate expressions of SE and BER in detail. As a result, a closed-form expression of SE is obtained. To simplify the numerical calculation of the accurate BER, a tight closed-form approximate expression is also derived by utilizing the theoretical BER and approximation of the complementary error function. This approximate BER is shown to match the accurate BER well. With these expressions, the system performances with perfect and imperfect CSI in Ricean channel will be effectively assessed, and some existing theoretical expressions in Rayleigh channel can be included. Simulation results show that the derived theoretical expressions are valid for evaluating the system performance, and have good agreement with the simulation results.

The notations we use throughout this paper are as follows. Bold upper case and lower case letters denote matrices and column vectors, respectively. The superscripts  $(\cdot)^H$ ,  $(\cdot)^T$  and  $(\cdot)^*$

denote the Hermitian transposition, transposition, and complex conjugation, respectively.  $\mathbf{A}_{ij}$  denotes the element in the  $i$ th row and the  $j$ th column of matrix  $\mathbf{A}$ .

## 2. System Model

In this section, we consider a wireless multi-antenna communication system with  $N$  transmit antennas and  $L$  receive antennas, which operates over a flat and quasi-static Ricean fading channel represented by an  $L \times N$  fading channel matrix  $\mathbf{H} = \{h_{ln}\}$ . The complex element  $h_{ln}$  denotes the channel gain from the  $n^{\text{th}}$  transmit antenna to the  $l^{\text{th}}$  receive antenna, which is assumed to be constant over a frame of  $P$  symbols and varies from one frame to another. For Ricean fading channels, the channel gains  $\{h_{ln}\}$  are modeled as independent complex Gaussian random variables with respective means  $m_I$  and  $m_Q$  for the real and imaginary parts and variance of 0.5 per dimension [11] [22]. In this case,  $|h_{ln}|^2$  will be a noncentral chi-square distribution with 2 degrees of freedom. A complex orthogonal space-time block code, which is represented by a  $T \times N$  transmission matrix  $\mathbf{D}$ , is used to encode  $P$  input symbols into an  $N$ -dimensional vector sequence of  $T$  time slots. The matrix  $\mathbf{D}$  is a linear combination of  $P$  symbols satisfying the complex orthogonality:  $\mathbf{D}^H \mathbf{D} = \varepsilon(|d_1|^2 + \dots + |d_p|^2) \mathbf{I}_N$ , where  $\mathbf{I}_N$  is  $N \times N$  identity matrix,  $\{d_p\}_{p=1, \dots, P}$  are the  $P$  input symbols, and  $\varepsilon$  is a constant which depends on the STBC transmission matrix [9][10]. Hence, the transmission rate of the STBC is  $r = P/T$ . In this paper, the conventional STBC,  $G_2$ ,  $G_3$ ,  $G_4$ ,  $H_3$  and  $H_4$  codes are used for performance evaluation in section 5. Their code matrices are defined as [9]:

$$G_2 = \begin{bmatrix} d_1 & d_2 \\ -d_2^* & d_1^* \end{bmatrix}, \quad G_3 = \begin{bmatrix} d_1 & d_2 & d_3 \\ -d_2 & d_1 & -d_4 \\ -d_3 & d_4 & d_1 \\ -d_4 & -d_3 & d_2 \\ d_1^* & d_2^* & d_3^* \\ -d_2^* & d_1^* & -d_4^* \\ -d_3^* & d_4^* & d_1^* \\ -d_4^* & -d_3^* & d_2^* \end{bmatrix}, \quad G_4 = \begin{bmatrix} d_1 & d_2 & d_3 & d_4 \\ -d_2 & d_1 & -d_4 & d_3 \\ -d_3 & d_4 & d_1 & -d_2 \\ -d_4 & -d_3 & d_2 & d_1 \\ d_1^* & d_2^* & d_3^* & d_4^* \\ -d_2^* & d_1^* & -d_4^* & d_3^* \\ -d_3^* & d_4^* & d_1^* & -d_2^* \\ -d_4^* & -d_3^* & d_2^* & d_1^* \end{bmatrix},$$

$$H_3 = \begin{bmatrix} d_1 & d_2 & d_3/\sqrt{2} \\ -d_2^* & d_1^* & d_3/\sqrt{2} \\ \frac{d_3^*}{\sqrt{2}} & \frac{d_3^*}{\sqrt{2}} & \frac{d_2 - d_2^* - d_1 - d_1^*}{2} \\ \frac{d_3^*}{\sqrt{2}} & \frac{-d_3^*}{\sqrt{2}} & \frac{d_2 + d_2^* + d_1 - d_1^*}{2} \end{bmatrix}, \quad \text{and} \quad H_4 = \begin{bmatrix} d_1 & d_2 & d_3/\sqrt{2} & d_3/\sqrt{2} \\ -d_2^* & d_1^* & d_3/\sqrt{2} & -d_3/\sqrt{2} \\ \frac{d_3^*}{\sqrt{2}} & \frac{d_3^*}{\sqrt{2}} & \frac{d_2 - d_2^* - d_1 - d_1^*}{2} & \frac{d_1 - d_1^* - d_2 - d_2^*}{2} \\ \frac{d_3^*}{\sqrt{2}} & \frac{-d_3^*}{\sqrt{2}} & \frac{d_2 + d_2^* + d_1 - d_1^*}{2} & \frac{d_2^* - d_2 - d_1 - d_1^*}{2} \end{bmatrix}.$$

Utilizing the complex orthogonality of STBC, the effective SNR after space-time decoding is expressed as [10][11]

$$\rho = \bar{S} / (rN\sigma_n^2) \|\mathbf{H}\|_F^2 = \bar{\rho} / (rN) \sum_{n=1}^N \sum_{l=1}^L |h_{ln}|^2 = \bar{\rho} / (rN) \beta \quad (1)$$

where  $\|\mathbf{H}\|_F^2$  is the Frobenius norm defined as  $\|\mathbf{H}\|_F^2 = \sum_{n=1}^N \sum_{l=1}^L |h_{nl}|^2$ , and  $\beta = \|\mathbf{H}\|_F^2$ .  $\bar{S}$  is the transmitted power radiated from the  $N$  transmit antennas,  $\sigma_n^2$  is the noise power, and  $\bar{\rho} = \bar{S} / \sigma_n^2$  is the average SNR per receive antenna.

For Ricean fading channel,  $\beta = \sum_{n=1}^N \sum_{l=1}^L |h_{nl}|^2$  is noncentral chi-square distributed with  $2NL$  degrees of freedom. Using transformation of random variable and Eq.(2-1-118) in [22], the pdf of  $\beta$  can be obtained as

$$f(\beta) = (\beta / s^2)^{(NL-1)/2} (2\sigma^2)^{-1} e^{-(s^2+\beta)/(2\sigma^2)} I_{NL-1}(\sqrt{s^2\beta} / \sigma^2), \quad \beta \geq 0 \quad (2)$$

where  $s^2 = NL(m_t^2 + m_q^2)$  is the noncentrality parameter,  $\sigma^2=0.5$ , and  $I_\nu(x)$  is the  $\nu$ th-order modified Bessel function of the first kind [22][23]. Using (2) and changing variable, the pdf of  $\rho$  in (1) with Rice factor  $K = (m_t^2 + m_q^2)/(2\sigma^2)$  can be obtained as

$$f(\rho) = \left(\frac{Nr}{\bar{\rho}}\right)^{(NL+1)/2} \left(\frac{\rho}{NLK}\right)^{(NL-1)/2} e^{-NLK - Nr\rho/\bar{\rho}} I_{NL-1}\left(2\sqrt{\frac{rLKN^2}{\bar{\rho}}}\rho\right) \quad (3)$$

$$= \sum_{i=0}^{\infty} (Nr/\bar{\rho})^{NL+i} \rho^{NL+i-1} (NLK)^i e^{-NLK - Nr\rho/\bar{\rho}} / [i!\Gamma(NL+i)], \quad \rho \geq 0 \quad (4)$$

The second equation utilizes

$$I_n(x) = \sum_{u=0}^{\infty} (x/2)^{n+2u} / [u!\Gamma(n+u+1)] \quad (5)$$

where  $\Gamma(\cdot)$  is the Gamma function. The cumulative distribution function (cdf) of  $\rho$  can be expressed in terms of the generalized Marcum  $Q$ -function [22] as

$$F(\rho) = 1 - Q_{NL}(\sqrt{2NLK}, \sqrt{2rN\rho/\bar{\rho}}) \quad (6)$$

where the generalized Marcum  $Q$ -function,  $Q_m(\cdot, \cdot)$ , is defined as

$$Q_k(\mu, \nu) = \mu^{1-k} \int_{\nu}^{\infty} x^k I_{k-1}(\mu x) \exp[-(x^2 + \mu^2)/2] dx \quad (7)$$

Substituting (5) into (7) gives

$$Q_k(\mu, \nu) = \sum_{i=0}^{\infty} \mu^{2i} 2^{-i} e^{-\mu^2/2} \Gamma(k+i, \nu^2/2) / [i!\Gamma(k+i)] \quad (8)$$

With (4), and setting  $K=0$ , we can obtain the following pdf as

$$f(\rho) = (Nr\rho/\bar{\rho})^{NL} \exp(-Nr\rho/\bar{\rho}) / [\rho \cdot \Gamma(NL)], \quad \rho \geq 0 \quad (9)$$

Eq.(9) is the pdf of  $\rho$  in Rayleigh fading channel [13]. The above result indicates that (9) is a special case of (3), i.e., the derived pdf of  $\rho$  in Ricean fading includes Rayleigh fading as a special case.

### 3. Adaptive Modulation with Space-time Block Coding

In this section, we will give an adaptive modulation scheme for MIMO system with space-time block coding, and square MQAM is considered for modulation in the system due to its inherent SE and ease of implementation. For discrete-rate MQAM, the constellation size  $M_j$  is defined as  $\{M_0=0, M_1=2, \text{ and } M_j=2^{2^{j-2}}, j=2, \dots, J\}$ , where  $M_0$  means no data transmission. The effective SNR range is divided into  $J+1$  regions with switching thresholds  $\{\rho_0, \rho_1, \dots, \rho_J, \rho_{J+1}; \rho_0=0, \rho_{J+1}=\infty\}$ . The MQAM of modulation size  $M_j$  is used for modulation when  $\rho$  falls in the  $j$ -th region  $[\rho_j, \rho_{j+1})$ . Consequently, the data rate is  $b_j=\log_2 M_j$  bits/symbol with  $b_0=0$ .

According to [22] and [24], the BER of MQAM with Gray code over additive white Gaussian noise (AWGN) channel for the received SNR  $\rho$  and constellation size  $M_j$  is approximately given by

$$BER_\rho \approx 0.2 \exp\{-1.5\rho / (M_j - 1)\} \quad (10)$$

The exact BER is bounded from above by this approximated BER for  $M_j \geq 4$ . But when  $M_j=2$ , which is often used at low SNR, (10) is no longer an upper bound for the exact BER. For this reason, we employ the following very tight BER expression [22][24][25] for switching thresholds

$$BER_\rho \cong a_j \text{erfc}\{\sqrt{g_j \rho}\}, \quad a_1=0.5, g_1=1; a_j = \frac{2}{b_j} \left(1 - \frac{1}{\sqrt{M_j}}\right), g_j = \frac{1.5}{M_j - 1}, j=2, \dots, J. \quad (11)$$

where  $\text{erfc}\{\cdot\}$  denotes the complementary error function.

Let the target BER be  $BER_0$ . To meet this target, (11) can be rewritten as

$$a_j \text{erfc}\{\sqrt{g_j \rho}\} = BER_0, \quad \rho \in [\rho_j, \rho_{j+1}), j=1, \dots, J. \quad (12)$$

Using (12), the switching thresholds can be obtained as

$$\rho_j = [\text{erfc}^{-1}\{BER_0 / a_j\}]^2 / g_j, \quad j=1, \dots, J. \quad (13)$$

where  $\text{erfc}^{-1}\{\cdot\}$  denotes the inverse complementary error function which can be evaluated by table look-up. When the switching thresholds are chosen according to (13), the system will operate with a BER below target  $BER_0$ , as will be confirmed in the following numerical results in Section 5.

Based on the switching thresholds described in (13), and using (6), we can calculate the probability that the SNR,  $\rho$ , falls in the  $j$ -th region  $[\rho_j, \rho_{j+1})$ , denoted by  $P_j$ , as

$$P_j = \int_{\rho_j}^{\rho_{j+1}} f(\rho) d\rho = Q_{NL}(\sqrt{2NLK}, \sqrt{2Nr\rho_j / \bar{\rho}}) - Q_{NL}(\sqrt{2NLK}, \sqrt{2Nr\rho_{j+1} / \bar{\rho}}) \quad (14)$$

For discrete-rate adaptive scheme, the SE is defined as the average transmission rate. So the SE of the adaptive MQAM scheme with STBC can be expressed as

$$\eta = \sum_{j=1}^J rb_j \int_{\rho_j}^{\rho_{j+1}} f(\rho) d\rho = \sum_{j=1}^J rb_j P_j \quad (15)$$

Using (14), the average SE as defined in (15) can be expressed as

$$\eta = r \sum_{j=1}^J b_j [Q_{NL}(\sqrt{2NLK}, \sqrt{2Nr\rho_j / \bar{\rho}}) - Q_{NL}(\sqrt{2NLK}, \sqrt{2Nr\rho_{j+1} / \bar{\rho}})] \quad (16)$$

By setting  $K=0$ , and using (8), (16) can be reduced to

$$\eta = r \sum_{j=1}^J b_j [\Gamma(NL, Nr\rho_j / \bar{\rho}) - \Gamma(NL, Nr\rho_{j+1} / \bar{\rho})] / \Gamma(NL) \quad (17)$$

This is a closed-form expression of the system SE in Rayleigh fading channel. Thus, (17) is a special case of (16).

We define average BER for MIMO systems with AM as

$$\begin{aligned} \overline{BER} &= \sum_{j=1}^J rb_j \overline{BER}_j / [\sum_{j=1}^J rb_j \int_{\rho_j}^{\rho_{j+1}} f(\rho) d\rho] \\ &= \sum_{j=1}^J b_j \int_{\rho_j}^{\rho_{j+1}} \overline{BER}_j f(\rho) d\rho / \sum_{j=1}^J b_j P_j \end{aligned} \quad (18)$$

where  $\overline{BER}_j$  is the bit error rate of MQAM of modulation size  $M_j$  with Gray coding. An accurate expression of  $\overline{BER}_j$  is given by [25][26]

$$\overline{BER}_j = \sum_i \zeta_{ji} \operatorname{erfc} \left\{ \sqrt{\kappa_{ji} \rho} \right\} \quad (19)$$

where  $\zeta_{ji}$  and  $\kappa_{ji}$  are constants which depend on the modulation size  $M_j$  [25][26].

Substituting (13) and (18) into (16), the average BER can be evaluated as

$$\overline{BER} = \frac{\sum_{j=1}^J b_j \sum_i \zeta_{ji} \int_{\rho_j}^{\rho_{j+1}} \operatorname{erfc} \left\{ \sqrt{\kappa_{ji} \rho} \right\} \left( \frac{Nr}{\bar{\rho}} \right)^{(NL+1)/2} \left( \frac{\rho}{NLK} \right)^{(NL-1)/2} e^{-NLK - Nr\rho/\bar{\rho}} I_{NL-1} \left( 2\sqrt{\frac{rLKN^2}{\bar{\rho}} \rho} \right) d\rho}{\sum_{j=1}^J b_j [Q_{NL}(\sqrt{2NLK}, \sqrt{2Nr\rho_j / \bar{\rho}}) - Q_{NL}(\sqrt{2NLK}, \sqrt{2Nr\rho_{j+1} / \bar{\rho}})]} \quad (20)$$

The above equation is an accurate expression of the average BER of the AM system with STBC (referred to as AM-STBC). This theoretical formula will be shown to be in good agreement with simulation results. By setting  $K=0$ , and using (8), we can obtain the theoretical average BER of the AM-STBC in Rayleigh fading channel as follows:

$$\overline{BER} = \frac{\sum_{j=1}^J b_j \sum_i \zeta_{ji} \int_{\rho_j}^{\rho_{j+1}} \text{erfc}\{\sqrt{\kappa_{ji}\rho}\} (Nr / \bar{\rho})^{NL} \rho^{NL-1} \exp(-Nr\rho / \bar{\rho}) d\rho}{\sum_{j=1}^J b_j [\Gamma(NL, Nr\rho_j / \bar{\rho}) - \Gamma(NL, Nr\rho_{j+1} / \bar{\rho})]} \quad (21)$$

Considering that (20) needs numerical integral, in the following, we will give a closed-form approximate expression of average BER by means of mathematical manipulation.

According to Ref.[27], the complementary error function  $\text{erfc}(x)$  can be approximately expressed as  $\text{erfc}(x) \cong \sum_{m=1}^2 u_m \exp(-v_m x^2)$ , where  $\{u_m\}=\{1/6, 1/2\}$ ,  $\{v_m\}=\{1, 4/3\}$ . By using this approximation and the generalized Marcum  $Q$ -function (7), the numerator in (20) can be given by

$$\begin{aligned} \text{I} &\cong \sum_{j=1}^J b_j \sum_i \zeta_{ji} \sum_{m=1}^2 u_m \left(\frac{Nr}{Nr + v_m \kappa_{ji} \bar{\rho}}\right)^{NL} \exp\left(-\frac{NLKv_m \kappa_{ji} \bar{\rho}}{Nr + v_m \kappa_{ji} \bar{\rho}}\right) \\ &\times [Q_{NL}\left(\sqrt{\frac{2rLKN^2}{Nr + v_m \kappa_{ji} \bar{\rho}}}, \sqrt{2\left(\frac{Nr}{\bar{\rho}} + v_m \kappa_{ji}\right)\rho_j}\right) - Q_{NL}\left(\sqrt{\frac{2rLKN^2}{Nr + v_m \kappa_{ji} \bar{\rho}}}, \sqrt{2\left(\frac{Nr}{\bar{\rho}} + v_m \kappa_{ji}\right)\rho_{j+1}}\right)] \end{aligned} \quad (22)$$

Substituting (22) and (16) into (20) yields

$$\begin{aligned} \overline{BER} &\cong \frac{r}{\eta} \sum_{j=1}^J b_j \sum_i \zeta_{ji} \sum_{m=1}^2 u_m \left(\frac{Nr}{Nr + v_m \kappa_{ji} \bar{\rho}}\right)^{NL} \exp\left(-\frac{NLKv_m \kappa_{ji} \bar{\rho}}{Nr + v_m \kappa_{ji} \bar{\rho}}\right) \\ &\times [Q_{NL}\left(\sqrt{\frac{2rLKN^2}{Nr + v_m \kappa_{ji} \bar{\rho}}}, \sqrt{2\left(\frac{Nr}{\bar{\rho}} + v_m \kappa_{ji}\right)\rho_j}\right) - Q_{NL}\left(\sqrt{\frac{2rLKN^2}{Nr + v_m \kappa_{ji} \bar{\rho}}}, \sqrt{2\left(\frac{Nr}{\bar{\rho}} + v_m \kappa_{ji}\right)\rho_{j+1}}\right)] \end{aligned} \quad (23)$$

where  $\eta$  is defined as (16). Eq.(23) is a tightly approximate expression of the average BER of the AM-STBC system. It avoids the integration operator of (20), and will obtain the results close to the actual simulation. When Ricean factor  $K$  is equal to 0, (23) will be reduced to the approximate average BER of AM-STBC in Rayleigh fading channel.

#### 4. Performance of Adaptive Modulation with STBC and Imperfect CSI

In the previous section, we analyze the performance of AM-STBC system assuming that the knowledge of CSI is perfectly known at the receiver and transmitter. In practice, the CSI is imperfect due to channel estimation errors. So in this section, we investigate the effect of imperfect CSI on SE and average BER with channel estimation errors modeled as complex Gaussian random variables [6][7] at the receiver. The channel estimation matrix  $\hat{\mathbf{H}}$  is related to the actual channel matrix  $\mathbf{H}$  as

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E} \quad (24)$$

where  $\mathbf{E}$  is estimation error matrix. The entries of  $\hat{\mathbf{H}}$  and  $\mathbf{E}$ ,  $\{\hat{\mathbf{H}}_m\}$  and  $\{\mathbf{E}_m\}$  are modeled as

independent complex Gaussian variables [6] with  $\hat{\mathbf{H}}_{ln} \sim CN(m_l + jm_l, c^2)$  and  $\mathbf{E}_{ln} \sim CN(0, \sigma_e^2)$ , where ‘ $CN$ ’ denotes complex Gaussian distribution,  $\sigma_e^2$  is estimation error and  $c^2=1-\sigma_e^2$ .

Because the actual channel  $\mathbf{H}$  is not available, we use its estimate  $\hat{\mathbf{H}}$  in the space-time decoding at the receiver to perform the signal detection. The corresponding decision variable is written as

$$\hat{d}_p = \varepsilon \|\hat{\mathbf{H}}\|_F^2 d_p + \tilde{n}_p, p=1, \dots, P. \quad (25)$$

where  $\{d_p\}$  are the  $P$  input symbols and  $\tilde{n}_p = h_{ep} + \hat{n}_p$  is the equivalent noise including the channel estimation error and noise. The channel estimation error,  $h_{ep}$ , can be regarded as an uncorrelated complex Gaussian noise [24][6][7] with zero mean and variance  $\varepsilon \|\hat{\mathbf{H}}\|_F^2 \sigma_e^2 \bar{S}$ . The complex noise,  $\hat{n}_p$ , is Gaussian distributed with zero mean and variance  $\varepsilon \|\hat{\mathbf{H}}\|_F^2 \sigma_n^2$ . As a result, the received SNR,  $\hat{\rho}$ , can be written as

$$\hat{\rho} = \frac{\varepsilon^2 \|\hat{\mathbf{H}}\|_F^4 \bar{S} / (\varepsilon Nr)}{\varepsilon \|\hat{\mathbf{H}}\|_F^2 \sigma_e^2 \bar{S} + \varepsilon \|\hat{\mathbf{H}}\|_F^2 \sigma_n^2} = \frac{\bar{\rho} \hat{\beta}}{Nr(\bar{\rho} \sigma_e^2 + 1)} \quad (26)$$

where  $\hat{\beta} = \sum_{n=1}^N \sum_{l=1}^L |\hat{h}_{ln}|^2$  is noncentral chi-square distributed with  $2NL$  degrees of freedom. According to the analysis in [22] and using transformation of random variable, the pdf of  $\hat{\rho}$  can be obtained as

$$f(\hat{\rho}) = \left(\frac{Nr}{\bar{\rho}'}\right)^{(NL+1)/2} \left(\frac{\hat{\rho}}{NLK'}\right)^{(NL-1)/2} \exp\left(-NLK' - \frac{rN\hat{\rho}}{\bar{\rho}'}\right) I_{NL-1}\left(2\sqrt{\frac{rLK'N^2}{\bar{\rho}'}} \hat{\rho}\right), \quad \hat{\rho} \geq 0 \quad (27)$$

where  $\bar{\rho}' = \bar{\rho}(1-\sigma_e^2) / (\bar{\rho}\sigma_e^2 + 1)$  and  $K' = K / (1-\sigma_e^2)$ . For perfect estimation ( $\sigma_e^2=0$ ),  $\bar{\rho}' = \bar{\rho}$ , and  $K' = K$ . Thus, (26) and (27) are reduced to (1) and (3) respectively.

Using (27) and (15), we can obtain a closed-form expression of SE of the AM-STBC with imperfect CSI as follows:

$$\eta = r \sum_{j=1}^J b_j \left[ Q_{NL}\left(\sqrt{2NLK'}, \sqrt{2rN\rho_j / \bar{\rho}'}\right) - Q_{NL}\left(\sqrt{2NLK'}, \sqrt{2rN\rho_{j+1} / \bar{\rho}'}\right) \right] \quad (28)$$

By substituting (28), (27) and (19) into (18), the average BER can be obtained as

$$\overline{BER} = \frac{\sum_{j=1}^J b_j \sum_i \zeta_{ji} \int_{\rho_j}^{\rho_{j+1}} \text{erfc}\left\{\sqrt{\kappa_{ji} \hat{\rho}}\right\} f(\hat{\rho}) d\hat{\rho}}{\sum_{j=1}^J b_j \left[ Q_{NL}\left(\sqrt{2NLK'}, \sqrt{2rN\rho_j / \bar{\rho}'}\right) - Q_{NL}\left(\sqrt{2NLK'}, \sqrt{2rN\rho_{j+1} / \bar{\rho}'}\right) \right]} \quad (29)$$

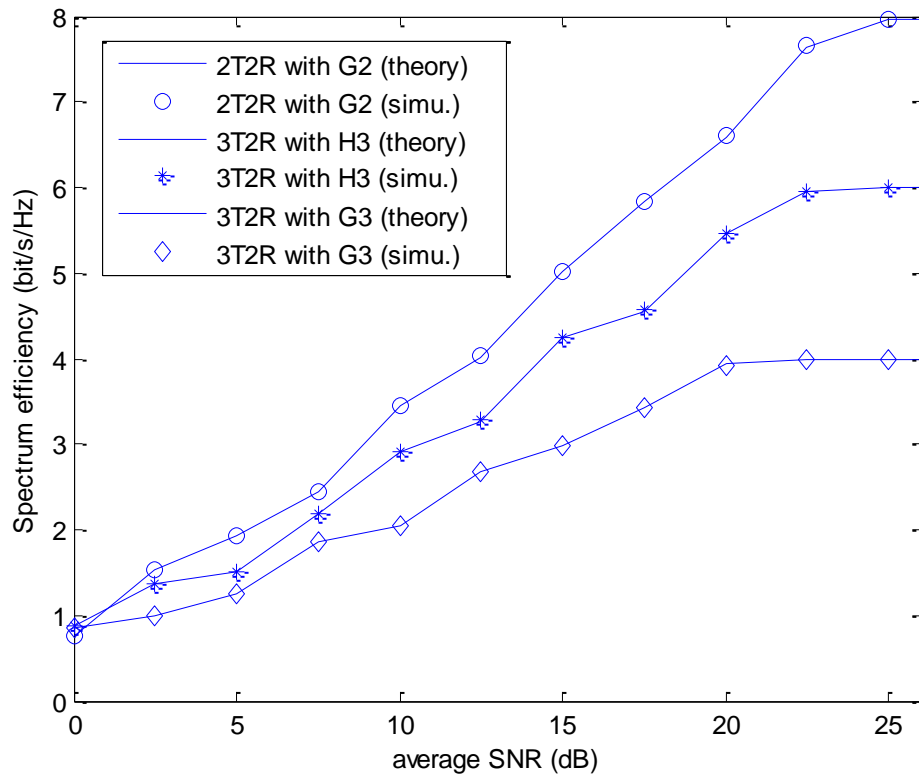
Following the derivation procedure for perfect CSI, an approximate expression of average



BER can be achieved as

$$\begin{aligned} \overline{BER} &\cong \frac{r}{\eta} \sum_{j=1}^J b_j \sum_i \zeta_{ji} \sum_{m=1}^2 u_m \left( \frac{rN}{rN + v_m \kappa_{ji} \bar{\rho}'} \right)^{NL} \exp\left(-\frac{NLK' v_m \kappa_{ji} \bar{\rho}'}{rN + v_m \kappa_{ji} \bar{\rho}'}\right) \\ &\times [Q_{NL}\left(\sqrt{\frac{2rLK'N^2}{rN + v_m \kappa_{ji} \bar{\rho}'}}\right), \sqrt{2\left(\frac{Nr}{\bar{\rho}'} + v_m \kappa_{ji}\right)\rho_j}] - Q_{NL}\left(\sqrt{\frac{2rLK'N^2}{rN + v_m \kappa_{ji} \bar{\rho}'}}\right), \sqrt{2\left(\frac{Nr}{\bar{\rho}'} + v_m \kappa_{ji}\right)\rho_{j+1}}) \end{aligned} \quad (30)$$

where  $\eta$  is defined as (28). Eq.(30) is a closed-form expression of the average BER of the AM-STBC system for imperfect CSI. When the CSI is perfectly known with  $\sigma_e^2=0$ , (28), (29) and (30) will be reduced to (16), (20) and (23), respectively. These results show that the SE and average BER for perfect CSI are special cases of those for imperfect CSI.



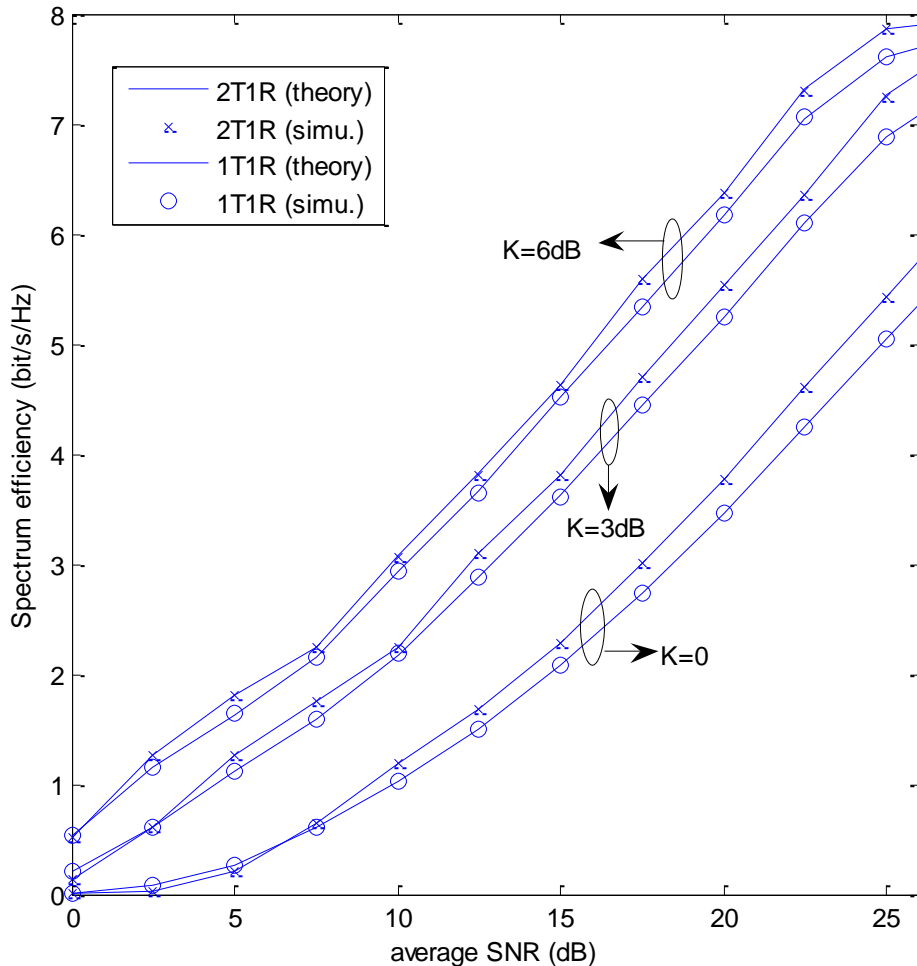
**Fig. 1.** SE of AM-STBC with multiple transmit antennas and two receive antennas for  $K=3$ dB under perfect CSI.

## 5. Simulation Results and Analysis

In this section, we use the derived theoretical formulae and computer simulation to evaluate the spectrum efficiency and average bit error rate of the AM-STBC over Ricean fading channel. In simulation, the channel is assumed to be quasi-static flat fading. Gray code is employed to map the data bits to MQAM constellations. The set of MQAM constellations is

$\{M_j\}_{j=0,1,\dots,5} = \{0, 2, 4, 16, 64, 256\}$ . The target BER is set as  $BER_0=0.001$ . Different space-time block codes, such as  $G_2$ ,  $G_3$ ,  $H_3$ ,  $G_4$ , and  $H_4$  [9] are adopted for evaluation and comparison. In the following figures,  $xT yR$  denotes a MIMO system with  $x$  transmit antennas and  $y$  receive antennas.

In Fig. 1, we plot the theoretical SE and simulation results of the AM-STBC systems with different transmit antennas and two receive antennas under perfect CSI, where  $G_2$ ,  $G_3$  and  $H_3$  code are used for STBC, and the Ricean factor  $K$  is equal to 3dB. The theoretical SE is computed by (16). As shown in Fig. 1, AM is able to increase SE with SNR. The 2T2R system of using  $G_2$  code provides larger SE than the 3T2R system of using half-rate  $G_3$  code or 3/4-rate  $H_3$  code because the  $G_2$  is a full rate code. Due to the same reason, the system using  $H_3$  code has larger SE than that of using  $G_3$  code. Moreover, the theoretical analysis of average SE is in agreement with the corresponding simulation. The above results show that the derived SE expression under perfect CSI is effective.



**Fig. 2.** SE of AM-STBC with different transmit antennas and one receive antenna for  $K=0$ ,  $K=3$ dB and 6dB under perfect CSI.

In Fig.2, we give the theoretical SE and simulation results of the AM-STBC systems with 1T1R and 2T1R under perfect CSI, where  $G_2$  code is used for space-time coding, and Ricean factor  $K$  is set as to be 3 dB and 6dB. For comparison, Rayleigh fading channel (i.e.,  $K=0$ ) is also considered. The theoretical SE is calculated by (16) and shown in good agreement with the simulation results. It is shown that the SE of AM with  $G_2$  code is higher than that of AM with single transmit antenna because more antennas are used. Moreover, we can see that the bigger the value of  $K$ , the larger the SE is. This is because the fading severity decreases as the Ricean factor  $K$  increases. Besides, the SE in Ricean fading channel is obviously higher than that in Rayleigh fading channel due to the direct-path propagation. The former having better SE than the latter verifies the effectiveness of existence of direct path. By comparing the results of Fig.1 and Fig.2, it is found that the SE of the system can be increased effectively as the number of the receive antenna increases. The above results further verify that the derived SE is valid for performance evaluation.

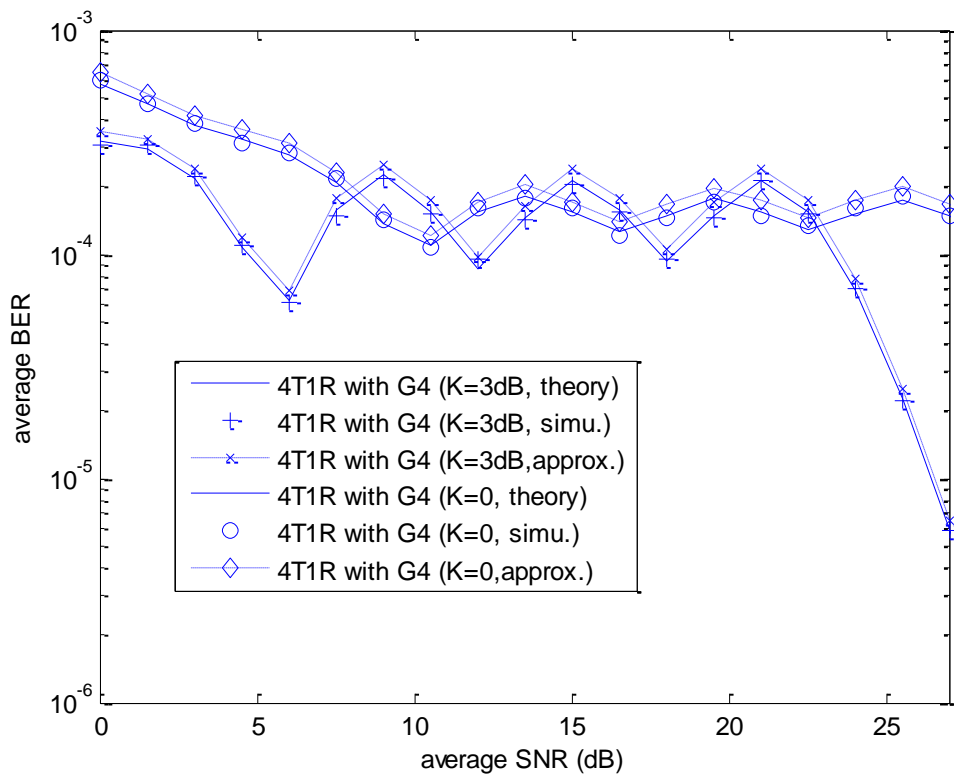
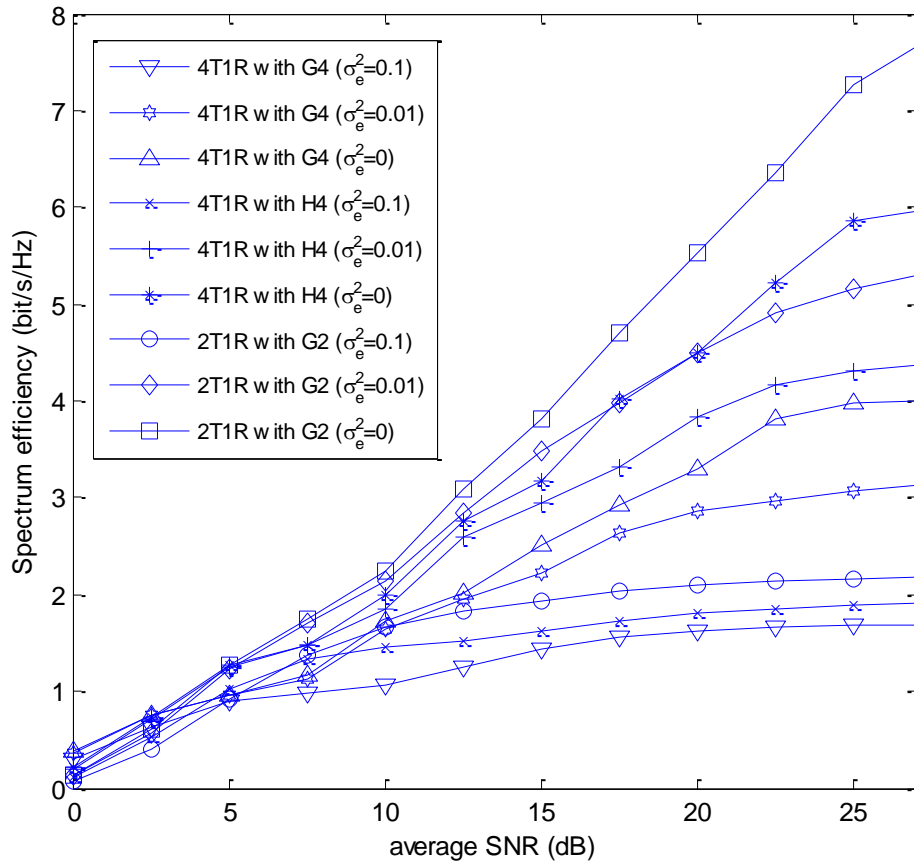


Fig. 3. Average BER of AM-STBC for four transmit antennas and one receive antenna with  $K=3$ dB and  $K=0$  under perfect CSI.

In Fig.3, we plot the theoretical average BER and simulation results of AM-STBC systems with four transmit antennas and one receive antenna for  $K=0$ , 3dB under perfect CSI.  $G_4$  code is employed for STBC. We use (20) and (23) to calculate the theoretical and approximate BER, respectively. From Fig.3, it is observed that the average BER of the AM-STBC systems is always less than the target  $BER_0$  while the SE effectively increases with SNR. This result verifies the effectiveness of the AM-STBC system on SE. Moreover, we can see that the accurate theoretical analysis of (20) is in good agreement with the simulation result, and the derived approximate BER (23) is also close to the simulated values because of the close

approximation. Besides, the average BER in Ricean fading channel is obviously lower than that in Rayleigh fading channel ( $K=0$ ) due to the existence of direct-path. These results show that our derived BER expressions are effective for perfect CSI.



**Fig. 4.** SE of AM-STBC with different transmit antennas and one receive antenna for different estimation errors under imperfect CSI. ( $K=3\text{dB}$ )

In **Fig.4** and **Fig.5**, we respectively evaluate the SE and average BER of AM-STBC system for imperfect CSI. In **Fig.4**, we plot the SE of AM-STBC with imperfect CSI for different transmit antennas and one receive antennas, where  $G_2$ ,  $G_4$  and  $H_4$  code are used for STBC. The Rice factor  $K=3\text{dB}$ . Under imperfect CSI, the estimation error  $\sigma_e^2=0, 0.01, 0.1$  is considered for evaluation. **Fig.4** shows that the AM scheme is capable to increase SE with SNR for imperfect CSI, but the increment of SE will decrease for large SNR due to the influence of imperfect CSI. Under the same CSI, the 2T1R system provides larger SE than the 4T1R systems because  $G_2$  code is a full rate code while  $G_4$  and  $H_4$  codes are having code rate less than one. Moreover, the SE of the 4T1R system with  $H_4$  code is higher than that with  $G_4$  code because the former has higher code rate. From this figure, it is also observed that the smaller the estimation error, the bigger the SE is. The SE for imperfect CSI is obviously lower than that for perfect CSI due to the estimation error. The above results show that the derived SE is also valid for performance evaluation under imperfect CSI.

In **Fig.5**, we plot the average BER of AM-STBC under imperfect CSI for three transmit

antennas and different receive antennas, where  $H_3$  is employed for STBC and the Ricean factor  $K=3\text{dB}$ . We set the estimation error variance  $\sigma_e^2$  to be 0 and 0.1, respectively. It is found that the behaviors of the average BER under different system configurations are similar, and the BER performance with imperfect CSI is obviously worse than that with perfect CSI due to the estimation error. In these systems, the BER performance of 3T1R system is worse than the 3T2R system. This is because the latter employs multiple receive antennas and has greater diversity than the former. Moreover, the average BERs of the system for different estimation errors are all maintained less than the target BER for different SNRs. It means that the AM under imperfect CSI is successful in increasing SE while the target BER is maintained. Besides, the average BER has ripples phenomenon (which can also be seen in Fig.3) due to the use of the individual discrete constellations. These results indicate that the derived BER expressions under imperfect CSI are also effective.

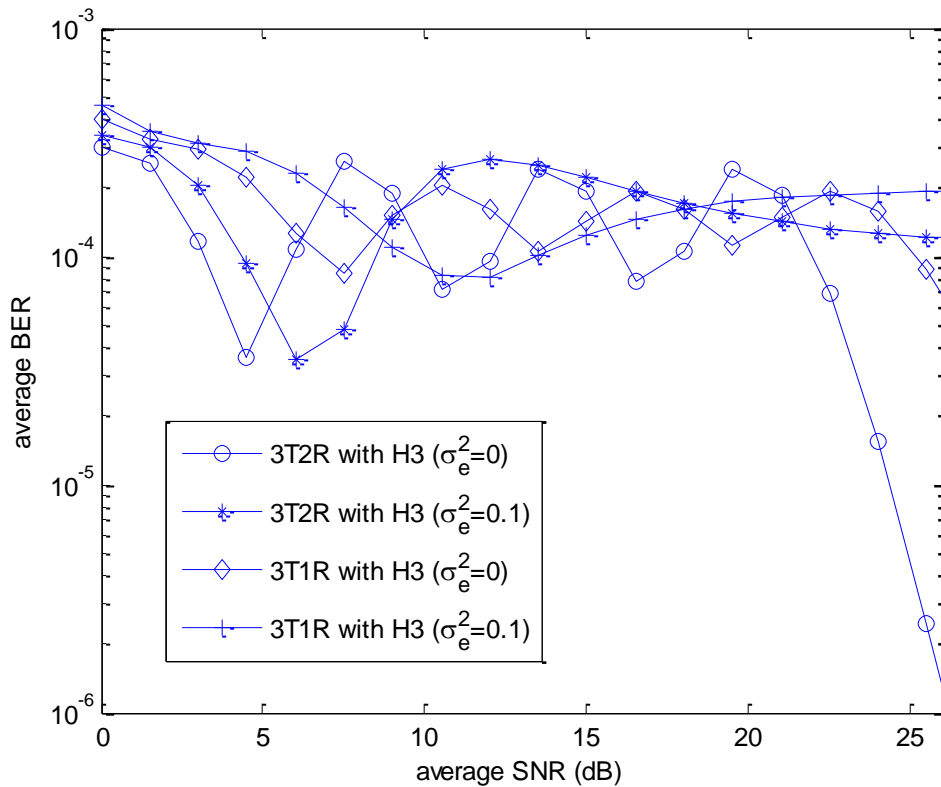


Fig. 5. Average BER of AM-STBC with three transmit antennas and different receive antennas for different estimation errors under imperfect CSI. ( $K=3\text{dB}$ )

## 6. Conclusion

The performance of AM-MIMO system with space-time block code over Ricean fading channels for both perfect and imperfect CSI is presented. By using a very tight BER formula, the switching thresholds subject to target BER constraint are obtained for AM. With the obtained switching thresholds, the system can satisfy the target BER requirement. Based on imperfect CSI, the probability density function of the effective SNR is derived. With these results, and utilizing the generalized Marcum  $Q$ -function, the accurate theoretical expressions

for SE and BER are derived. As a result, a closed-form SE expression is obtained. By using the approximation of the complementary error function, a tightly closed-form approximate expression of BER is also derived to simplify the numerical calculation of the accurate BER. Using the derived expressions, the system performances with perfect and imperfect CSI in Ricean channel can be evaluated effectively. Moreover, some existing theoretical expressions in Rayleigh channel are included. Simulation results verify the validity of the derived theoretical expressions, that is, the theoretical values are close to the corresponding simulated values. The results show that the AM-STBC in Ricean fading channel has higher SE than that in Rayleigh fading channel, and the performance of AM-STBC with imperfect CSI is worse than that with perfect CSI due to the imperfection of channel information.

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