Design and Optimization for Distributed Compress-and-Forward System based on Multi-Relay Network

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Received Septebmer 1, 2018; revised October 22, 2018; accepted December 7, 2018; published May 31, 2019

Abstract

A novel distributed compress-and-forward (CF) system based on multi-relay network is presented. In this system, as the direct link between the source and destination is invalid due to some reasons, such as the limited power, special working environment, or even economic factors, relays are employed to receive analog signals and carry on distributed compressed encoding. Subsequently, the digital signals are transmitted to the destination via wireless channel. Moreover, a theoretical analysis for the system is provided by utilizing the Chief Executive Officer (CEO) theory and Shannon channel capacity theory, and the rate-distortion function as well as the connection between the transmission rate and the channel capacity are constructed. In addition, an optimal signal-to-noise ratio (SNR) -based power allocation method is proposed to maximize the quantization SNR under the limited total power. Simulation result shows that the proposed CF system outperforms the amplify-and-forward (AF) system versus the SNR performance.

Keywords: compress-and-forward, multi-relay, amplify-and-forward, the CEO problem, SNR, power allocation

1. Introduction

In recent years, wireless relay technology has attracted much attention in wireless communications, for it can combat channel fading, promote spectrum utilization and enhance wireless network coverage in the case of bad direct link. Now, it is widely used in various wireless communication systems, such as satellite communication systems, mobile communication systems, etc.

Numerous researches have been addressed for different kinds of wireless relay technology, such as amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF). For AF system, the relays are utilized to amplify and forward signal, whereas the noise is also amplified[1]-[5]. For DF system, the relays firstly demodulate and decode the received signal, and then re-encode and transmit it to the destination. However, it will cause error propagation if a decoding error occurs[6]-[8]. For CF system, the compressed signals from relay nodes are jointly decoded at the destination with the direct signal from the source. That is, an extra direct channel is involved between the source and destination [9][10]. In addition, wireless relay technology has also been studied from other aspects, such as outage probability[11][12], relay selection[13]-[15], minimizing the bit error ratio (BER)[16][17] and spatial channel pairing strategy [18]. It should be noted that power allocation has long been playing an important role in wireless relay technology [3][5], [19]-[22], since a better system performance can be achieved by appropriately allocating limited power between the source and relays.

In this paper, a new CF system model based on multi-relay network is presented, which is different from the traditional AF and CF system in two aspects. Firstly, the proposed CF system consists of two parts. One part is the analog sensor network from the source to the relays, which generates and transmits analog signals. The other part is the digital communication network from the relays to the destination, which transmits digital signals to the destination. Specifically, the source can only yield analog signals and the destination can only receive digital signals, where no direct link exists between the source and destination. As a result, the relays are utilized to transform analog signals into digital signals and forward them. Furthermore, quantization SNR criterion is superior to BER criterion to assess the performance of the CF system. It is due to that the exactly recovered digital signals at destination are sampled from the original analog signals, some information will be dropped during sampling and compressing process and the generated distortion is similar to the noise that disturbs analog signals over noisy channel. In addition, an optimized method, which allocates the power among the source and relays under sum SNR constraint, is proposed to maximize quantization SNR at the destination. The proposed CF model suits to various fields, such as industrial monitoring, sewage treatment, home life and so on. The system is different from multi-input multi-output (MIMO) system [23], for it is based on the technology of multi-relay rather than multi-antenna. Meanwhile, the main idea of this paper is to provide a new theoretical framework, and many details of technology are not touched.

The rest of the paper is organized as follows. Section 2 provides the models of traditional AF system and the proposed CF system. In section 3, theoretical analysis about AF and CF systems are conducted. For CF system, theoretical analysis is based on CEO problem and then power allocation method is proposed to maximize the quantization SNR performance. SNR performance comparisons of the proposed CF system and AF system are given in Section 4. Section 5 concludes this paper.

2. System Model

2.1 Model of AF system

Fig. 1 presents the model of traditional AF system. The source node S transmits analog signals $X(t)$ to L relays $R_i (i=1,2,\cdots,L)$ and no direct link exists between source S and destination D. Assume that $X(t)$ follows Gaussian distribution $N(0,\sigma_x^2)$, and the channels between source and relays as well as relays and destination are all additive white Gaussian noise (AWGN) channels. The corrupted analog signals $Y_{ri}(t)$ received at each relay R_i can be represented as

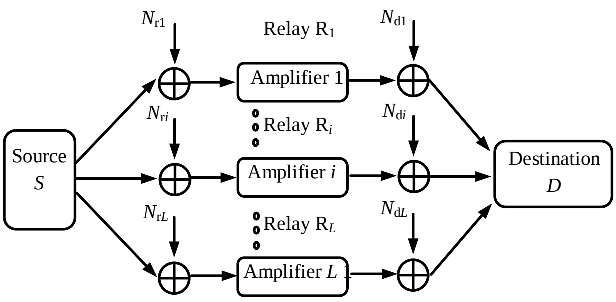

Fig. 1. Model of AF system

$$Y_{ri}(t) = X(t) + N_{ri}(t), \quad (i=1,2,\cdots,L) \tag{1}$$

where $N_{ri}(t)$ is independently and identically distributed (i.i.d.) Gaussian random noise with zero mean and variance σ_i^2, the power of the received signal $Y_{ri}(t)$ is σ_x^2 for channel fading is not taken into account. SNR γ_{ri} of signal $Y_{ri}(t)$ in the ith relay is

$$\gamma_{ri} = \frac{\sigma_x^2}{\sigma_i^2}, \quad (i=1,2,\cdots,L) \tag{2}$$

Then the signal $Y_{ri}(t)$ is amplified in relay R_i and forwarded to destination D with power P_{ri}, where

$$P_{ri} = \beta^2(\sigma_x^2 + \sigma_i^2), \quad (i=1,2,\cdots,L) \tag{3}$$

β is magnification factor of power. If the channel fading of relay-to-destination link is not considered, the signal that destination received from relay R_i is

$$Y_{di}(t) = \beta Y_{ri}(t) + N_{di}(t), \quad (i=1,2,\cdots,L) \tag{4}$$

where $N_{di}(t)$ is i.i.d. Gaussian random noise that follows $N(0,\sigma_i^2)$. SNR of signal $Y_{di}(t)$ in the destination is

$$\gamma_{di} = \frac{P_{ri}}{\sigma_i^2}, \quad (i=1,2,\cdots,L) \tag{5}$$

When the number of relay nodes L is more than 1, each source-relay-destination sublink is similar to a AF system with single one relay. Assume that the signal power P_{ri} of each sublink is equal to the others, the received signal at the destination is

$$Y_d(t) = \frac{1}{L}\sum_{i=1}^{L}\left[\beta Y_{ri}(t) + N_{di}(t)\right] = \frac{1}{L}\sum_{i=1}^{L}\left[\beta\left(X(t) + N_{ri}(t)\right) + N_{di}(t)\right] \quad (6)$$

and the SNR γ_A of the signal $Y_d(t)$ is

$$\gamma_A = \frac{L^2\beta^2\sigma_x^2}{L\sigma_i^2 + L\beta^2\sigma_i^2} = \frac{L\beta^2\sigma_x^2}{\left(1+\beta^2\right)\sigma_i^2} = \frac{L\sigma_x^2 P_{ri}}{\left(\sigma_x^2 + P_{ri} + \sigma_i^2\right)\sigma_i^2} \quad (7)$$

2.2 Model of the Proposed Distributed CF System

The model of the proposed distributed CF system based on multi-relay network is depicted in **Fig. 2**. In the CF system, no direct link exists between source and destination. The source yields and transmits analog signals, while the destination can only receive digital signals. Encoder is assembled at each relay node to encode separately the corrupted analog signals, and subsequently the encoded digital signals are transmitted to the destination D through the channel with additive white Gaussian noise. The destination receives and jointly decodes the digital signals from all relays and yields the estimation of the original analog signals. Until now, the communication between source and destination is completed.

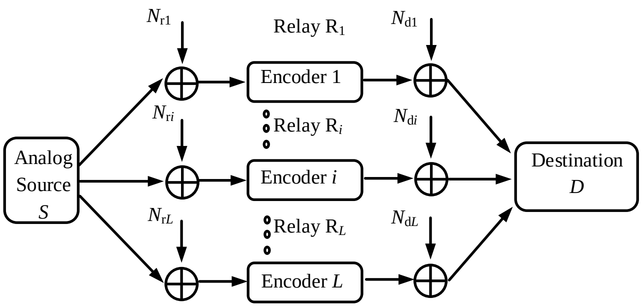

Fig. 2. Model of distributed CF system

We also assume that the analog signals $X(t)$ transmitted from source follows the Gaussian distribution $N(0,\sigma_x^2)$, the channels between source and relays as well as relays and destination are all AWGN channels, the noises in relays and destination are i.i.d. noises and follow the Gaussian distribution $N(0,\sigma_i^2)$. Similar to the AF system mentioned above, the received analog signals $Y_{ri}(t)$ in each relay R_i and its SNR are

$$Y_{ri}(t) = X(t) + N_{ri}(t), \quad (i = 1,2,\cdots,L)$$
$$\gamma_{ri} = \frac{\sigma_x^2}{\sigma_i^2}, \quad (i = 1,2,\cdots,L) \quad (8)$$

Relay R_i receives the analog signals $Y_{ri}(t)$ and transforms it into digital signals $Y'_{ri}(t)$. Assume that digital signals $Y'_{ri}(t)$ is transmitted to the destination with power P_{ri}, neglecting the channel fading of relay-to-destination link, the received digital signals $Y_{di}(t)$ in the destination and its SNR are

$$Y_{di}(t) = Y'_{ri}(t) + N_{di}(t), \quad (i = 1, 2, \cdots, L)$$
$$\gamma_{di} = \frac{P_{ri}}{\sigma_i^2}, \quad (i = 1, 2, \cdots, L) \tag{9}$$

3. Theoretical Analysis

3.1 Theoretical Analysis of AF System

Denote the sum power of signal $X(t)$ and all signals $Y'_{ri}(t)$ by P

$$\sigma_x^2 + LP_{ri} = P \tag{10}$$

Consider that the average powers σ_i^2 of all noise are identical, we can get

$$\frac{\sigma_x^2}{\sigma_i^2} + L\frac{P_{ri}}{\sigma_i^2} = \frac{P}{\sigma_i^2} \tag{11}$$

$$\gamma_{ri} + L\gamma_{di} = \gamma_T \tag{12}$$

where $\gamma_T = \dfrac{P}{\sigma_i^2}$. Take it into (7)

$$\gamma_A = \frac{L\gamma_{ri}\gamma_{di}}{1 + \gamma_{ri} + \gamma_{di}} = \frac{\gamma_{ri}(\gamma_T - \gamma_{ri})}{1 + \gamma_{ri} + \dfrac{\gamma_T - \gamma_{ri}}{L}} \tag{13}$$

It means that γ_A is function of γ_{ri}. When L = 1, we can get from (13)

$$\gamma_A = \frac{\gamma_{ri}(\gamma_T - \gamma_{ri})}{1 + \gamma_T} \tag{14}$$

Take its derivative versus γ_{ri} as $\dfrac{d\gamma_A}{d\gamma_{ri}} = 0$, we find that the AF system can reach maximum SNR $(\gamma_A)_{\max}$ when

$$\gamma_{ri} = \frac{1}{2}\gamma_T \tag{15}$$

When L is more than 1, we take the derivative of γ_A with respect to γ_{ri} and let $\dfrac{d\gamma_A}{d\gamma_{ri}} = 0$,

$$(L-1)\gamma_{ri}^2 + 2(L + \gamma_T)\gamma_{ri} - \gamma_T(L + \gamma_T) = 0 \tag{16}$$

$$\gamma_{ri} = \frac{\sqrt{(L + \gamma_T)^2 + \gamma_T(L-1)(L + \gamma_T)} - (L + \gamma_T)}{L - 1} \tag{17}$$

Take (17) into (13), the maximum SNR $(\gamma_A)_{\max}$ of AF system in destination can be obtained.

3.2 Theoretical Analysis of the CF System Based on the CEO Problem

In the proposed CF system, the received signals are compressed and encoded at each relay, and then forwarded to the destination. It is an analogy to the CEO problem and the source coding problem.

The CEO problem is a special case of multi-terminal source coding problem which was presented by Toby, Zhang, and Viswanathan[24]. It describes a fact that, if a firm's CEO is interested in reconstructing a data sequence that he cannot observe directly, he deploys a team of L agents to encode their observations with no cooperation with each other. The main aim of the CEO problem is to seek a trade-off between a rate and distortion when L tends to infinity, where the rate refers to total rate that the agents communicate with the CEO, and the distortion is generated from reconstructing information. It characterizes the code rate of L relays which can support a desired fidelity so that the source signal can be accurately recovered at the destination. From the perspective of the analog source and channel noise which follows the Gaussian distribution, the CEO problem verifies that the code rate after distributed compressed coding in relay nodes as well as rate-distortion region follow a certain distortion constraint [25]. In [26], the expression of rate-distortion function versus the quadratic Gaussian source is provided. It shows that, the compressed communication of analog Gaussian source in multi-relay network is similar to the CEO problem. Xu and Wang established a new extremal inequality to formulate a complete characterization for the rate region of the vector Gaussian CEO problem with the trace distortion constraint [27]. According to [27], the rate-distortion function $R(d)$ of the vector Gaussian CEO problem under the constraint of distortion d is similar to the Berger-Tung [28][29] inner bounds $R^{BT}(d)$

$$R(d)=R^{BT}(d)=\min_{(b_1,\cdots,b_L)}\frac{1}{2}\sum_{i=1}^{L}\log\frac{\sigma_i^{-2}}{\left|\sigma_i^{-2}-b_i\right|}+\frac{1}{2}\log\frac{\sigma_X^{-2}+\sum_{j=1}^{L}b_j}{\sigma_X^{-2}}$$

$$st.\begin{cases}\left(\sigma_X^{-2}+\sum_{i=1}^{L}b_i\right)^{-1}\le d \\ \sigma_i^{-2}\ge b_i\ge 0,\ \ (i=1,2,\cdots,L)\end{cases} \tag{18}$$

where $R(d)$ is rate-distortion function and denotes the minimum transmission rate after the distributed compressed coding in each relay, $b_i(i=1,2,\cdots,L)$ is optimized intermediate variable for getting $R(d)$. From (18) and Appendix we can get

$$R_i(d)=\frac{1}{2}\log\frac{L\gamma_{ri}}{1+\gamma-\gamma_D}+\frac{1}{2L}\log\gamma_D \quad (i=1,2,\cdots,L) \tag{19}$$

$$L\gamma_{ri}\ge 1+\sum_{i=1}^{L}\gamma_{ri}-\gamma_D=1+\gamma-\gamma_D \quad (i=1,2,\cdots,L) \tag{20}$$

Where γ_D is the quantization SNR of $X(t)$ after it is encoded at the relays, and $\gamma=\sum_{i=1}^{L}\gamma_{ri}$.

3.3 Joint Design and Formula Optimization

In the CF system, the connections between the relays and the destination can be regarded as digital communication network. If the SNR of the received signal in the destination D is γ_{di}, according to Shannon channel capacity theory, the Gaussian channel capacity C_i can be expressed as

$$C_i = \frac{1}{2}\log(1+\gamma_{di}) \tag{21}$$

The destination can recover the relay information without distortion when the transmission rate $R_i(d)$ of each relay is less than the channel capacity C_i, that is

$$\frac{1}{2}\log\frac{L\gamma_{ri}}{1+\gamma-\gamma_D}+\frac{1}{2L}\log\gamma_D \le \frac{1}{2}\log(1+\gamma_{di}) \quad (i=1,2,\cdots,L) \tag{22}$$

In some wireless communication systems, especially the systems powered by battery rather than information signal [30], the available power is typically limited. It is of significance to optimize the power allocation for improved reliability and extended service life. In this paper, the power allocation among the source and relays is conducted with limited sum SNR constraint. As in some multi-relay networks, especially sensor network, the amount of system information which needs to be transmitted in unit time is constant, and excessive SNR will take more power and resultantly shorten the service life. Furthermore, due to the uncertainty and time-varying property of the channel state or circumstance noise, the power allocation among source and relays has to be changed simultaneously for long service life and stable system performance [31]. Therefore, we can improve the system performance according to the result of the SNR constraint method which allocates the limited power among the source and relays more reasonably.

Meanwhile, the digital signals that are decoded correctly at the destination D come from source analog signals, and some informations are discarded during sampling and compressing. It means that the system performance can be assessed by the quantization SNR γ_D. Consider the sum of SNR of the received analog signals $Y_{ri}(t)$ in each relay R_i and SNR of the received digital signals $Y_{di}(t)$ in the destination is limited, the optimization problem to resolve a high quantization SNR γ_D can be formulated as

$$\begin{aligned}&\max : \gamma_D\\ &s.t.\begin{cases}\frac{1}{2}\log\frac{L\gamma_{ri}}{1+\gamma-\gamma_D}+\frac{1}{2L}\log\gamma_D \le \frac{1}{2}\log(1+\gamma_{di})\\ \frac{1}{L}\sum_{i=1}^{L}\gamma_{ri}+\sum_{i=1}^{L}\gamma_{di}=\gamma_T\\ L\gamma_{ri}\ge 1+\gamma-\gamma_D\\ \gamma_{ri},\gamma_{di}\ge 0\end{cases}\end{aligned} \tag{23}$$

where γ_T is the sum SNR constrain of the CF system, it means that the sum of SNR γ_{ri} and γ_{di} is limited, where $i=1,2,\cdots,L$. For the state of the channels between the source and relays, as well as the quality of the received signal at each relay are different, γ_{ri} is unique and the ratio is assumed as $\gamma_{r1}:\gamma_{r2}:\cdots:\gamma_{rL}=a_1:a_2:\cdots:a_L$, where $\sum_{i=1}^{L}a_i=1$. Meanwhile, the system achieves optimal performance when the channel capacity is fully utilized. As a result, the first constraint can take the equality and the optimization problem can be reconstructed as

$$\max : \gamma_D$$
$$s.t. \begin{cases} L^2(\gamma_T + L - s)\gamma_D^{1/L} - s\left[1 - \gamma_D + L(\gamma_T + L - s)\right] = 0 \\ \gamma_D \ge 1 + L(\gamma_T + L - s) - L^2(\gamma_T + L - s)a_m \\ \dfrac{1}{a_m} \le s \le \gamma_T + L \end{cases} \tag{24}$$

where s is a parameter and

$$s = \frac{1+\gamma_{di}}{a_i}$$
$$a_m = \min_{i \in N} a_i, \quad (N = 1, 2, \cdots, L) \tag{25}$$

From (24) we can see that, γ_D will increase with γ_T.

4. Experimental Classification Results and Analysis

In this section, we propose an iterative algorithm to allocate the corresponding SNR among source and relays to obtain the optimal quantization SNR γ_D. The detailed steps (see **Fig. 3**) are shown as follows:

i. Determine whether the interval $s \in \left[a_m^{-1}, \gamma_T + L\right]$ is empty. If it is empty, the process jumps to step *iii*, or turns to step *ii*.

ii. By resolving the derivative of γ_D with respect to s, the maximum of γ_D and the corresponding s, L can be obtained. Subsequently, determine whether $\{\gamma_D, s, L\}$ can satisfy the second and third condition of (24). If not, jumps to step *iii*; otherwise, saves $\{\gamma_D, s, L\}$ and jumps to step *iv*.

iii. Remove the relay node R_m and increases a_m, let $L = L - 1$ and then returns to step *i*.

iv. Outputs the optimal allocation solution $\{\gamma_D, s, L\}$.

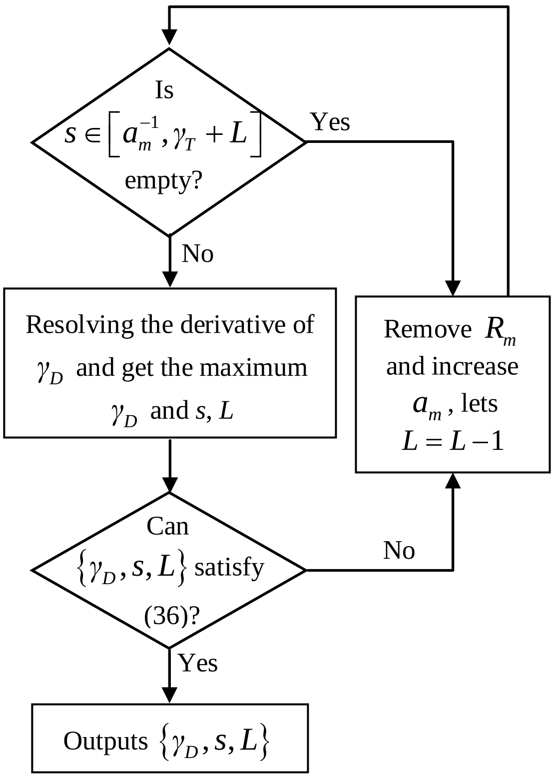

Fig. 3. Flow chart of the simulation

Numerical simulation results are given in this part to validate the effectiveness of the optimized distributed CF system. As each relay works under different conditions, it will provide different SNR with the same total power. In this paper, we assume that there are two cases involved, one is that the SNR of the signal in each relay is equal to others, and the other is that the SNRs are different. To reduce the complexity, the SNR ratios among relays of the latter case are assumed as $a_1 : a_2 : \cdots : a_L = 1 : 2 : \cdots : L$. **Fig. 4** depicts the comparison of SNR performance for the two cases. For simplicity, the coding scheme has not been take into account in this paper.

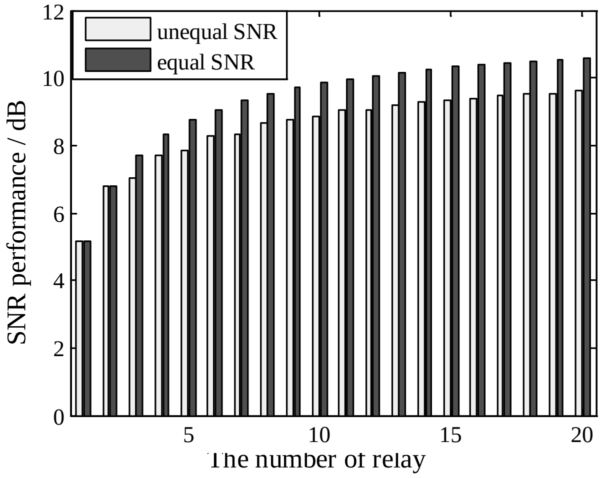

Fig. 4. SNR performance comparison when SNR of each relay is equal or different to the others

It is illustrated in **Fig. 4** that the SNR performance of the two cases is almost the same when the number of relays is less than 3, while the latter one is about 1dB higher than the former one when the number of relay is in the range of 3 and 20. That is, the SNR performance of the proposed system with similar SNR allocation is superior to that of the system with different SNR.

Subsequently, we give the comparison of SNR performance between the distributed CF system and the AF system mentioned above, where the SNR of signal in each relay is equal to others. **Fig. 5** provides the SNR performance of the two systems with the same relay number and sum SNR constraint. It is obviously shown that the SNR performance of the two systems increase with the number of relay, where the sum SNR γ_T is set to 10dB and 20dB. This is due to that, the increase of the number of relay nodes takes more transmission gain. Specifically, the SNR performance of the distributed CF system outperforms that of AF system. For example, when the number of relay is 5 and sum SNR constraint γ_T is 20dB, the SNR performance of the CF system is about 6.5dB higher than that of the AF system, and it reaches 9.5dB when the number of relay is 15. The reason is that, the increase of the number of relay nodes also takes more noise, and the anti-interferece ability of CF system is more powerful than that of AF system.

Fig. 6 describes the plots of SNR performance of the two systems versus sum SNR constraint. We can see that, the SNR performance of the two systems is improved with the increase of sum SNR γ_T, where the number of relay is 5 and 10. Meanwhile, the SNR performance of the distributed CF system outperforms that of AF system with the same conditions. When sum SNR γ_T is increasing from 0dB to about 8dB, the differece of the SNR performances between the two systems will witness a reduction of 5dB. However, the difference will increase when sum SNR γ_T is beyond 8dB.

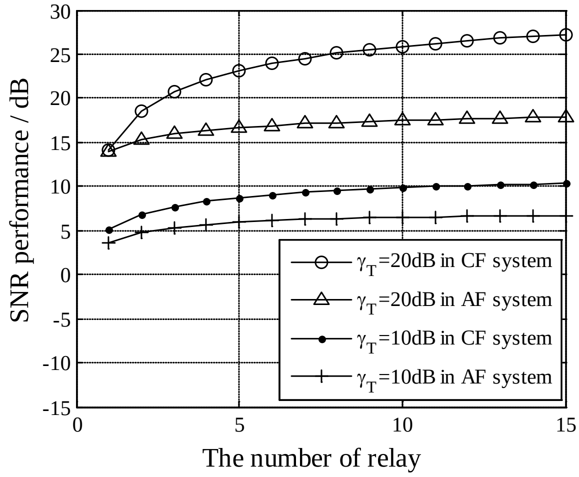

Fig. 5. SNR performance comparison in different relay numbers

It is obviously shown that, with the increase of sum SNR or the number of relay, the SNR performances of AF and CF sytem will be improved at different degree, and the latter one grows faster. On the other hand, SNR performance of the distributed CF system outperforms that of AF system, when the condition is the same. The main reason is that, in AF system, the signals are simply amplified in relays and the noises are amplified at the same time, what the destination received are the noise seriously disturbed signals. While in CF system, the source analog signals are transformed into digital signals in relays, and forwarded to destination with the transmission rate not bigger than channel capacity, this decrease the effection of the channel noise. That is to say, compared to AF system, the distributed CF system can promote transmission performance of system and reliability of information transmission; or in other words, it can decrease transmit power and extend system service life while ensuring system performance.

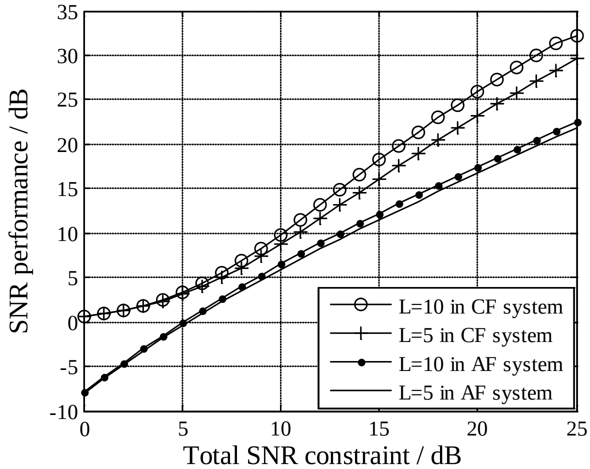

Fig. 6. SNR performance comparison in different SNR constraint

5. Conclusion

In this paper, we propose a distributed CF system based on multi-relay network. Meanwhile, from its theoretical analysis and the CEO problem, we establish an optimization method aiming to attain the maximum quantization SNR at the destination by allocating the power among the source and relays under sum SNR constraint. Specifically, the extensive simulation result verifies that the performance of the CF system outperforms that of different SNR when SNR of each relay is equvalent to others. Furthermore, the distributed CF system outperforms the AF system versus system performance with the same conditions. In practical applications, the proposed optimization method is still of significance even though the relay has a specific coding scheme and rate without consideration in this paper.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (no. 61471192).

Appendix

According to (18) and base on Lagrange multipliers, the Lagrange function can be written as

$$f(b_1,\cdots,b_L)=\frac{1}{2}\sum_{i=1}^{L}\log\frac{\sigma_i^{-2}}{\left|\sigma_i^{-2}-b_i\right|}+\frac{1}{2}\log\frac{\sigma_x^{-2}+\sum_{i=1}^{L}b_i}{\sigma_x^{-2}}+\lambda\left(\sigma_x^{-2}+\sum_{i=1}^{L}b_i-d^{-1}\right)$$
$$=-\frac{1}{2}\sum_{i=1}^{L}\log\left(\sigma_i^{-2}-b_i\right)+\frac{1}{2}\log\left(\sigma_x^{-2}+\sum_{i=1}^{L}b_i\right)+\lambda\sum_{i=1}^{L}b_i \tag{26}$$

where λ is the Lagrange multiplier, f is a function of $b_i(i=1,2,\cdots,L)$. Take the partial derivative of f versus b_i and let $\frac{df}{db_i}=0$, we can get

$$\frac{\partial f(b_1,\cdots,b_L)}{\partial b_i}=-\frac{1}{2}\frac{-1}{\left(\sigma_i^{-2}-b_i\right)}+\frac{1}{2}\frac{1}{\sigma_x^{-2}+\sum_{i=1}^{L}b_i}+\lambda=0 \tag{27}$$

When $R(d)$ reaches the minimum compress rate, it will be the maximum distortion. From (18)

$$\left(\sigma_x^{-2}+\sum_{i=1}^{L}b_i\right)^{-1}=d \tag{28}$$

Take (28) into (27), we can obtain

$$\frac{1}{\sigma_i^{-2}-b_i}+d+2\lambda=0 \tag{29}$$

According to (29)

$$b_i=\frac{1}{d+2\lambda}+\sigma_i^{-2} \tag{30}$$

$$\frac{1}{b_i-\sigma_i^{-2}}=d+2\lambda \tag{31}$$

When the number of relay node L is more than 1, we can get from the sum of (30)

$$\sum_{i=1}^{L} b_i = \frac{L}{\mathrm{d}+2\lambda} + \sum_{i=1}^{L} \sigma_i^{-2} \tag{32}$$

Take (32) into (28)

$$d^{-1} - \sigma_x^{-2} = \frac{L}{d+2\lambda} + \sum_{i=1}^{L} \sigma_i^{-2} \tag{33}$$

According to (33), we can obtain

$$d + 2\lambda = \frac{L}{d^{-1} - \sigma_x^{-2} - \sum_{i=1}^{L} \sigma_i^{-2}} \tag{34}$$

Take (34) into (31),

$$\frac{1}{b_i - \sigma_i^{-2}} = \frac{L}{d^{-1} - \sigma_x^{-2} - \sum_{i=1}^{L} \sigma_i^{-2}} \tag{35}$$

then we take (35) into (18) and combine with (28), the rate-distortion function can be obtained

$$R(d) = \frac{1}{2}\sum_{i=1}^{L} \log \frac{L\sigma_i^{-2}}{\sum_{i=1}^{L} \sigma_i^{-2} + \sigma_x^{-2} - d^{-1}} + \frac{1}{2}\log\frac{\sigma_x^2}{d} \tag{36}$$

The transmission rate of each relay R_i is represented by

$$\begin{aligned} R_i(d) &= \frac{1}{2}\log \frac{L\sigma_i^{-2}}{\sum_{i=1}^{L} \sigma_i^{-2} + \sigma_x^{-2} - d^{-1}} + \frac{1}{2L}\log\frac{\sigma_x^2}{d} \\ &= \frac{1}{2}\log \frac{L\sigma_x^2\sigma_i^{-2}}{\sigma_x^2\sum_{i=1}^{L} \sigma_i^{-2} + 1 - \sigma_x^2 d^{-1}} + \frac{1}{2L}\log\frac{\sigma_x^2}{d} \quad (i = 1, 2, \cdots, L) \end{aligned} \tag{37}$$

Take $\frac{\sigma_x^2}{\sigma_i^2} = \gamma_{ri}$ and $\frac{\sigma_x^2}{d} = \gamma_D$ into (37)

$$R_i(d) = \frac{1}{2}\log\frac{L\gamma_{ri}}{1+\gamma-\gamma_D} + \frac{1}{2L}\log\gamma_D \quad (i = 1, 2, \cdots, L) \tag{38}$$

Where γ_D is the quantization SNR of $X(t)$ after it is encoded at the relays, and $\gamma = \sum_{i=1}^{L}\gamma_{ri}$. In addition, combine (35) with (18), we can get

$$L\gamma_{ri} \ge 1 + \sum_{i=1}^{L}\gamma_{ri} - \gamma_D = 1 + \gamma - \gamma_D \quad (i = 1, 2, \cdots, L) \tag{39}$$

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