Transmit Precoder Design for Two-User Broadcast Channel with Statistical and Delayed CSIT

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Abstract

Recent studies have revealed the efficacy of incorporating delayed channel state information at transmit side (CSIT) in transmission scheme design. This paper focuses on transmit precoder design to maximize the ergodic sum-rate in a two-user Multiple-Input Single-Output (MISO) system with delayed and statistical CSIT. A new transmit strategy which precodes signals in all transmit slots is proposed in this paper, denoted as all time-slots precoding Alternative MAT (AAMAT). There is a common procedure in conventional delayed-CSIT based schemes, which is retransmitting the overheard interferences. Since the retransmitting signal is intended to both users, all previous schemes tend to use only one antenna. We however figure out an improvement in spectral efficiency could be realized if all antennas can be utilized. In this paper, we detail the design of the procoder which enabling all antennas and also we compute a lower bound of the ergodic sum-rate in an ideal condition. In addition, simulation results demonstrate the superiority of our proposed scheme.

Keywords: MISO system, statistical CSIT, delayed CSIT, beamforming, precoder

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1. Introduction

With the rapid development in wireless communication, multiple-input multiple-output (MIMO) has drawn considerable attention from relevant researchers. Many works focus on the multi-user systems under perfect instantaneous channel state information at transmit side [1]-[3]. Despite the fact that in point to point channels, CSIT only provides power gains via water-filling, it can also provide multiplexing gains in multiuser channels. However, it is not easy to achieve accurate current CSIT in practice, especially in frequency division duplex systems where the CSI has to be measured at the receiver and fed back to the transmitter[4]-[6]. Most multi-user precoding techniques require the transmitter to know the perfect CSI to maintain orthogonality among users. However, due to quantization errors and feedback delay, the CSIT is inevitably imperfect. As long as the feedback time is larger than the coherence time of the channel, precoding cannot offer multiplexing gain. Therefore, in our study we consider the transmitter side only has the knowledge of delayed and statistical CSI. There are number of works that study transmission schemes incorporating imperfect CSI [7]-[12].

In [13], Maddah-Ali and Tse (MAT) proposed a transmit strategy which confirms that completely delayed CSIT is still very useful. They showed that in MIMO broadcast channel with M transmit antennas and K receivers each with one receive antenna, $\frac{K}{1+\frac{1}{2}+\cdots+\frac{1}{K}}(> 1)$degrees of freedom (Dof) can be achieved. The transmission scheme of MAT is divided in two stages, which takes three time-slots in total. By MAT, multiple data streams can be resolved while all interference can be eliminated at the receiver side. However, there is a critical issue in terms of the implementation complexity. As the number of user increases, the MAT scheme needs more time slots to achieve the optimal DoF [14].

On the other hand, statistical CSI (SCSI) is rather static and varies at a much slower rate. SCSI is characterized by the covariance matrix and it can be accurately and easily acquire through long-term feedback. [15] indicates that by precoding the transmitted signals along the weakest eigenvector of the unintended user's channel covariance matrix, the inter-user interference can be effectively suppressed. [16] focuses on a statistical-eigenmode space-division multiple-access (SE-SDMA) transmission approach which maximizes the ergodic signal to leakage and noise ratio (SLNR). On the other hand, **[17]** studies multi-user MISO broadcast channels with SCSI at the transmit side. By scheduling users over different time slot, the average group-rate of the whole system can be improved.

A virtual-MAT (VMAT) transmission scheme is proposed in [18], which exploits both statistical and delayed CSIT. Though statistical precoding in the first two transmission slots, the VMAT scheme reduces the original K-user MAT system to a two-user virtual MAT system. Not only does the VMAT yield a higher sum-rate per slot at finite SNRs, it also lowers the implementation complexity.

The [19] generalizes the MAT scheme of the two-user case with precoding in the third slot as GMAT and achieved a higher sum-rate at finite SNR, which strike a balance between desired signal enhancement and interference alignment. Being similar to GMAT that only exploits the delayed CSIT and improves original MAT, an alternative MAT (AMAT) transmission strategy is introduced in paper [20]. It is different from MAT which transmits the desired signals to each user during the first two slots and the overheard interference is sent in the third slot. In order to maximize the sum-rate performance in MU-MISO systems with partial CSIT, the authors combine rate-splitting strategy with precoder design and optimization techniques [21].

In [22], under equal power allocation, with precoding in the first slot, it has shown that additional statistical CSIT enable a higher achievable sum-rate compared to the original AMAT. Although [22] improves the sum-rate, it still does not make the full use of transmit antennas in the second and third slot. In our paper, the key contributions are as follows.

• We design an AMAT-style transmission scheme, which precodes in all time-slots denoted as AAMAT. The proposed scheme exploits both statistical and delayed CSIT, so that the system is able to provide higher capacity by the novel procoder design.

• We focus on the two users case which has a tractable structure. With the novel precoder, we improve the antenna utilization and enhance the sum-rate performance.

• Besides, we also approximate the ergodic sum-rate under the ideal condition. We consider the special case where two users have the same dominant eigenvector. Then an approximated expression of the achievable ergodic sum-rate is derived.

The rest of this paper is organized as follows. Section 2 introduces the system model and transmission scheme. Precoder design and achievable ergodic sum-rate for the proposed AAMAT scheme are shown in section 3. Our numerical results are shown in section 4, while section 5 concludes the paper.

Notations: Bold lower case and upper case letters denote vectors and matrices, respectively. The conjugate transpose and transpose are donated by $(\cdot)^H$ and $(\cdot)^T$, respectively. And $\mathrm{diag}(\cdot)$ is a diagonal matrix whereas $\mathbb{E}[\cdot]$ is the expectation operator. $\mathbf{u}_{\max}$ and $\mathbf{u}_{\min}$ indicate the largest and smallest eigenvectors of a matrix, respectively. Operators $\mathrm{Tr}(\cdot)$ and $\det(\cdot)$ refer to the trace and determinant of a matrix. We denote $\mathrm{U}(\mathrm{a},\mathrm{b})$ as the uniform distribution.

2. System Model and Transmission Schemes

We assume a Rayleigh fading broadcast channel model with M-antenna base station and single-antenna two users. The transmit side can obtain the statistical CSI (SCSI) of each user by feedback channels. The SCSI can be depicted by the second-order moments of the channel [23], which can be measured by the correlation matrix over time, i.e.

$$\mathbf{R}_k = \mathbb{E}\left[\mathbf{h}_k^H \mathbf{h}_k\right], \tag{1}$$

where $\mathbf{h}_k$ is the channel vector towards user k. Therefore, the channel vector between the transmitter and user A in time slot j can be modeled as $\mathbf{h}_j$ and $\mathbf{g}_j$ respectively as,

$$\mathbf{h}_j = \mathbf{R}_A^{1/2} \mathrm{h}_{\omega,j} \tag{2}$$

$$\mathbf{g}_j = \mathbf{R}_B^{1/2} \mathrm{g}_{\omega,j}, \tag{3}$$

where $\mathrm{h}_{\omega,j}$ and $\mathrm{g}_{\omega,j}$ are $M \times 1$-sized vector of independent and identical distribution $(\mathrm{i.i.d}) \sim \mathbb{CN}(0,1)$ entries. Because of that feedback delay and processing latency exits in most communication systems, the transmitter can only achieve the information of $\mathrm{h}_{\omega,j}$ in slot j, where $j' < j$. We call it delayed CSIT. $\mathbf{R}_A$ and $\mathbf{R}_B$ are the transmit correlation matrix for user A and B respectively, which can be decomposed as $\mathbf{R}_k = \mathbf{v}_k \mathbf{\Lambda}_k \mathbf{v}_k^H$, k=A,B,$\mathbf{v}_k \in \mathbb{C}^{M \times M}$ is a unitary matrix whose columns are eigenvectors of $\mathbf{R}_k$, and the diagonal $\mathbf{\Lambda}_k$ that contains the eigenvalues of $\mathbf{R}_k$ is normalized as $\mathrm{Tr}(\mathbf{\Lambda}_k) = M$. $\mathbf{\Lambda}_k = \mathbf{I}$ shows that the k-th channel is spatially uncorrelated and $\mathrm{rank}(\mathbf{\Lambda}_k) = 1$ indicates that the channel is perfectly correlated.

2.1 AMAT

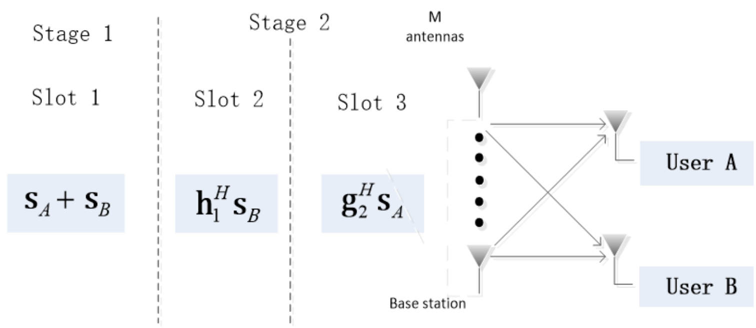

Fig. 1. AMAT transmit strategy

For two users MISO case, the original AMAT scheme is performed in two stages, spanning three time slots [20]. It can be shown in **Fig. 1**. In the first slot, the desired signals $\mathbf{s}_A$, $\mathbf{s}_B$ are superposed simply at transmit side and then send to both users simultaneously. In the second slot, the transmitter sends the interference overheard by user A in the first slot. The function of this stage is twofold: to resolve the interference of user A and reinforce the signal for user B. In the third slot, the interference overheard by user B is sent by transmitter to help both users the other way around. It can be described by the following equations, help both users the other way around.

$$\mathbf{x}_1 = \sqrt{\rho}(\mathbf{s}_A + \mathbf{s}_B)$$
$$\mathbf{x}_2 = \sqrt{\rho}[\mathbf{h}_1^H \mathbf{s}_B \quad 0]^T \tag{4}$$
$$\mathbf{x}_3 = \sqrt{\rho}[\mathbf{g}_1^H \mathbf{s}_A \quad 0]^T,$$

in which $\mathbf{x}_i \epsilon \mathbb{C}^{M\times 1}$ represents the transmit signals in each slot and ρ is the individual transmit power of a signal. $\mathbf{s}_k$ k=A, B are $M \times 1$Gaussian signal vectors intended to user A and B, respectively, satisfying $\mathbf{s}_k = [s_{k1} \quad s_{k2}]^T$, $\mathbb{E}[\mathbf{s}\mathbf{s}_i^H] = \mathbf{I}$.

2.2 SAMAT

The SAMAT also contains two stages/three time slots. By precoding the original AMAT in the first slot, the transmitted signals in the SAMAT can be denoted as,

$$\mathbf{x}_1 = \sqrt{\rho}(\mathbf{W}\mathbf{s}_A + \mathbf{Q}\mathbf{s}_B) \tag{5}$$

$$\mathbf{x}_2 = \sqrt{\rho}[\mathbf{h}_1^H \mathbf{Q}\mathbf{s}_B \quad 0]^T \tag{6}$$

$$\mathbf{x}_3 = \sqrt{\rho}[\mathbf{g}_1^H \mathbf{W}\mathbf{s}_A \quad 0]^T, \tag{7}$$

$\mathbf{W}$, $\mathbf{Q}$ are the precoders in the first slot [22]. With statistical beamforming design, $\mathbf{W} = [\mathbf{w}_1 \quad \mathbf{w}_2]$, $\mathbf{Q} = [\mathbf{q}_1 \quad \mathbf{q}_2]$ are listed as:

$$\mathbf{w}_1 = \mathbf{u}_{\min}(\mathbf{R}_B^{-1}\mathbf{R}_A), \mathbf{w}_2 = \mathbf{u}_{\max}(\mathbf{R}_B^{-1}\mathbf{R}_A)$$
$$\mathbf{q}_1 = \mathbf{u}_{\min}(\mathbf{R}_A^{-1}\mathbf{R}_B), \mathbf{q}_2 = \mathbf{u}_{\max}(\mathbf{R}_A^{-1}\mathbf{R}_B). \tag{8}$$

Finally the received signal can be expressed as

$$\mathbf{y}_A = \sqrt{\rho}\begin{bmatrix} \mathbf{h}_1^H \mathbf{W} \\ \mathbf{0} \\ \mathbf{h}_3^H \mathbf{g}_1^H \mathbf{W} \end{bmatrix} \mathbf{s}_A + \sqrt{\rho}\begin{bmatrix} \mathbf{h}_1^H \mathbf{Q} \\ \mathbf{h}_2^H \mathbf{h}_1^H \mathbf{Q} \\ \mathbf{0} \end{bmatrix} \mathbf{s}_B + \mathbf{n}_A, \tag{9}$$

where $\mathbf{y}_A = [y_{A1} \quad y_{A2} \quad y_{A3}]^T$ expresses the received signals over three time slots, $\mathbf{n}_A \sim \mathbb{CN}(0, \mathbf{I})$ denotes the complex Gaussian noise added at the receiver. As can be easily seen from (9) that original AMAT can be regarded as a special case of SAMAT when

$\mathbf{W} = \mathbf{I}$ and $\mathbf{Q} = \mathbf{I}$. In other words SAMAT actually generalized the idea of original AMAT.

Although the SAMAT scheme with precoding in the first slot outperforms the original AMAT, the overheard interference is reconstructed and broadcast only via a single antenna in the following two slots. In contrast with [22], the SAMAT discussed in this paper is a simplified SAMAT scheme which exploits statistical CSIT in first slot and also uses the delayed CSIT. The original SAMAT can reduce to SBF [15] at low SNR. We explore the full usage of transmit antennas when delivering overheard interference.

3.The Proposed AAMAT Scheme

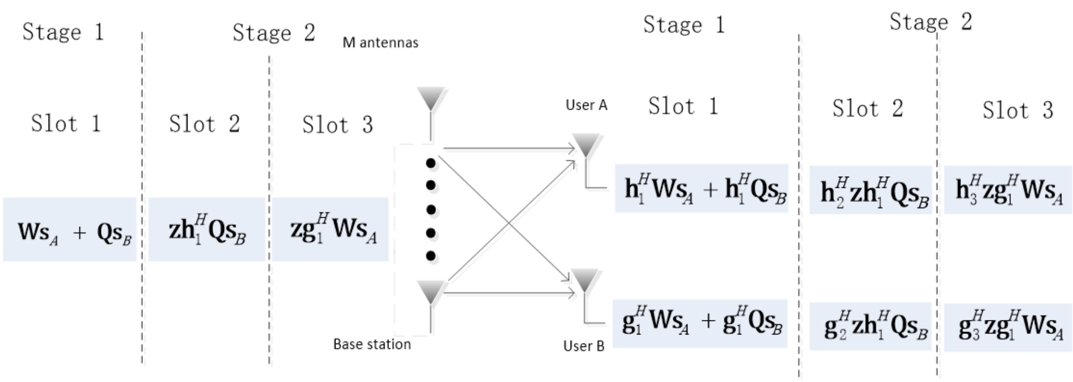

Fig. 2. AAMAT transmit strategy

In previous section, we briefly introduce the SAMAT scheme which precodes the transmit signals with statistical CSIT in the first slot under equal power allocation. We keep this design but propose to precode the overheard interference before transmission in slot 2 and 3. Therefore, each user can receive its desired signal as well as the overheard interference at the end of stage one. Due to feedback delay and processing latency, the transmitter can only obtain the information of h_1 in the second slot. As can be seen from **Fig. 2**, by applying such a precoder $\mathbf{z}$ in stage 2, the overheard interference is transmitted via all antennas at transmits side instead of a single antenna. In short, the AAMAT scheme also consists of two stages: 1) broadcasting the desired signals; 2) multicasting the overheard interference. Consequently the received signal at user A is given by

$$\mathbf{y}_A = \sqrt{\rho}\begin{bmatrix} \mathbf{h}_1^H\mathbf{W} \\ \mathbf{0} \\ \mathbf{h}_3^H\mathbf{z}\mathbf{g}_1^H\mathbf{W} \end{bmatrix} \mathbf{s}_A + \sqrt{\rho}\begin{bmatrix} \mathbf{h}_1^H\mathbf{Q} \\ \mathbf{h}_2^H\mathbf{z}\mathbf{h}_1^H\mathbf{Q} \\ \mathbf{0} \end{bmatrix} \mathbf{s}_B + \mathbf{n}_A, \tag{10}$$

where we have $\mathbf{z} \in \mathbb{C}^{M\times\mathbf{1}}$ and $\mathbf{z}^H\mathbf{z} = 1$. It is plain to see that if $\mathbf{z} = [1 \quad 0]^T$, then our proposed scheme boils down to SAMAT which means AAMAT is a more generalized form of original AMAT. User *A* and *B* share the same precoding matrix $\mathbf{z}$ in slot 2 and 3 because we consider the overheard interferences are equally important to both users.

3.1 Precoder Design

We now introduce the precoder design of $\mathbf{z}$. In stage 2, in order to make full use of the overheard interference, our scheme precodes $\theta_A = \varepsilon_A \mathbf{h}_1^H \mathbf{Q}\mathbf{s}_B, \theta_B = \varepsilon_B \mathbf{g}_1^H \mathbf{W}\mathbf{s}_A$ respectively, and then sends to the both users via all transmit antennas. Note that before precoding the overheard interference, θ_A and θ_B should be normalized namely, $\| \theta_A\|^2 = 1$ and $\| \theta_B\|^2 = 1$. Otherwise, the transmit power might exceed the power constraint due to the uncontrollable CSI term. We achieve the normalization by adjust the scalars ε_A and ε_B. By precoder design, the overheard interference can be more effectively retransmitted to both users, who can utilize it to resolve intended signals and eliminate unwanted signals.

Since in the first stage we apply the precoder in [22], we solely focus on stage 2 i.e. the interference retransmission process. The insight to find our precoder is derived from maximizes the ergodic SNR to both users time-slot 2 and 3. For user *A*, the ergodic SNR in slot 2 is

$$\overline{SNR}_A = \mathbb{E}[\mathbf{h}_2^H \mathbf{z} \theta_A \theta_A^H \mathbf{z}^H \mathbf{h}_2] = \mathbb{E}[\mathbf{h}_2^H \mathbf{z}\mathbf{z}^H \mathbf{h}_2] = \mathbf{z}^H \mathbf{R}_A \mathbf{z} \ . \tag{11}$$

Therefore, by Jensen's inequality we have

$$R_A \leq \log_2(\overline{SNR}_A). \tag{12}$$

Even though high the ergodic SNR cannot precisely indicate high data-rate, some studies still intend to enhance system performance by maximizing transmitted signals power incorporating with CSI from the "ergodic" view [24–25]. Similarly, the ergodic SNR of user *B* in slot 2 can be expressed as

$$\overline{SNR}_B = \mathbf{z}^H \mathbf{R}_B \mathbf{z}. \tag{13}$$

In order to maximize the ergodic SNR, the optimized precoded signals to user *A* should be along the dominant eigenvector of $\mathbf{R}_A$ namely $\mathbf{v}_A = \mathbf{U}_{\max}(\mathbf{R}_A)$. The same goes for user *B* and we denote the dominant eigenvector of $\mathbf{R}_B$ as $\mathbf{v}_B = \mathbf{U}_{\max}(\mathbf{R}_B)$. However in practical communication systems, users are diverse which means it is hard to find two users with the same eigenmode. Thus, we have to make a compromise between $\mathbf{v}_A$ and $\mathbf{v}_B$ and it can be written as the following maximization problem,

$$\arg\max_{\mathrm{z},\alpha} \boldsymbol{\mathcal{F}} = \left\|\mathbf{z}^H\left(\mathbf{v}_A + \mathbf{v}_B \mathrm{e}^{j\alpha}\right)\right\|^{\mathbf{2}}$$

$$\boldsymbol{s.t.}\,\|\mathbf{z}\|^2 = 1 \tag{14}$$

Note that the dominant eigenvector of $\mathbf{R}_B$ is invariant to transformations of the form $\mathbf{v}_B \longmapsto \mathbf{v}_B \mathrm{e}^{j\alpha}$. By denoting the Hermitian matrix

$$\mathbf{R_V} = \left(\boldsymbol{v_A} + \boldsymbol{v_B}\mathbf{e}^{\boldsymbol{j\alpha}}\right)(\boldsymbol{v_A} + \boldsymbol{v_B}\mathbf{e}^{\boldsymbol{j\alpha}})^H. \tag{15}$$

We can further write the objective function as,

$$\boldsymbol{\mathcal{F}} = \mathbf{z}^H\mathbf{R_V}\mathbf{z} = \mathbf{z}^H\mathbf{U}_\mathbf{v}^H\boldsymbol{\Lambda}_\mathbf{v}\mathbf{U}_\mathbf{v}\mathbf{z} \tag{16}$$

where $\mathbf{U_v}$ and $\boldsymbol{\Lambda}_\mathbf{v}$ are the eigenmode decomposition of $\mathbf{R_V}$.The diagonal matrix has only one non-zero entry, we define it as λ, specifically, $\boldsymbol{\Lambda}_\mathbf{v} = \mathrm{diag}[\lambda, 0, \ddots, 0]$,

$$\begin{aligned}\lambda &= (\boldsymbol{v_A} + \boldsymbol{v_B}\mathbf{e}^{\boldsymbol{j\alpha}})^H\left(\boldsymbol{v_A} + \boldsymbol{v_B}\mathbf{e}^{\boldsymbol{j\alpha}}\right)\\ &= |\mathbf{v_A}|^2 + |\mathbf{v_B}|^2 + \mathbf{e}^{-\mathbf{j}\boldsymbol{\alpha}}\mathbf{v_B}^\mathbf{H}\mathbf{v_A} + \mathbf{e}^{\mathbf{j}\boldsymbol{\alpha}}\mathbf{v_A}^\mathbf{H}\mathbf{v_B}\\ &= |\boldsymbol{v_A}|^2 + |\boldsymbol{v_B}|^2 + 2\Re\left(\mathbf{e}^{-j\alpha}\mathbf{v_B}^H\boldsymbol{v_A}\right)\end{aligned} \tag{17}$$

It is not hard to see that the optimal precoder $\mathbf{z}$ should along the dominant eigenvector. By doing so, we can get the maximum objective value $\boldsymbol{\mathcal{F}} = \lambda$. To maximize the eigenvalue λ, we can adjust $\boldsymbol{\alpha}$ to maximize the term $\boldsymbol{\Re}\left(\mathbf{e}^{-j\alpha}\mathbf{v_B}^{\boldsymbol{H}}\boldsymbol{v_A}\right)$.Therefore we obtained the solution of (14), namely,

$$\mathbf{z} = \frac{\boldsymbol{v_A}+\boldsymbol{v_B}\mathbf{e}^{\boldsymbol{j\alpha}}}{\sqrt{(\boldsymbol{v_A}+\boldsymbol{v_B}\mathbf{e}^{\boldsymbol{j\alpha}})^H(\boldsymbol{v_A}+\boldsymbol{v_B}\mathbf{e}^{\boldsymbol{j\alpha}})}}, \boldsymbol{\alpha} = \mathbf{angle}(\boldsymbol{v}_{\boldsymbol{B}}^H\boldsymbol{v_A}) \tag{18}$$

In this way, we expect that the precoder could not only fully utilize all antennas but also maximize the SNR to both users. The efficacy of this design will be shown in the result part.

3.2. Rate Approximation

Hereafter, we focus on the performance of user *A* and user *B* can derive the similar results as user *A*. In order to resolve $\boldsymbol{s_A}$ from (10), we use a zero-forcing equalizer by which the interference from user *B* is eliminated through left multiplying (10) with a transformation matrix $\mathbf{L}$

$$\mathbf{L}\mathbf{y}_A = \mathbf{L}\left(\sqrt{\rho}\begin{bmatrix}\mathbf{h}_1^H\mathbf{W}\\ \mathbf{0}\\ \mathbf{h}_3^H\mathbf{z}\mathbf{g}_1^H\mathbf{W}\end{bmatrix}\mathbf{s}_A + \sqrt{\rho}\begin{bmatrix}\mathbf{h}_1^H\mathbf{Q}\\ \mathbf{h}_2^H\mathbf{z}\mathbf{h}_1^H\mathbf{Q}\\ \mathbf{0}\end{bmatrix}\mathbf{s}_B + \mathbf{n}_A\right), \tag{19}$$

where the transformation matrix $\mathbf{L}$ is $\mathbf{L} \triangleq \begin{bmatrix}\mathbf{h}_2^H\mathbf{z} & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$, which satisfies $\mathbf{L}\begin{bmatrix}\mathbf{h}_1^H\mathbf{Q}\\ \mathbf{h}_2^H\mathbf{z}\mathbf{h}_1^H\mathbf{Q}\\ \mathbf{0}\end{bmatrix} = 0$.

With this approach, the equation (19) can be simplified as,

$$\tilde{\mathbf{y}}_A = \sqrt{\rho}\begin{bmatrix}\mathbf{h}_2^H\mathbf{z}\mathbf{h}_1^H\mathbf{W}\\ \mathbf{h}_3^H\mathbf{z}\mathbf{g}_1^H\mathbf{W}\end{bmatrix}\mathbf{s}_A + \begin{bmatrix}\mathbf{h}_2^H\mathbf{z}\mathrm{n}_{A1} - \mathrm{n}_{A2}\\ \mathrm{n}_{A3}\end{bmatrix}. \tag{20}$$

Then by using a Minimum Mean-square Error (MMSE) receiver with Successive Interference Cancellation, the ergodic rate achieved by user A per slot is

$$R_A = \frac{1}{3}\mathbb{E}\left[\log_2\det(\mathrm{I} + \rho\widetilde{\mathbf{H}}^H\mathbf{\Psi}^{-1}\widetilde{\mathbf{H}})\right], \tag{21}$$

where $\widetilde{\mathbf{H}} = \begin{bmatrix}\mathbf{h}_2^H\mathbf{z}\mathbf{h}_1^H\mathbf{W}\\ \mathbf{h}_3^H\mathbf{z}\mathbf{g}_1^H\mathbf{W}\end{bmatrix}$, $\mathbf{\Psi}$ is the covariance matrix of the noise vector in (16) and $\mathbf{\Psi} = \mathrm{diag}(1 + \left|\mathbf{h}_2^H\mathbf{z}\right|^2, 1)$.

It is hard to solve the closed-form expression of the ergodic sum-rate, especially for M > 2 case. So, we analyze the analytical approximation of R_A, which is given by the following theorem.

Theorem 1: In spatially correlated Rayleigh fading channel, we consider the ideal condition that $\mathbf{R}_A$ and $\mathbf{R}_B$ have the same unitary matrix. The ergodic rate of user A can be approximated as

$$R_A \approx \frac{2}{3}\log_2\left(1 + \rho\sqrt{\mathrm{e}^{\beta}\emptyset_A}\right), \tag{22}$$

where

$$\beta = 2(\ln(\lambda_{max}) - \gamma) - \mathrm{e}^{\frac{1}{\lambda_{max}}}\mathrm{E_i}(-\frac{1}{\lambda_{max}})) \tag{23}$$

and γ is the Euler constant, λ_{max} is the maximum entry of the eigenmode decomposition of $\mathbf{R}_B$, $\mathrm{Ei}(x) = -\int_{-x}^{\infty}\frac{\mathrm{e}^{-t}}{t}\mathrm{d}t$ is the exponential integral. Besides, in equation (22) we deduce

$$\emptyset_A = \mathrm{Tr}(\mathbf{W}^H\mathbf{R}_\mathrm{A}\mathbf{W})\mathrm{Tr}(\mathbf{W}^H\mathbf{R}_B\mathbf{W}) - \mathrm{Tr}(\mathbf{W}^H\mathbf{R}_A\mathbf{W}\mathbf{W}^H\mathbf{R}_B\mathbf{W}). \tag{24}$$

Then we can conclude

$$R_{\text{sum}} = R_A + R_B \approx \frac{2}{3}\log_2\left(1+\rho\sqrt{e^{\beta}\emptyset_A}\right) + \frac{2}{3}\log_2\left(1+\rho\sqrt{e^{\beta}\emptyset_B}\right). \tag{25}$$

It is plain to see that as the transmit power increases, the proposed AAMAT scheme exploiting both delayed CSIT and SCSI enables a DoF of $\frac{4}{3}$. So, we can conclude AAMAT has the same DoF gain with SAMAT as well as original AMAT.

4. Performance Evaluation

This section provides results to confirm the efficacy of the proposed AAMAT strategy. We model the SCSI of each user by the single parameter exponential correlation fashion which is also used in [26]

$$\mathbf{R}_k = \begin{bmatrix} 1 & t_k & \cdots & t_k^{M-1} \\ t_k^H & 1 & \cdots & t_k^{M-2} \\ \vdots & \ddots & \ddots & \vdots \\ (t_k^H)^{M-1} & \cdots & t_k^H & 1 \end{bmatrix}, \tag{26}$$

where t_k expresses the transmit correlation coefficient, and $t_k = |t_k|e^{j\varphi_k}$, $\varphi_k \in [0,2\pi]$, k=*A*,*B*. Large (small) $|t_k|$corresponds to strong (weak) correlation between channels.

4.1 Rate Comparison in Different SCSI

We show in **Fig. 3** for the *M* = 4 case in which the ergodic sum-rate is compared among various schemes. SSAMAT denotes simplified SAMAT that only precodes the AMAT in first slot by optimal linear beamforming vectors developed in section 2, the proposed AAMAT. We set $|t_A| = |t_B| = |t|$ that varies from 0 to 1 which also means it is from uncorrelated to highly correlated channels. $\varphi_A, \varphi_B \in U(0,2\pi)$ are randomly generated and SNR is fixed at 20 dB. From the **Fig. 2** we can conclude that, the three schemes have the same ergodic data-rate per slot at $|t| = 0$, and in between $|t| = 0 \sim 0.9$, the original AMAT and the SSAMAT basically remain unchanged. In contrast, the ergodic rate of the AAMAT scheme increases, as $|t|$ increases and a sharp rise occurs in the regime of $|t| > 0.9$. This phenomenon arises from that the SCSI-based linear beamforming can keep the residual interference sufficiently small in a highly correlated spatial channel. **Fig. 3** indicates that the proposed AAMAT scheme obtains strictly higher rate than SSAMAT and original AMAT by exploiting both statistical and delayed CSIT.

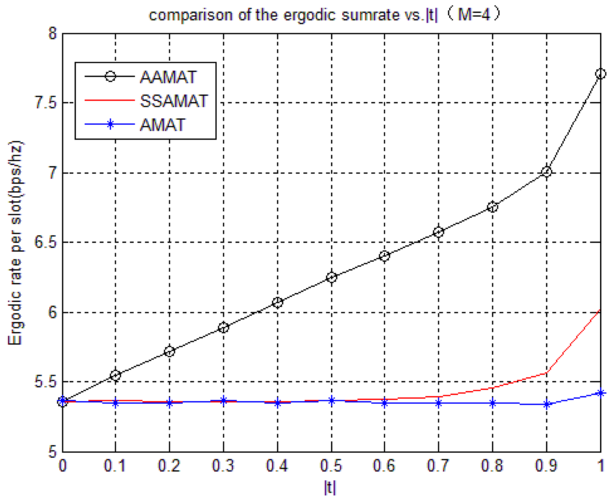

Fig. 3. Ergodic rate per slot vs.$|\mathrm{t}|$ for various schemes

4.2 Ergodic Sum-Rate Comparison in Different SNR

In addition, for given channel covariance matrices, the ergodic sum-rate of these schemes can be compared versus SNR. The transmit antennas *M*=2, *M*=4 are considered, respectively.$|\mathrm{t}_A| = 0.95$, $|\mathrm{t}_B| = 0.7$, $\varphi_A, \varphi_B \in \mathrm{U}(0,2\pi)$ are randomly generated. In **Fig. 4** and **Fig. 5**, in order to make a fair comparison among the three schemes, we generate the correlated channels as described in section II and use a MMSE receiver. We find that in the SNR regime of 0 ~ 5dB, the original AMAT and the SAMAT almost have the same ergodic sum-rate. It is also shown in **Fig. 3** and **Fig. 4** that the proposed AAMAT outperforms the SSAMAT as well as the original AMAT scheme at arbitrary SNR levels. We also observe that AAMAT finally achieve a $\frac{4}{3}$ DoF as we analyzed before.

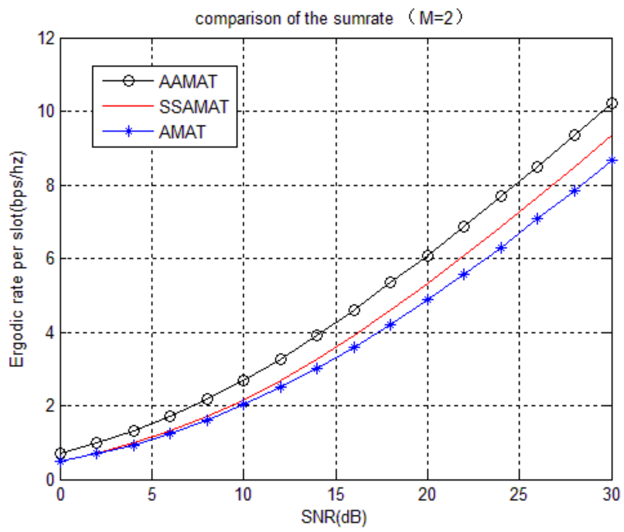

Fig. 4. Ergodic sum-rate vs.SNR for various schemes (M=2)

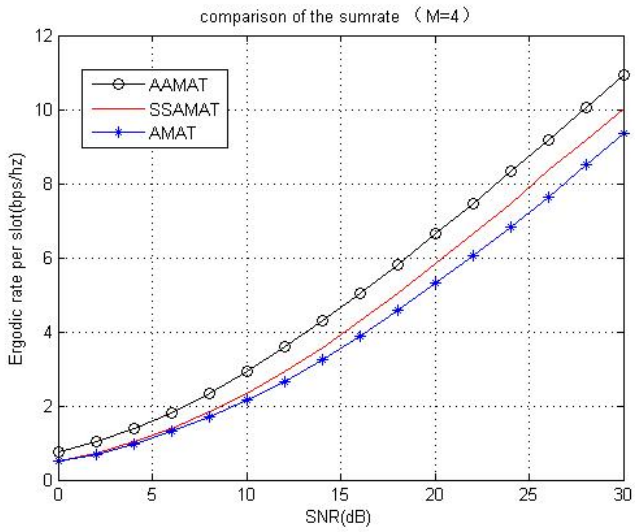

Fig. 5. Ergodic sum-rate vs.SNR for various schemes (M=4)

5. Conclusion

We propose a novel transmission scheme which exploits both statistical and delayed CSIT in a two-user broadcast channel to maximize the ergodic sum-rate. Moreover the precoder design in the second and third slot is elaborated, activating all transmit antennas. Our numerical results show that the proposed precoding algorithm outperforms the original AMAT and the SAMAT at arbitrary SNRs. Besides an approximation of the ergodic sum-rate is computed in closed form in terms of the two users with same dominant eigenvectors.

APPENDIX A

PROOF OF PROPOSITION 1

The lower bound of the mutual information in (21) can apply *Minkowski Determinant Theorem* [27]

$$\mathrm{I}_A = \log_2 \det(\mathbf{I}_{2\times 2} + \rho\mathbf{N}) \tag{A.1}$$

$$\geq \log_2 (1 + \rho \det(\mathbf{N})^{1/2})^2 \tag{A.2}$$

$$= 2\log_2[1 + \rho\exp(\tfrac{1}{2}\ln\det(\mathbf{N}))], \tag{A.3}$$

where

$$\mathbf{N} \triangleq \widetilde{\mathbf{H}}^H \boldsymbol{\Psi}^{-1} \widetilde{\mathbf{H}} \tag{A.4}$$

$$= [\mathbf{W}^H \mathbf{h}_1 \quad \mathbf{W}^H \mathbf{g}_1] \begin{bmatrix} \frac{\mathbf{z}^H \mathbf{h}_2 \mathbf{h}_2^H \mathbf{z}}{1+|\mathbf{h}_2^H \mathbf{z}|^2} & 0 \\ 0 & \mathbf{z}^H \mathbf{h}_3 \mathbf{h}_3^T \mathbf{z} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1^H \mathbf{W} \\ \mathbf{g}_1^H \mathbf{W} \end{bmatrix} \tag{A.5}$$

$$= [\mathbf{W}^H \mathbf{h}_1 \quad \mathbf{W}^H \mathbf{g}_1] \begin{bmatrix} \frac{|\mathbf{h}_2^H \mathbf{z}|^2}{1+|\mathbf{h}_2^H \mathbf{z}|^2} & 0 \\ 0 & |\mathbf{h}_3^H \mathbf{z}|^2 \end{bmatrix} \begin{bmatrix} \mathbf{h}_1^H \mathbf{W} \\ \mathbf{g}_1^H \mathbf{W} \end{bmatrix} \tag{A.6}$$

$$= \widetilde{\mathbf{G}}_{2\times 2} \boldsymbol{\Omega} \widetilde{\mathbf{G}}_{2\times 2}^H. \tag{A.7}$$

Through using the convexity of $\log_2(1 + \mathrm{re}^x), r > 0$ and Jensen's inequality, the ergodic rate of user A per slot can be lower bound as

$$R_A \geq \frac{2}{3} \mathbb{E}\left\{\left[\log_2 1 + \rho \exp\left(\frac{1}{2} \ln \det(\mathbf{N})\right)\right]\right\} \tag{A.8}$$

$$\geq \frac{2}{3} \log_2 \left[1 + \rho \exp(\frac{1}{2} \mathbb{E}[\ln \det(\mathbf{N})])\right], \tag{A.9}$$

where

$$\mathbb{E}[\ln \det(\mathbf{N})] = \mathbb{E}[\ln \det(\boldsymbol{\Omega})] + \mathbb{E}\left[\ln \det\left(\widetilde{\mathbf{G}} \widetilde{\mathbf{G}}^H\right)\right]. \tag{A.10}$$

The second term of (A.10) was solved in [22] by

$$\mathbb{E}\left[\ln \det\left(\widetilde{\mathbf{G}} \widetilde{\mathbf{G}}^H\right)\right] = \ln(\emptyset_A). \tag{A.11}$$

Then $\mathbb{E}[\ln \det(\boldsymbol{\Omega})] = \mathbb{E}\left[\ln\left(\frac{|\mathbf{h}_2^H \mathbf{z}|^2}{1+|\mathbf{h}_2^H \mathbf{z}|^2}\right)\right] + \mathbb{E}\left[\ln\left|\mathbf{h}_3^H \mathbf{z}\right|^2\right]$, and since we consider the ideal condition that user A and user B share the same unitary matrix, according (18), $\mathbf{z} = \mathbf{v}_A$. So

$$\mathbb{E}[\ln \det(\boldsymbol{\Omega})] = \mathbb{E}\left[\ln\left(\frac{|\mathbf{h}_2^H \mathbf{v}_A|^2}{1+|\mathbf{h}_2^H \mathbf{v}_A|^2}\right)\right] + \mathbb{E}\left[\ln\left|\mathbf{h}_3^H \mathbf{v}_A\right|^2\right] \tag{A.12}$$

according to the Lemma 1 which is given by [22], where the first term can be expressed as,

$$\mathbb{E}\left[\ln\left(\frac{|\mathbf{h}_2^H \mathbf{v}_A|^2}{1+|\mathbf{h}_2^H \mathbf{v}_A|^2}\right)\right] = \mathbb{E}\left[\ln\left(\frac{\left|\mathbf{h}_w^H \mathbf{R}_A^{1/2} \mathbf{v}_A\right|^2}{1+\left|\mathbf{h}_w^H \mathbf{R}_A^{1/2} \mathbf{v}_A\right|^2}\right)\right]$$

$$= \mathbb{E}\left[\ln \frac{\mathbf{h}_w^H \mathbf{R}_A^{1/2} \mathbf{v}_A \mathbf{v}_A^H \mathbf{R}_A^{1/2} \mathbf{h}_w}{1+\mathbf{h}_w^H \mathbf{R}_A^{1/2} \mathbf{v}_A \mathbf{v}_A^H \mathbf{R}_A^{1/2} \mathbf{h}_w}\right]$$

$$= \mathbb{E}\left[\ln \frac{\mathbf{h}_w^H \mathbf{U}_A^H \mathbf{\Lambda}_A \mathbf{U}_A \mathbf{h}_w}{1+\mathbf{h}_w^H \mathbf{U}_A^H \mathbf{\Lambda}_A \mathbf{U}_A \mathbf{h}_w}\right]$$

$$\stackrel{\text{def}}{=} \mathbb{E}\left[\ln \frac{\mathbf{h}_w^H \mathbf{\Lambda}_A \mathbf{h}_w}{1+\mathbf{h}_w^H \mathbf{\Lambda}_X \mathbf{h}_w}\right]$$

$$= \mathbb{E}\ln[\frac{\lambda_{max}|\mathrm{h}_{w,m}|^2}{1+\lambda_{max}|\mathrm{h}_{w,m}|^2}]. \tag{A.13}$$

where $\stackrel{\text{def}}{=}$ holds because $\mathbf{h}_w \mathbf{U}_A$ and $\mathbf{h}_w$ have the same distribution $\mathbf{\Lambda}_A = \mathrm{diag}([\lambda_{max}, 0])$ .Then we have

$$\mathbb{E}[\ln\det(\mathbf{\Omega})] = 2\mathbb{E}\left[\ln\left(\lambda_{max}|\mathrm{h}_{w,m}|^2\right)\right] - \mathbb{E}\left[\ln\left(1+\lambda_{max}|\mathrm{h}_{w,m}|^2\right)\right]$$

$$= 2(\ln(\lambda_{max}) - \gamma) - e^{\frac{1}{\lambda_{max}}}\mathrm{E_i}\left(-\frac{1}{\lambda_{max}}\right), \tag{A.14}$$

where (A.14) is obtained by using the fact that $|\mathrm{h}_{w,m}|^2 \sim Exp(1)$ and the equations in [28].

Finally, substituting (A.11) and (A.14) into (A.9) renders an analytical approximation of the ergodic rate of user *A* and then complete the proof.

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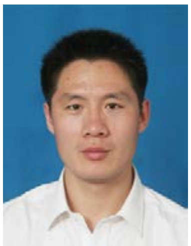

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