Optimal Cooperation and Transmission in Cooperative Spectrum Sensing for Cognitive Radio

Xian Zhang[1], Qihui Wu[1], Xiaoqiang Li[1] and Zi Yun[2]
[1] Institute of Communications Engineering, PLA University of Science and Technology
NanJing, P. R. China
[2] PLA Unit of 94860. NanJing, P. R. China

[e-mail: icexianzhang@gmail.com, wqhtxdk@yahoo.cn, lxq123lxq@yahoo.com.cn, yunzizi@126.com]

Received September 21, 2012; revised November 19, 2012; revised December 31, 2012; accepted January 25, 2013; published February 26, 2013

Abstract

In this paper, we study the problem of designing the power and number of cooperative node (CN) in the cooperation phase to maximize the average throughput for secondary user (SU), under the constraint of the total cooperation and transmission power. We first investigate the scheme of cooperative spectrum sensing without a separated control channel. Then, we prove that there indeed exist an optimal CN power when the number of CNs is fixed and an optimal CN number when CN power is fixed. The case without the constraints of the power and number of CN is also studied. Finally, numerical results demonstrate the characteristics and existences of optimal CN power and number. Meanwhile, Monte Carlo simulation results match to the theoretical results well.

Keywords: Cognitive radio, cooperative spectrum sensing, tradeoff of cooperation and transmission, throughput maximization.

This work was supported by the National Basic Research Program (973) of China under grant No. 2009CB320400, the National Science Foundation of China (No.60932002), the National Science Fund of China (No. 61172062), and Natural Science Fund of Jiangsu (No.BK2011116).

http://dx.doi.org/10.3837/tiis.2013.02.001

1. Introduction

With the increasing popularity of wireless devices, wireless networks have experienced rapid growth. The traditional approach of fixed spectrum allocation to licensed networks leads to spectrum under-utilization. The Federal Communications Commission (FCC) reported that temporal and spatial variations in the utilization of the assigned spectrum range from 15%－85%. This motivates the concept of cognitive radio that allows secondary user (SU) to opportunistically exploit under-utilized spectrum [1].

In order to exploit the under-utilized licensed spectrum efficiently in cognitive radio networks, SU is required to sense the licensed spectrum before transmissions and detect any possible activities of primary user (PU). If PU is occupying the spectrum, SU should defer its transmissions. The performance of spectrum sensing depends heavily on the signal strength at SU. Meanwhile, there also exists the hidden terminal problem which makes the detection for the primary activity lost so that PU suffers from interference by SU. Hence, the spectrum sensing and hidden terminal problems are two major challenges in the development of cognitive radio [2]. To improve the accuracy of spectrum sensing in a fading environment and alleviate the hidden terminal problem, cooperative spectrum sensing [3-6] has been presented. In cooperative spectrum sensing, multiple cooperative nodes (CNs) sense the spectrum independently, and send sensing results to SU, which will make the final detection on whether there are PU' activity in the sensing channel. [7] studied the spectrum sharing between cognitive radio system and digital broadcasting services using wireless link based on Global communication channel. It aimed to enhance the spectrum sensing and geolocation database spectrum sharing. In [8], the power allocation between SU and PU was proposed to maximize the sum rate. It considered the mutual effect between SU and PU.

In recent years, there are many studies dedicated to the researches on cooperative spectrum sensing. Cooperative spectrum sensing with multiple CNs was presented in [3-6], and [6] proposed a practical algorithm which allows cooperation between SUs. [9] analyzed the cooperative diversity to quantify the gain of cooperation performance in spectrum sensing, and different diversity quantities in different system performance metrics were also analyzed. [2] assumed that the spectrum sensing was replaced by separated sensing devices, and a new cognitive cycle was proposed. The problem of optimal location of separated sensing devices was studied for single-user and multi-user cases. These studies all assumed that sensing results of CNs were transmitted over an ideal channel from CNs to SU, i.e., prefect reporting channel, which is impractical in the real environment.

In [10], an optimal linear cooperation framework for spectrum sensing was proposed to accurately detect the weak primary signal. It assumed that the sensing results of CNs were transmitted with amplify-and-forward over a nonideal fading channel to a fusion center where they were combined to generate an estimate of the primary activities. The optimal power allocation strategies were then considered for different classes of cognitive radio systems. The objective of [10] is to maximize detection probability while satisfying a requirement on false alarm probability, rather than considering the average throughput for SU. Moreover, in order to obtain the optimal combining weight vector, the instantaneous channel state information must be available, which is difficult in practice.

Conventionally, the probabilities of miss-detection and false alarm are used to evaluate the performance of spectrum sensing. However, many pervious works mainly focused on how to improve the performance of spectrum or compromises the probabilities of miss-detection and false alarm, but ignored the relationship between spectrum sensing and transmission.

Specifically, the performance of spectrum sensing will make a great impact on SU transmission. When SU wrongly senses that the licensed channel that is being used by PU is available, it will transmit and interfere to PU transmission. Accordingly, SU and PU will interfere with each other so that both transmissions are unsuccessful. Hence, the relationship between spectrum sensing and transmission should be considered seriously in practical systems [11].

As we know that CNs send the sensing results to SU through the report channel, namely control channel, in traditional cooperative spectrum sensing scheme. Generally, the control channel is always a separated channel for sending the sensing results of CNs. Accordingly, the separated control channel is at the expense of utilization efficiency. In this paper, we propose a simple handshake between CNs and SU without a separated control channel, which improve the spectrum efficiency and is easy to implement.

The key contribution of this paper lies in formulation and optimization of tradeoff between spectrum sensing and transmission. We formulate the tradeoff between spectrum sensing and transmission by maximizing the average throughput of SU. Firstly, we consider the fundamental problem of designing the optimal cooperation power of CN to maximize the average throughput for SU, under the constraints of CN number and the total cooperation and transmission power, where only the mean channel gain information is available. Then, the optimal number of CNs is also considered when the CN power is given. Finally, the optimization problem without the constraints of CN power and number is investigated.

The most difference from our pervious work [11] is that [11] considered the mean channel gains between CNs and SU are different from each other. Different channel gains would result in the occurrence of CN selection. Meanwhile, the fraction of total power should also satisfy some requirements on the spectrum sensing performance. In this paper, we study the fundamental optimization problem for CN power and number through maximizing the average throughput of SU, and focus on the case where all the mean channel gains are the same. Through analyzing the optimal CN power and number, we can obtain some fundamental characteristic in different cases, which can provide some directions in the design of cognitive radio system.

This paper is organized as follows. In Section 2, the system model is introduced and formulations of cooperation and transmission phases are investigated. In Section 3, we study the optimization problem of power allocation and node selection for cooperation and transmission in different cases. The numerical results are presented and discussed in Section 4. Finally, Section 5 concludes this paper.

2. System Model

2.1 Cooperative Spectrum Sensing

We consider a cognitive radio network where cooperative spectrum sensing is implemented by multiple CNs. These CNs can be other SUs or separated sensing devices. To make the scenario concrete, we consider a typical secondary link consisting of a secondary transmitter (ST) and a secondary receiver (SR) in the cognitive radio network, as depicted in **Fig.1** [11].

With cooperative spectrum sensing, CNs sense spectrum independently and send the sensing results to ST, which will make the final decision on whether PU is occupying the channel or not. In order to reduce the cooperation overhead, CNs only report their final 1-bit hard decisions (i.e., idle or occupied) rather than actual measurements. This motivates a simple handshake between CNs and ST without a separated control channel: CN sends a beacon signal to ST for handshake through the sensing channel when it assumes the channel is idle; otherwise, CNs will keep silence. ST will make the final decision based on whether receives the beacon signal from CNs or not. If ST detects the beacon signal from CNs, it will decide that the channel is available and transmit over it; otherwise, ST decides that the channel is occupied by PU and can not be used for transmission.

We assume both PU and SU are with a synchronous frame structure, and the frame structure for cognitive radio network consists of sensing phase, cooperation phase and transmission phase. The lengths of cooperation phase and transmission phase are denoted as t_1 and t_2, respectively. In this paper, we focus on the power allocation problem among CNs and ST during the cooperation and transmission phases, and ignore the effect of different phases' lengths. Thus, the frame structure of cognitive radio network is assumed to be known as prior, namely t_1 and t_2 are fixed.

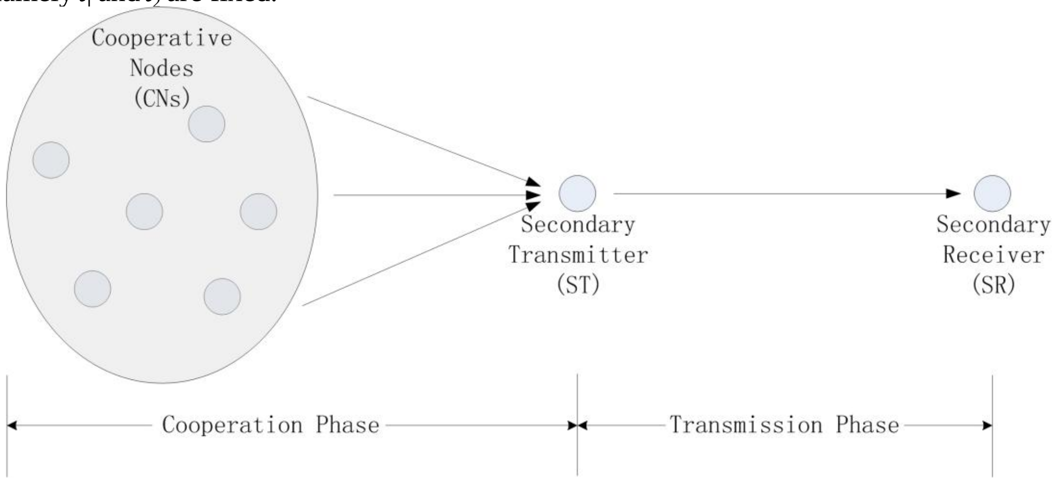

Fig. 1. The processes of cooperation phase and transmission phase [11].

2.2 Formulations of the Average Throughput for SU

Let h_i denotes the instantaneous channel power gain (the amplitude square of the instantaneous channel gain) between CN i and ST, and keeps constant during the cooperation phase. It is assumed that the channel power gain h_i is an exponential random variable with mean m_i. Without loss of generality, the noise variance is normalized to 1.

For cooperative spectrum sensing scheme, when CNs sense the channel and it is idle, CN will send signals to ST for the handshake. At ST, it applies the optimal signal processing and combining the received signals from n CNs, and the received SNR γ at ST is

$$\gamma = \sum_{i=1}^{n} p_i h_i , \tag{1}$$

where p_i denotes the power of CN i, h_i is an exponential random variable with mean m_i, i.e., the probability density function (PDF) of h_i is

$$f\left(h_i\right) = \frac{1}{m_i} e^{-\frac{h_i}{m_i}} . \tag{2}$$

We use false alarm probability to evaluate the performance of spectrum sensing. False alarm means when channel is sensed idle by CNs, then CNs send signal to ST, but ST does not detect CNs signal and decides the channel is occupied. Accordingly, the channel opportunity for ST will be lost.

We assume that SNR-based detection is adopted in ST since it is one of the most common methods. SNR-base detection is tractable for theoretical analysis and can be implemented easily in practical systems [12]. Note that the length of cooperation phase t_1 has no effect on the detection performance of SNR-based detector, thus we can ignore the effect of t_1. The SNR-based detection mainly reflects the effect of the transmitted power and random channel rather than detection time. For other detection methods, the cooperation phase length may have an effect on the detection performance. For instance, matched filter and cyclostationary detection are sensitive to detection time. Even though the detection time has an effect on the detection performance, when cooperation phase time is fixed, the effect of CN transmitted power on detection performance will be dominating. Thus, the optimization process in this paper can also be suitable for other detection methods. For the SNR-based detection scheme [13], η denotes the detection threshold at ST, false alarm is said to occur if the received SNR $\gamma < \eta$, and the false alarm probability can be defined as

$$P_f = \Pr\{\gamma < \eta\} = \Pr\left\{\sum_{i=1}^{n} p_i h_i < \eta\right\}. \tag{3}$$

We assume that with independent identically distributed (i.i.d.) fading channels, namely $m_i = m$, the cooperation power is split equally among CNs. Thus, we have $p_i = p_0$. Applying characteristic functions and partial fraction techniques, the PDF of the received SNR at ST can be derived as below [12][13]:

$$\tilde{f}(\gamma) = \frac{\gamma^{n-1}}{(mp_0)^n (n-1)!} e^{-\frac{\gamma}{mp_0}}, \tag{4}$$

Hence, false alarm probability is expressed as

$$\begin{aligned} P_f &= \Pr\{\gamma < \eta\} = \int_0^{\eta} \tilde{f}(\gamma) d\gamma \\ &= \int_0^{\eta} \frac{\gamma^{n-1}}{(mp_0)^n (n-1)!} e^{-\frac{\gamma}{mp_0}} d\gamma . \\ &= \sum_{i=n}^{\infty} \frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}} \end{aligned} \tag{5}$$

and channel available probability P_a for ST can be denoted as

$$P_a = 1 - P_f = 1 - \sum_{i=n}^{\infty} \frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}} = \sum_{i=0}^{n-1} \frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}} . \tag{6}$$

Denote C as the throughput of secondary pair when ST detects the cooperation signal successfully and transmits in transmission phase. Let p_s be ST transmitted power, h instantaneous channel power gain between ST transmitter and receiver, and σ^2 the noise power. Then C can be formulated as

$$C = t_2 \log_2\left(1 + \frac{hp_s}{\sigma^2}\right). \tag{7}$$

Note that SNR-based detection is adopted at ST, thus the probability of the miss-detection caused by the external noise can be ignored. In addition, when miss-detection occurs, i.e., ST wrongly assumes the channel is free, both SU and PU transmissions will coexist and interfere with each other. Consequently, the transmissions for both SU and PU will be unsuccessful. We can conclude that the occurrence of miss-detection does not affect SU throughput. Therefore, we only consider the impact of false alarm probability on SU throughput in the transmission phase. From (6)(7), we have the average throughput (R) for SU denoted as

$$R = CP_a = t_2 \log_2\left(1+\frac{hp_s}{\sigma^2}\right)\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}}. \quad (8)$$

3. Optimization of CN Power and Number

In wireless communication, the energy in terminals is precious resource [14]. As a whole secondary network, we consider the cooperation-throughput tradeoff under the constraint of the total cooperation and transmission power is fixed. It is reason to assume the constraint of the total power since CNs and ST can be considered as a whole system. In the secondary network, the throughput for SU is related with the performance of cooperation phase. In this section, we study the fundamental tradeoff [15][16] between cooperation and average throughput. Using cooperative diversity and SNR-based detection scheme, we will prove that there exists the optimal transmission power of CN when the number of CNs is fixed and the optimal node number when the transmission power of CN is fixed. The optimization without the constraints of CN power and number is also considered.

3.1 Optimal CN Power with Fixed CN Number

In this part, we consider the optimization of CN power with the given number of CNs. Let E be the total energy for the whole cognitive network in once cooperation and transmission, and n be the number of CNs which is fixed. Obviously, from (6), we know that since (6) is monotonically increasing function of CN power p_0, for a given total power p_t and number of CNs n, the more CN power p_0, the higher channel available probability P_a, which corresponds to the case that SU can use the channel with a higher probability. On the other hand, the more CN power p_0, the less power for SU, which corresponds to the case that the average throughput will be less. The objective of cooperation-throughput tradeoff is to identify the optimal CN power that the achievable throughput for SU is maximized under the constraints of total power p_t and CN number n. The optimization problem can be formulated as

$$\begin{aligned} &\max_{p_0, p_s}\left\{t_2 \log_2\left(1+\frac{hp_s}{\sigma^2}\right)\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}}\right\} \\ &\quad s.t. \quad p_0, p_s \ge 0, \\ &\quad t_2 p_s + n t_1 p_0 = E \end{aligned} \quad . \quad (9)$$

In this paper, we mainly focus on the effect of CN number and power on system performance. For the simplification, we set $t_1 = t_2 = 1$. Thus, we can rewrite the (9) as follows

$$\begin{aligned} &\max_{p_0, p_s}\left\{\log_2\left(1+\frac{hp_s}{\sigma^2}\right)\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}}\right\} \\ &\quad s.t. \quad p_0, p_s \ge 0, \\ &\quad p_s + n p_0 = p_t \end{aligned} \quad , \quad (10)$$

where the total "power" p_t is set as $p_t = E$. The optimization problem (10) is general for arbitrary t_1 and t_2 since p_s and p_0 can be replaced by $t_2 p_s$ and $t_1 p_0$ respectively. Accordingly, the results for the optimization problem (10) can also instruct the general case.

For a given CN number n , we have

$$\max_{p_0} \left\{ \log_2\left(1+\frac{h\left(p_t - np_0\right)}{\sigma^2}\right)\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}} \right\}. \quad s.t.\ \ 0 \le np_0 \le p_t \tag{11}$$

Denote $1/\alpha$ as CN power proportion in the total power. $p_0 = p_t/\alpha$, $\alpha p_0 = p_t$. We have

$$\max_{\alpha} \left\{ \log_2\left(1+\gamma\left(1-\frac{n}{\alpha}\right)\right)\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\alpha\eta}{mp_t}\right)^i e^{-\frac{\alpha\eta}{mp_t}} \right\}, \quad s.t.\ \ \alpha \ge n \tag{12}$$

where $\gamma = hp_t/\sigma^2$.

Theorem 1: Under the constraint of total power for the cooperation and transmission phases, if the number of CNs is fixed, there exists an optimal CN power which yields the maximum average throughput for SU.

Proof: We define

$$C = \log_2\left(1+\left(1-\frac{n}{\alpha}\right)\gamma\right) > 0, \tag{13}$$

and

$$P_a = \sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\alpha\eta}{mp_t}\right)^i e^{-\frac{\alpha\eta}{mp_t}} > 0. \tag{14}$$

It can be verified from (13) (14)that

$$\frac{\partial C}{\partial \alpha} = \frac{n\gamma}{\alpha^2(1+\gamma)-\alpha n\gamma} > 0, \tag{15}$$

and

$$\begin{aligned}\frac{\partial P_a}{\partial \alpha} &= \sum_{i=1}^{n-1}\frac{\alpha^{i-1}}{(i-1)!}\left(\frac{\eta}{mp_t}\right)^i e^{-\frac{\alpha\eta}{mp_t}} - \sum_{i=0}^{n-1}\frac{\alpha^i}{i!}\left(\frac{\eta}{mp_t}\right)^{i+1} e^{-\frac{\alpha\eta}{mp_t}} \\ &= \sum_{i=0}^{n-2}\frac{\alpha^i}{i!}\left(\frac{\eta}{mp_t}\right)^{i+1} e^{-\frac{\alpha\eta}{mp_t}} - \sum_{i=0}^{n-1}\frac{\alpha^i}{i!}\left(\frac{\eta}{mp_t}\right)^{i+1} e^{-\frac{\alpha\eta}{mp_t}} \\ &= -\frac{\alpha^{n-1}}{(n-1)!}\left(\frac{\eta}{mp_t}\right)^n e^{-\frac{\alpha\eta}{mp_t}} < 0\end{aligned}. \tag{16}$$

Hence, the first-order derivative of R is:

$$\frac{\partial R}{\partial \alpha} = \frac{\partial C}{\partial \alpha}P_a + C\frac{\partial P_a}{\partial \alpha}. \tag{17}$$

Obviously, we have

$$\alpha \to n \Rightarrow \lim_{\alpha \to n} C = 0 \Rightarrow \lim_{\alpha \to n} \frac{\partial R}{\partial \alpha} > 0, \tag{18}$$

$$\alpha \to \infty \Rightarrow \lim_{\alpha \to \infty} P_a = 0 \Rightarrow \lim_{\alpha \to \infty} \frac{\partial R}{\partial \alpha} < 0. \tag{19}$$

From (18), we know that R is an increasing function of α when α approaches n and (19) means R is a decreasing function when α is large. Hence, there exists a maximum point of R within interval (n, ∞), i.e., there exists an optimal cooperative node power which yields the maximum average throughput for the secondary user [17].

This completes the proof.

Theorem 2: Under the constraint of total power for cooperation and transmission phases, when the number of CNs is fixed, R is concave for the range of CN power, and the optimal CN power which yields to the maximum average throughput is unique in this range.

Proof: From (13) and (15), the second-order derivatives are:

$$\frac{\partial^2 C}{\partial \alpha^2} = \frac{-n\gamma\left(2\alpha(1+\gamma) - n\gamma\right)}{\left(\alpha^2(1+\gamma) - \alpha n\gamma\right)^2} < 0, \tag{20}$$

and

$$\begin{aligned} \frac{\partial^2 P_a}{\partial \alpha^2} = &-\frac{\eta}{mp_t}\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\alpha\eta}{mp_t}\right)^i e^{-\frac{\alpha\eta}{mp_t}} \\ &+\sum_{i=2}^{n-1}\frac{\alpha^{i-2}}{(i-2)!}\left(\frac{\eta}{mp_t}\right)^i e^{-\frac{\alpha\eta}{mp_t}} - \sum_{i=1}^{n-1}\frac{\alpha^{i-1}}{(i-1)!}\left(\frac{\eta}{mp_t}\right)^{i+1} e^{-\frac{\alpha\eta}{mp_t}}, \end{aligned} \tag{21}$$

Similar to (20), we have

$$\frac{\partial^2 P_a}{\partial \alpha^2} < 0. \tag{22}$$

Hence, the second-order derivative of R is

$$\frac{\partial^2 R}{\partial \alpha^2} = \frac{\partial^2 C}{\partial \alpha^2} P_a + C\frac{\partial^2 P_a}{\partial \alpha^2} + 2\frac{\partial C}{\partial \alpha}\frac{\partial P_a}{\partial \alpha} < 0. \tag{23}$$

Form (23), we know that the second-order derivative of R is negative, thus R is concave for the range of α and it also means that R is concave for the range of P_0. From Theorem 1, we know that there is the maximum point of R within interval (n, ∞), and the concavity makes the maximum point of R to be unique in the interval (n, ∞). Therefore, the optimal CN power is unique in this range.

This completes the proof.

Due to the concavity of R, the optimal CN power proportion α can be obtained by efficient search algorithms, and the optimal CN power will also be obtained.

When the total power for cooperation and transmission is low, the cooperative diversity will vanish, and the condition of fixed number of CNs will be degenerated. This issue will be discussed in Section 3.3.

3.2 Optimal CN Number with Fixed CN Power

In this part, we consider the optimization of CN number with the fixed CN power. From (9), we know that the optimization problem can be formulated as

$$\max_{n,p_s}\left\{\log_2\left(1+\frac{hp_s}{\sigma^2}\right)\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}}\right\}$$
$$s.t.\quad n\ge 0, n\in \mathbf{Z}_+, \qquad , \tag{24}$$
$$p_s\ge 0,$$
$$p_s+np_0=p_t$$

where $\mathbf{Z}_+$ denotes the nonnegative integers set.

We can rewrite it as

$$\max_{n}\left\{\log_2\left(1+\frac{h(p_t-np_0)}{\sigma^2}\right)\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}}\right\}. \tag{25}$$
$$s.t.\quad 0\le n\le \frac{p_t}{p_0}, n\in \mathbf{Z}_+$$

Based on the Taylor series theory, the channel available probability can be denoted as

$$\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}} = 1-\frac{e^{-\frac{\eta}{mp_0}}e^{\xi}}{n!}\left(\frac{\eta}{mp_0}\right)^n,$$
$$0\le \xi \le \frac{\eta}{mp_0}. \tag{26}$$

The second term is Lagrange remainder of the Taylor series after n terms. We use the Gamma function to denote the continuous factorial,

$$n! = \Gamma(n+1) = \int_0^{\infty} x^n e^{-x} dx. \tag{27}$$

Thus, we can rewrite the channel available probability P_a with continuous variable n as below

$$P_a = 1-\frac{e^{-\frac{\eta}{mp_0}}e^{\xi}}{\Gamma(n+1)}\left(\frac{\eta}{mp_0}\right)^n. \tag{28}$$

Theorem 3: Under the constraint of total power for cooperation and transmission phases, if CN power is fixed, there exists an optimal CN number which yields to the maximum average throughput for SU.

Proof: Similar to the proof of Theorem 1, thus the detail of this proof is omitted.

Theorem 4: For $\ln(\eta/mp_0)\le t$, where t within $\left(0,\Gamma(n+1)'/\Gamma(n+1)\right]$, P_a is concave at the range of n, and t can be obtained by solving $\partial^2 P_a/\partial n^2 = 0$.

Proof: We can verify from (7) that, second-order derivative of C is

$$\frac{\partial^2 C}{\partial n^2} = \frac{-\gamma^2 p_0^2}{\left(p_t+\gamma\left(p_t-np_0\right)\right)^2} < 0. \tag{29}$$

The second-order derivative of P_a is

$$\frac{\partial^2 P_a}{\partial n^2} = \frac{e^{-\frac{\eta}{mp_0}} e^{\xi} \left(\frac{\eta}{mp_0}\right)^n}{\Gamma(n+1)^3} \Big(\Gamma(n+1)\Gamma(n+1)'' - \ln^2\left(\frac{\eta}{mp_0}\right)\Gamma(n+1)\Gamma(n+1) - 2\Gamma(n+1)'^2 + 2\ln\left(\frac{\eta}{mp_0}\right)\Gamma(n+1)\Gamma(n+1)'\Big) . \quad (30)$$

Notice that for

$$\ln\left(\frac{\eta}{mp_0}\right) = 0 , \quad (31)$$

we have

$$\frac{\partial^2 P_a}{\partial n^2} = e^{-\frac{\eta}{mp_0}} e^{\xi} \left(\frac{\eta}{mp_0}\right)^n \frac{\Gamma(n+1)\Gamma(n+1)'' - 2\Gamma(n+1)'^2}{\Gamma(n+1)^3} . \quad (32)$$

Based on the characteristic of recursion, we can obtain

$$\Gamma(n+1)\Gamma(n+1)'' - 2\Gamma(n+1)'^2 < 0 , \quad (33)$$

thus

$$\frac{\partial^2 P_a}{\partial n^2} < 0 . \quad (34)$$

On the other hand, for

$$\ln\left(\frac{\eta}{mp_0}\right) = \frac{\Gamma(n+1)'}{\Gamma(n+1)} , \quad (35)$$

we have

$$\frac{\partial^2 P_a}{\partial n^2} = \frac{e^{-\frac{\eta}{mp_0}} e^{\xi} \left(\frac{\eta}{mp_0}\right)^n}{\Gamma(n+1)^3} \left(\Gamma(n+1)\Gamma(n+1)'' - \Gamma(n+1)'^2\right) . \quad (36)$$

Because $\Gamma(n+1)$ is log-convex for $n \geq 0$, thus we can obtain

$$\Gamma(n+1)\Gamma(n+1)'' - \Gamma(n+1)'^2 \geq 0 , \quad (37)$$

i.e.,

$$\frac{\partial^2 P_a}{\partial n^2} > 0 . \quad (38)$$

Therefore, there exists a point t within $\left(0, \Gamma(n+1)'/\Gamma(n+1)\right]$ which makes $\partial^2 P_a / \partial n^2 = 0$, and P_a is concave when $\ln\left(\eta/mp_0\right) \leq t$, i.e., $\eta/mp_0 \leq e^t$. t can be calculated by solving $\partial^2 P_a / \partial n^2 = 0$ as a quadratic equation.

Furthermore, from the above analyses, we have

$$\frac{\partial^2 R}{\partial n^2} = C\frac{\partial^2 P_a}{\partial n^2} + P_a \frac{\partial^2 C}{\partial n^2} + 2\frac{\partial P_a}{\partial n}\frac{\partial C}{\partial n} < 0 . \quad (39)$$

Therefore, R is concave when $\eta/mp_0 \le e^t$. Hence, there is a maximum point of R within the interval $\left(0, p_t/p_0\right)$.

This completes the proof.

3.3 Optimization without the Constraints of CN Power and Number

In this part, we consider the scenario without the constraints of CN power and number. Accordingly, the optimization problem can be formulated as

$$\begin{aligned}&\max_{n,p_0}\left\{\log_2\left(1+\frac{h\left(p_t-np_0\right)}{\sigma^2}\right)\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}}\right\}\\ &s.t.\quad n\ge 0, n\in \mathbf{Z}_+, \\ &np_0 \ge p_t\end{aligned} \quad . \tag{40}$$

From preceding analysis, we know that the channel available probability P_a is a function of CN power p_0 and number n. Firstly, we study the channel available probability P_a when the total cooperation power is fixed. For a given total power p_c for cooperation phase, the problem of maximum channel available probability can be formulated as

$$\begin{aligned}&\max_{n,p_0}\left\{\sum_{i=0}^{n-1}\frac{1}{i!}\left(\frac{\eta}{mp_0}\right)^i e^{-\frac{\eta}{mp_0}}\right\}\\ &s.t.\quad n\ge 0, n\in \mathbf{Z}_+, \\ &np_0 = p_c\end{aligned} \quad . \tag{41}$$

Theorem 5: Without the constraints of power and number of CN, for $p_t \le \eta/m$, the maximum average throughput will be obtained at CN number $n=1$; for $p_t > \eta/m$ and p_t is large enough, the maximum average throughput will be obtained at CN number $n=+\infty$; otherwise, the larger throughput for $n=1$ and $n=+\infty$ is the maximum average throughput for SU.

Proof: Similar to the analyses of fixed cooperative node power, we can know the channel available probability is increasing function of n while $mp_c > \eta$, and is decreasing function of n for $mp_c < \eta$.

When $n \to \infty$, correspondingly the channel available probability is

$$P_a = \begin{cases}1 & mp_c > \eta, n=+\infty, n\in\mathbf{Z}_+\\ 0 & mp_c < \eta, n=+\infty, n\in\mathbf{Z}_+\end{cases}. \tag{42}$$

Thus, we can conclude that when the total cooperation power p_c can be set more than η/m, the channel available probability $P_a = 1$ will be obtained at the cooperative node number $n \to \infty$, meanwhile the total cooperation power $p_c \to \left(\eta/m\right)^+$ and the maximum power for secondary user transmission is $p_t - \left(\eta/m\right)^+$. When the total cooperation power p_c is less than η/m, the maximum channel available probability will be obtained at $n=1$ due to the decreasing function, and the maximum average throughput can be calculated from (20).

Therefore, the average throughput for the secondary user can be formulated as

$$R = \begin{cases}\log_2\left(1+\dfrac{h\left(p_t-\eta/m\right)}{\sigma^2}\right) & n=+\infty, mp_c>\eta\\ \max\limits_{p_c}\left\{\left(1-e^{-\frac{\eta}{mp_c}}\right)\log_2\left(1+\dfrac{h^2\left(p_t-p_c\right)}{\sigma^2}\right)\right\} & n=1, mp_c<\eta\end{cases}. \tag{43}$$

Obviously, for $p_t \le \eta/m$, the maximum average throughput will be obtained at $n=1$ due to the decreasing function; for $p_t > \eta/m$, the maximum average throughput for the secondary user can be formulated as

$$R = \max\left\{\log_2\left(1+\frac{h(p_t-\eta/m)}{\sigma^2}\right),\ \max_{p_c}\left\{\left(1-e^{-\frac{\eta}{mp_c}}\right)\log_2\left(1+\frac{h(p_t-p_c)}{\sigma^2}\right)\right\}\right\}. \tag{44}$$

This completes the proof.

4. Numerical Results

In this section, the Monte Carlo simulations and the theoretical results are derived to validate the preceding analyses. The Monte Carlo simulations are implemented by MATLAB. The corresponding parameters are set up as follows: the mean channel gain between CN and ST is $m=10$, and the SNR between ST and SR is $\gamma=10dB$. The SNR-based detection scheme is used at SU. We are interested in the analysis of different cases mentioned above.

4.1 Optimal CN Power with Fixed CN Number

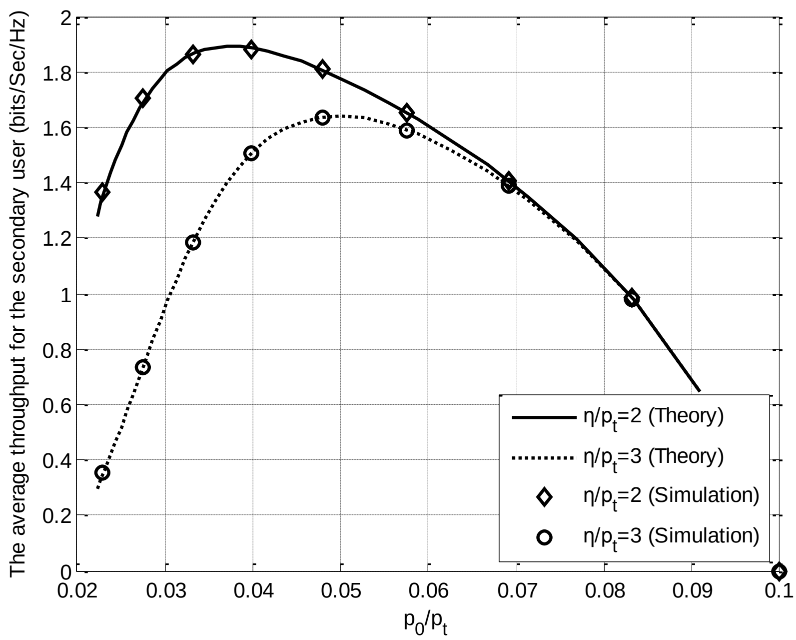

Fig. 2. The average throughput for SU versus p_0/p_t. The number of CNs is $n=10$.

We consider the optimal CN power with fixed number of CNs $n=10$. **Fig. 2** shows the average throughput for SU versus p_0/p_t, where p_0/p_t means that the power proportion for a CN. It is seen that for $\eta/p_t=2$, the maximum average throughput for SU is achieved at the power proportion of about $p_0/p_t=0.0385$; for $\eta/p_t=3$, the maximum average throughput for SU is achieved at the power proportion of about $p_0/p_t=0.05$. **Fig. 3** illustrates the relationship between throughput and inverse proportion α. We can calculate the optimal CN power more simply and accurately by searching α. From **Fig. 3**, we can see that for $\eta/p_t = 2$ and $\eta/p_t = 3$, the maximum average throughput for SU is achieved at $\alpha = 26$ and $\alpha = 20$, respectively.

Fig. 2 and **Fig. 3** show the average throughput for SU are concave for the range of CN power p_0, and the maximum point of R is unique in this range. Furthermore, the simulation results match to the theoretical results well.

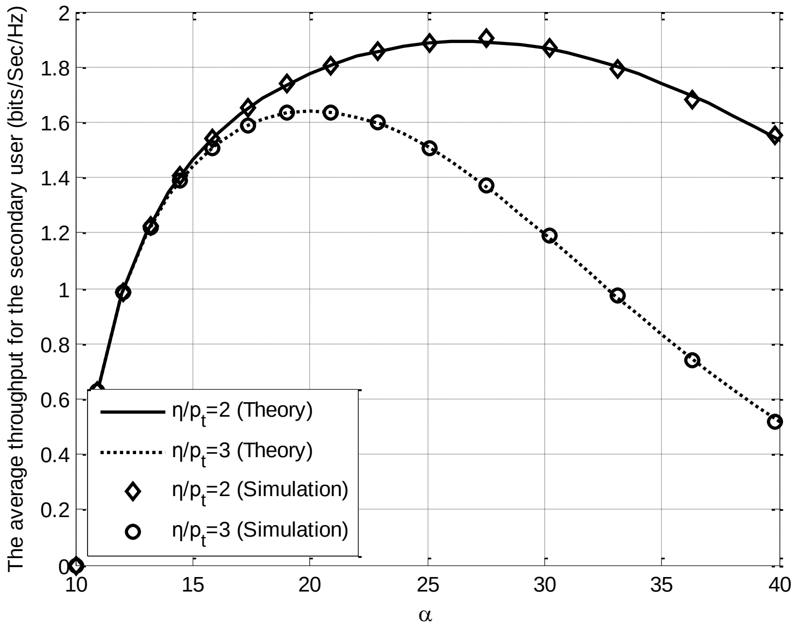

Fig. 3. The average throughput for SU versus α. The number of CNs is $n = 10$.

4.2 Number Selection with fixed CN Power

In this part, the scenario that the optimal number of CNs with fixed CN power $p_t/p_0 = 60$ is considered. The mean channel gain between CN and ST is $m = 10$, and the SNR between ST and SR is $\gamma = 10dB$. **Fig. 4** and **Fig. 5** show that the discrete simulation results match to the continuous theoretical results very well. **Fig. 4** shows that for $\ln(\eta/mp_0) \le \Gamma'(n+1)/\Gamma(n+1)$, the maximum throughput will be obtained at $n = 5$ and $n = 7$ while $\eta/p_0 = 10$ and $\eta/p_0 = 20$, respectively. For the simplicity, we can see that for $\ln(\eta/mp_0) = 0$, i.e., $\eta/p_0 = 10$, the average throughput for SU R is concave for the continuous number of CNs n, and the maximum point of R is unique in this continuous range. If the optimal continuous number of CNs falls into this range, efficient search algorithms can be developed.

Fig. 5 shows the average throughput for $\ln(\eta/mp_0) > \Gamma'(n+1)/\Gamma(n+1)$, where R is not concave for the whole range of continuous n. For $n > \hat{n}$, where $\hat{n}$ satisfies $\Gamma'(\hat{n}+1)/\Gamma(\hat{n}+1) \le t$, R is concave for the range of continuous $[\hat{n}, +\infty)$. We notice that the maximum point of R is unique in this range, and the maximum point is the global maximum point. Hence, we can search the optimal number from $\hat{n}$.

The optimal discrete number of CNs can be calculated from the optimal continuous number easily. However, the optimal discrete number may not be unique in the discrete range.

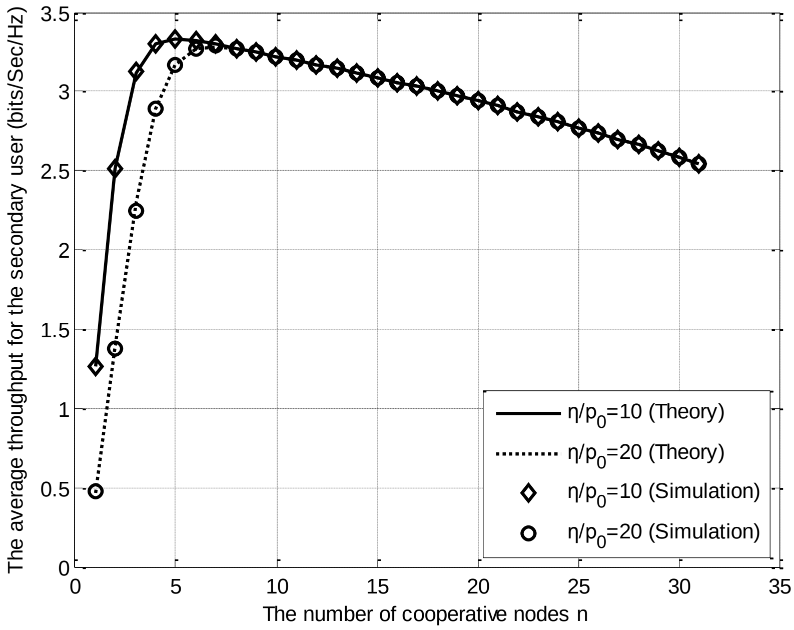

Fig. 4. The average throughput for SU versus the number of CNs n at $\eta/p_0 = 10$ and $\eta/p_0 = 20$, and $p_t/p_0 = 60$.

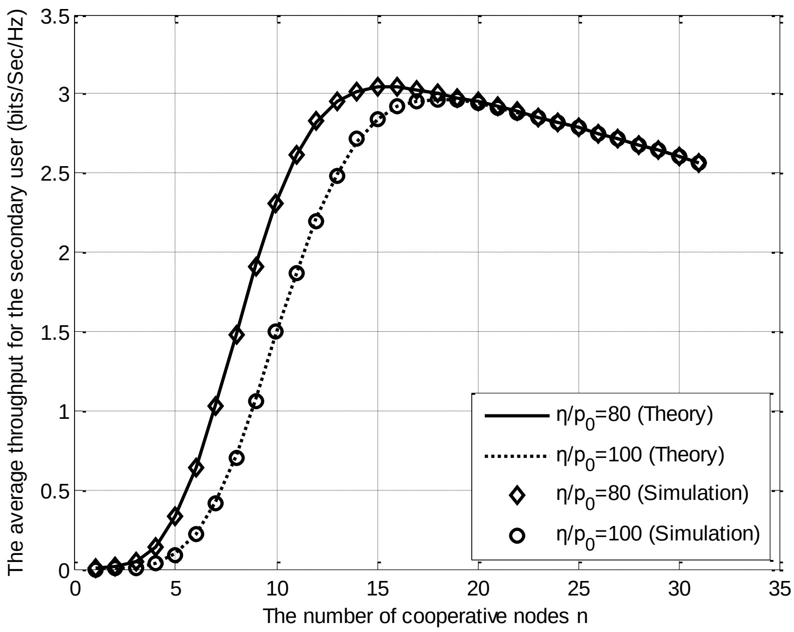

Fig. 5. The average throughput for SU versus the number of CNs n at $\eta/p_0 = 80$ and $\eta/p_0 = 100$, and $p_t/p_0 = 60$.

4.3 Power Allocation and Nodes Selection without CN constraint

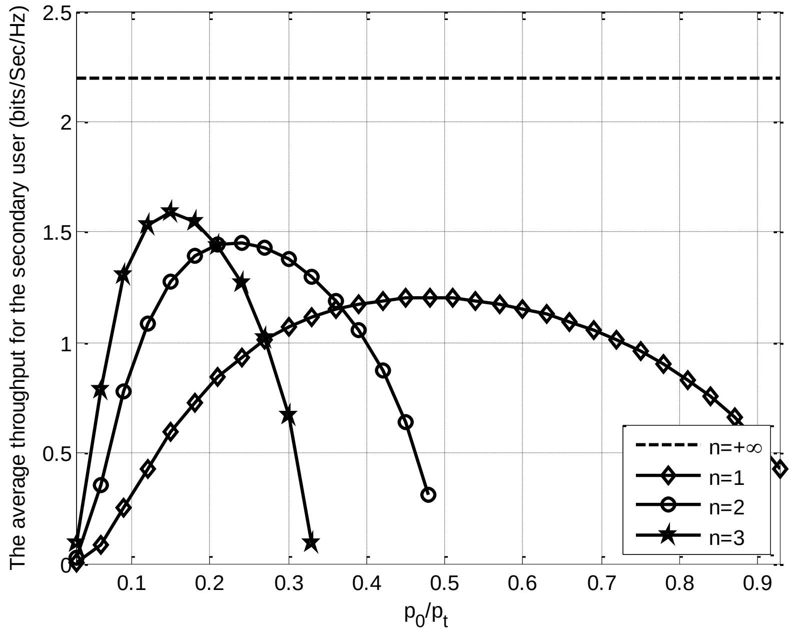

Fig. 6. The average throughput for SU versus p_0/p_t , and $\eta/p_t = 2$.

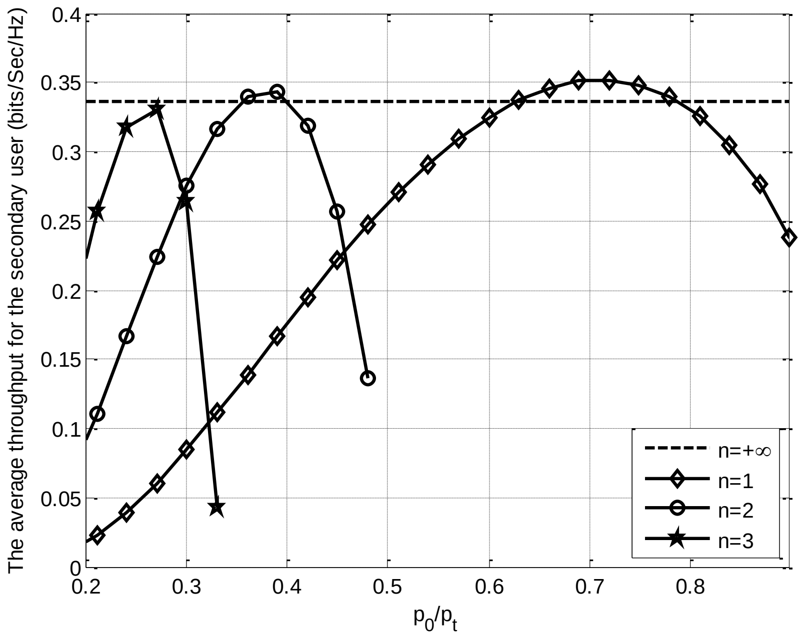

Fig. 7. The average throughput for SU versus p_0/p_t , and $\eta/p_t = 9.6$.

Fig. 6 and **Fig. 7** demonstrate the case of unfixed CN power and number. For $p_t \leq \eta/m$, the maximum average throughput will be obtained at $n=1$ due to the decreasing function. **Fig. 6** shows that the maximum average throughput is obtained at $\log_2(1+\gamma(1-\eta/mp_t))$ for $n=+\infty$. **Fig. 7** shows that the maximum average throughput is obtained when the number of CN is $n=1$ for $p_t > \eta/m$. We can further to validate that when the total power p_t is more than a certain threshold which is a function of η/m, the maximum average throughput will be obtained at $\log_2(1+\gamma(1-\eta/mp_t))$ for $n=+\infty$; otherwise, the maximum average throughput is obtained when the number of CN is $n=1$.

5. Conclusions

In cognitive radio networks, cooperative spectrum sensing has the potential to improve the accuracy of spectrum sensing and alleviate the hidden terminal problem. In this paper, we consider a cognitive radio network with CNs which detect the primary users' activities and cooperate with ST at the cooperation phase. A simple handshake between SU and CNs is proposed without a separate control channel. Then, we formulate the cooperation and transmission phases, and the problem of designing the power and number of CN in the cooperation phase to maximize the average throughput for SU under the constraint that the total cooperation and transmission power, where only mean channel gain is available. We prove the characteristics of the optimal CN power with fixed number of CNs, and the optimal CN number with fixed power. The case without constraints of CN power and number is also considered. Finally, the numerical results show the characteristics and existences of optimal CN power and number. Meanwhile, Monte Carlo simulation results match to the theoretical results well.

References

[1] J. Mitola III and G. Q. Maguire Jr., "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, Aug. 1999. Article (CrossRef Link)

[2] Z. Han, R. Fan, and H. Jiang, "Replacement of spectrum sensing in cognitive radio," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2819–2826, Jun. 2009. Article (CrossRef Link)

[3] J. Shen, S. Liu, L. Zeng, G. Xie, J. Gao and Y. Liu, "Optimisation of cooperative spectrum sensing in cognitive radio network," *IET Commun.*, vol. 3, no. 7, pp. 1170-1178, 2009. Article (CrossRef Link)

[4] R. Fan and H. Jiang, "Optimal multi-channel cooperative sensing in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 3, pp. 1128-1138, Mar. 2010. Article (CrossRef Link)

[5] G. Ganesan and Y. Li, "Cooperative spectrum sensing in cognitive radio—part I: two user networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2204-2213, June 2007. Article (CrossRef Link)

[6] G. Ganesan and Y. Li, "Cooperative spectrum sensing in cognitive radio—part II: multiuser networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2214-2222, June 2007. Article (CrossRef Link)

[7] W. A. Hassan, H. S. Jo, M. Nekovee, C. Y. Leow and T. A. Rahman, "Spectrum sharing method for cognitive radio in TV white spaces: enhancing spectrum sensing and geolocation database," *KSII Trans. Internet and Information Systems,* vol. 6, no. 8, pp. 1894-1912, Aug. 2012. Article (CrossRef Link)

[8] D. Xu, Z. Feng and P. Zhang, "Power allocation schemes for downlink cognitive radio networks with opportunistic sub-channel access," *KSII Trans. Internet and Information Systems,* vol. 6, no. 7, pp. 1777-1791, July 2012. Article (CrossRef Link)

[9] D. Duan, L. Yang, and J. C. Principe, "Cooperative diversity of spectrum sensing in cognitive

radio networks," in *Proc.of IEEE WCNC*, Budapest, pp. 1-6, Apr. 2009. Article (CrossRef Link)
[10] Z. Quan, S. Cui, A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE J. Select. Areas Signal Processing*, vol. 2, no. 1, pp. 28–40, Feb. 2008. Article (CrossRef Link)
[11] X. Zhang, Q. Wu and J. Wang, "Power allocation in cognitive radio networks with cooperative spectrum sensing," *Int. J. Electron. Commun*., vol. 66, no. 11, pp. 949-954, Nov. 2012. Article (CrossRef Link)
[12] J. Luo, R. Blum, L. Cimini, L. Greenstein and A. Haimovich, "Power allocation in a transmit diversity system with mean channel gain information," *IEEE Commun. Lett.*, vol. 9, no. 7, pp. 616-618, July 2005. Article (CrossRef Link)
[13] J. Luo, R. Blum, L. Cimini, L. Greenstein and A. Haimovich, "Decode-and-forward cooperative diversity with power allocation in wireless networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 3, pp.793-799, Mar. 2007. Article (CrossRef Link)
[14] J. Wei and X. Zhang, "Energy-efficient distributed spectrum sensing for wireless cognitive radio networks," in *Proc. IEEE INFOCOM*, San Diego, CA, pp. 1-6, Mar. 2010. Article (CrossRef Link)
[15] A. Ghasemi and E. S. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design trade-offs," *IEEE Commun. Magazine*, pp. 32-39, Apr. 2008. Article (CrossRef Link)
[16] Y.-C. Liang, Y. Zeng, E. C. Y. Peh, and A. T. Hoang, "Sensing throughput tradeoff for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1326-1337, Apr. 2008. Article (CrossRef Link)
[17] S. P. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004. http://www.optimization-online.org/DB_FILE/2007/10/1798.pdf

Xian Zhang received his B.S. degree in communications engineering, from Institute of Communications Engineering, Nanjing, China, in 2007. He is currently pursuing the Ph.D. degree in communications and information system in Institute of Communications Engineering, PLA University of Science and Technology. His research interests focus on cognitive radio, optimization of wireless network.

Qihui Wu received his B.S. degree in communications engineering, M.S. degree and Ph.D. degree in communications and information system from Institute of Communications Engineering, Nanjing, China, in 1994, 1997 and 2000, respectively. He is currently professor at the PLA University of Science and Technology, China. His current research interests are algorithms and optimization for cognitive wireless networks, soft-defined radio and wireless communication.

Xiaoqiang Li received the B. S. degree and the M. S. degree both from PLA University of Science and Technology, Nanjing, China in 1997 and 2007 respectively. He is currently pursuing for the Ph. D degree in Communications and Information System at the Institute of Communications Engineering, PLA University of Science and Technology, Nanjing, China. His current research interests include cognitive radio technology and digital communications.

Zi Yun received her B.S. degree in communications engineering, M.S. degree in communications and information system from Institute of Communications Engineering, Nanjing, China, in 2007 and 2010, respectively. She is currently working at the PLA Unit of 94860. Her research interests focus on HF communication and satlelite communication.