

Unified Model for Performance Analysis of IEEE 802.11 Ad Hoc Networks in Unsaturated Conditions

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Abstract

IEEE 802.11 standard has achieved huge success in the past decade and is still under development to provide higher physical data rate and better quality of service (QoS). An important problem for the development and optimization of IEEE 802.11 networks is the modeling of the MAC layer channel access protocol. Although there are already many theoretic analysis for the 802.11 MAC protocol in the literature, most of the models focus on the saturated traffic and assume infinite buffer at the MAC layer. In this paper we develop a unified analytical model for IEEE 802.11 MAC protocol in ad hoc networks. The impacts of channel access parameters, traffic rate and buffer size at the MAC layer are modeled with the assistance of a generalized Markov chain and an M/G/1/K queue model. The performance of throughput, packet delivery delay and dropping probability can be achieved. Extensive simulations show the analytical model is highly accurate. From the analytical model it is shown that for practical buffer configuration (e.g. buffer size larger than one), we can maximize the total throughput and reduce the packet blocking probability (due to limited buffer size) and the average queuing delay to zero by effectively controlling the offered load. The average MAC layer service delay as well as its standard deviation, is also much lower than that in saturated conditions and has an upper bound. It is also observed that the optimal load is very close to the maximum achievable throughput regardless of the number of stations or buffer size. Moreover, the model is scalable for performance analysis of 802.11e in unsaturated conditions and 802.11 ad hoc networks with heterogenous traffic flows.

Keywords: IEEE 802.11, Ad Hoc Networks, Performance Analysis, Markov Chain, M/G/1/K Model

A preliminary version of this paper appeared in IEEE Globecom 2008, Dec., New Orleans, USA. This version includes comparison of M/G/1/K and M/M/1/K queuing model, performance optimization of 802.11 MAC layer, the modeling of 802.11e and heterogenous traffic flows over 802.11 ad hoc networks.

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1. Introduction

IEEE 802.11 [1] standard is the dominant technology for wireless local area networks (WLANs). Distributed coordination function (DCF) is one of the channel access protocols specified for the media access control (MAC) layer of IEEE 802.11 networks. DCF is based on carrier sense multiple access with collision avoidance (CSMA/CA) with binary exponential backoff. There are two channel access methods specified for DCF: basic access and the Request-To-Send/Clear-To-Send (RTS/CTS) mechanism. Because of its low complexity and easy configuration, DCF contributes significantly to the popularity of 802.11 wireless networks and will continue to play important roles in the standards such as IEEE 802.11e for QoS enhancement and 802.11n for higher data rate [2]. In addition to the infrastructure WLAN, DCF is crucial for the widely studied wireless multi-hop ad hoc networks. It is obvious that efficient modeling of DCF is one of the most important problems for the 802.11 wireless ad hoc networks.

Up to now, there are a lot of works reported on the performance evaluation of 802.11 DCF protocol. Bianchi [4] proposes the famous Markov chain model for 802.11 DCF in saturation conditions. Using the Markov chain model, the performance of 802.11 DCF has been extensively studied for saturation conditions [4][5]. With respect to performance analysis, IEEE 802.11e can achieve relative service differentiation such as throughput or delay in saturated conditions [6][7][8][9]. Because the unsaturated case is more typical, analysis of IEEE 802.11 DCF in non-saturated case has attracted remarkable attention recently. In [10], an analytical model was presented for 802.11e in non-saturated conditions. Zhai, Chen and Fang [11] presented an inspiring discovery that the original 802.11 WLAN can support strict QoS under unsaturated traffic load. Zhai et al [12][13] also derives the probability distribution of the MAC layer service time by using the signal transfer function of generalized state transition diagram and then describes the 802.11 MAC layer by M/G/1/K and M/M/1/K model, the work has important merit but Bianchi's model is directly applied to the unsaturated case without extensions, in addition, there are obvious deviation between the analytical and simulation results. In [14][15] and [16], a non-Markovian model (i.e., G/G/1 queuing model) was developed for IEEE 802.11 MAC protocol but it was based on unsuitable assumption that the stations have infinite queue length. In [17], a novel analytical stochastic reward net model is derived for performance evaluation of the IEEE 802.11 DCF MAC protocol in multi-hop ad hoc networks in the presence of hidden nodes, taking into account the characteristics of the physical layer, different traffic loads, packet size and carrier sense range.

In addition, many researchers [18][19][20][21][22][23][26] focus on analysis of 802.11 DCF under limited load by extending Bianchi's Markov chain model [4]. Among these extensions, [17][18][19] was based on 2D Markov chain model. In [18] the detailed post backoff process is modeled for 802.11 DCF in non-saturated conditions. The model in [18] is complex and hard to be used for queuing analysis, in addition, neither analytical nor simulation results for packet size diversity are given. In [19], we develop a simple yet accurate Markov chain model for non-saturated throughput analysis of 802.11 DCF protocol. It's shown that basic mechanism can achieve almost the same maximum throughput as that of RTS/CTS mechanism in non-saturated conditions. However, both the models in [18] and [19] are based on the assumption that each station can buffer just one packet. In [20][21][22][23][26], 3-D Markov chain model is developed for IEEE 802.11 DCF under finite load but the models are complicated and lack of scalability for queuing analysis of IEEE 802.11e or traffic flow diversity. Note that in [22], the derivation of modeling on traffic flow diversity is not explicit. In addition, the non-Markovian approach which is combined with the

M/G/1/K model is developed for modeling 802.11 DCF networks with finite buffer capacity in [24] and [25]. The models have good originality and much merits. Comparing to the model in [24][25], because the virtual slot in unsaturated cases is well defined, our unified model has good scalability for performance optimization of 802.11 MAC layer, modeling of 802.11e and heterogenous traffic flows over 802.11 ad hoc networks.

To the authors' best knowledge, there is a lack of unified and scalable model for 802.11 ad hoc networks with arbitrary buffer capacity in unsaturated cases. The performance optimization of 802.11 ad hoc networks, modeling of 802.11e and heterogenous traffic flows over 802.11 ad hoc networks are still open problems. In this paper we have extended our previous work to investigate the impacts of channel access parameters, traffic rate and buffer size on the 802.11 DCF performances. There are three major contributions in this paper. Firstly, we develop a unified analytical model for 802.11 MAC protocol with arbitrary buffer size in unsaturated conditions. The model is a combination of the 2D Markov chain model in unsaturated conditions [19] and an M/G/1/K queuing model. The analytical model enables very accurate queuing analysis of IEEE 802.11 MAC layer (including the impact of buffer size on performance). Secondly, we derive the optimal value of total load and the optimal achievable performance metrics for 802.11 ad hoc networks at MAC layer, which has significant merit for statistical QoS provisioning in IEEE 802.11 wireless networks. Finally, the analytical model is extended for modeling of 802.11e in unsaturated conditions and IEEE 802.11 ad hoc networks with heterogenous traffic flows.

The remainder of the paper is organized as follows. The analytical model is described in Section 2. In Section 3, we give the performance evaluation. In Section 4, some important applications of the model are provided. In Section 5, we discuss the analytical model. Finally, we conclude this paper. Note that part of this paper was presented at IEEE Globecom 2008 [27].

2. Unified Analytical Model

In this paper we consider only the basic access mechanism. The analytical model is in the assumption of ideal channel conditions (i.e., no hidden terminals and capture). In the model we also set that: (i) the network consists of n contending stations which can communicate with each other, i.e., we focus on the single-hop IEEE 802.11 ad hoc networks in this paper. (ii) each station's transmission buffer can contain K data packets and a packet will be removed upon successfully transmission or reaching the retry limits, (iii) each station receives packets from upper layer based on Poisson process with arrival rate λ and packet size is L , (iiii) the probability p that a transmission from a station collides is assumed to be constant regardless of the transmission history. For the meaning of other symbols in the analytical model, please refer to the [Table 1](#).

2.1 Generalized Markov Chain Model

The Markov chain model in [4] is developed for modeling of state transition of backoff counter in each station under saturated conditions. Let $j \in [0, m]$ denote the backoff stage and W_j be the corresponding contention window size. Note that $W_j = 2^j W$, where W is the initial contention window and m is the maximum backoff stage, which is specified in 802.11 standard. Let $b(t)$ be the value of backoff time counter and $s(t)$ the backoff stage for a given station at time t . Since the probability p that a transmission from a station collides is assumed to be constant regardless of the transmission history, the bidimensional stochastic process $(s(i, t), b(i,$

t) forms a discrete time Markov chain. However, in unsaturated conditions the backoff counter is not activated when there is no packet in transmission buffer. In order to model the unsaturated scenarios, we introduce an extra state $(-1,0)$ for the backoff counter, representing the buffer is empty. Let η_0 be the stationary probability that the transmission queue is empty upon a departure (i.e., a data packet is removed from the buffer due to successfully transmission or reaching the retry limits) in steady state. Therefore, when a data packet is removed from the buffer, the station will enter the state $(-1,0)$ with probability η_0 or activate the exponential backoff process with probability $(1-\eta_0)$. If the buffer is empty, the following time is virtually slotted, once a new packet arrives during a virtual slot time the binary exponential backoff process will be activated at the end of the virtual slot time, otherwise the station will keep in the state $(-1,0)$ and wait for the next virtual slot time. The probability $(1-q)$ that no packet arrives during a virtual slot time depends on the packet arriving rate and the size of virtual slot time. Note that the size of virtual slot time used here is equal to the average interval between two consecutive backoff counter decrements (i.e., a “real” slot time), which is a function of system load. The generalized Markov chain model is shown in Fig.1. Note that if $\eta_0 = 0$, i.e., the transmission queue is always non-empty, the model becomes the same as that in [4]. If $K=1$, we have the probability $\eta_0=1$ and the model becomes the same as that in [19]. The transition probabilities for the generalized Markov chain are as follows:

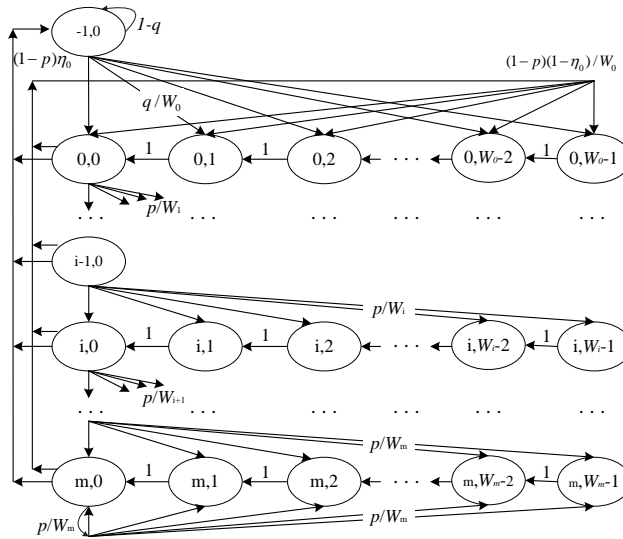


Fig.1. Generalized Markov Chain Model

$$\begin{cases}
 P\{i, k | i, k + 1\} = 1 & k \in [0, W_i - 2] \quad i \in [0, m] \\
 P\{i, k | i - 1, 0\} = p / W_i & k \in [0, W_i - 1] \quad i \in [1, m] \\
 P\{m, k | m, 0\} = p / W_m & k \in [0, W_m - 1] \\
 P\{-1, 0 | i, 0\} = (1 - p)\eta_0 & i \in [0, m] \\
 P\{-1, 0 | -1, 0\} = 1 - q \\
 P\{0, k | -1, 0\} = q / W_0 & k \in [0, W_0 - 1] \\
 P\{0, k | i, 0\} = \frac{(1 - p)(1 - \eta_0)}{W_0} & k \in [0, W_0 - 1] \quad i \in [0, m]
 \end{cases} \tag{1}$$

where the following short notation is adopted:

$$P\{j_1, k_1 | j_0, k_0\} = P\{s(t+1) = j_1, b(t+1) = k_1 | s(t) = j_0, b(t) = k_0\}$$

Let $b_{i,k} = \lim_{t \rightarrow \infty} P\{s(t) = i, b(t) = k\}$ be the stationary distribution of the chain. We get

$$b_{i,k} = \begin{cases} p^i b_{0,0} & i \in [0, m-1] & k = 0 \\ \frac{p^m}{1-p} b_{0,0} & i = m & k = 0 \\ \frac{w_i - k}{w_i} b_{i,0} & i \in [0, m] & k \in [0, w_i - 1] \\ (\frac{\eta_0}{q}) b_{0,0} & i = -1 & k = 0 \end{cases} \quad (2)$$

By imposing the normalization condition, we get $b_{0,0}$ as,

$$b_{0,0} = \frac{2(1-2p)(1-p)q}{(1-2p)[(W+1)q + 2\eta_0(1-p)] + pqW[1-(2p)^m]} \quad (3)$$

The probability τ that a station transmits in a randomly chosen slot time can be given by,

$$\begin{aligned} \tau &= \sum_{j=0}^m b_{j,0} = \frac{q \cdot b_{-1,0}}{(1-p)\eta_0} = \frac{b_{0,0}}{1-p} \\ &= \frac{2(1-2p)q}{(1-2p)[(W+1)q + 2\eta_0(1-p)] + pqW[1-(2p)^m]} \end{aligned} \quad (4)$$

The conditional probability p can be expressed as:

$$p = 1 - (1-\tau)^{n-1} \quad (5)$$

The average size of a “real” slot time for a station is given as

$$E[slot] = P_s \cdot T_s + P_{idle} \cdot \sigma + (1 - P_s - P_{idle}) \cdot T_c \quad (6)$$

Where P_s is the probability that the channel is sensed busy due to a successfully transmission among the other $n-1$ stations, P_{idle} is the probability that the channel is sensed idle by the tagged station We have

$$P_s = (n-1) \cdot \tau \cdot (1-\tau)^{n-2}, P_{idle} = (1-\tau)^{n-1} \quad (7)$$

and T_s is the time for a successfully transmission, T_c is the time that the channel sensed busy due to a collision, σ is the duration of an empty system slot time, which is given by

$$\begin{cases} T_s = \frac{H_{PHY}}{CR} + \frac{H_{MAC} + L}{R} + SIFS + \delta + \frac{ACK}{CR} + \delta + DIFS \\ T_c = \frac{H_{PHY}}{CR} + \frac{H_{MAC} + L}{R} + SIFS + \delta + EIFS \end{cases} \quad (8)$$

In fact, the length of a collision depends on whether a station is involved in the collision (including a vendor selected ACKTimeout) or is an onlooker (then using EIFS). Note that the 802.11 standards do not specify the length for ACKTimeout. According to the simulation environments, we choose $T_c = T_s$, Hence we have,

$$E[slot] = p \cdot T_s + (1-p) \cdot \sigma \quad (9)$$

The probability $1-q$ that no packet arrives from upper layer during a virtual slot time can be computed by:

$$1-q = \exp(-\lambda \cdot E[slot]) \quad (10)$$

If $K=1$, the probability η_0 is equal to 1, hence (4), (5) and (10) forms a nonlinear system with three unknown parameters τ , p and q , which can be solved using numerical methods. If $K>1$, η_0 depends on the queuing performance of 802.11 MAC layer, which will be analyzed in the next sub-section.

2.2 M/G/1/K Queuing Model

Since the packet transmission process at each station can be seen as a single “server”, each station can be modeled as an M/G/1/K. Let p_k be steady-state probability of k packets in the queuing system and η_k be the probability of k packets in the queuing system upon a departure at the steady state. Let X_k be the number of packets in the queue upon the k th departure, then the stochastic process X_k ($k=1,2,\dots$) forms a discrete time Markov chain which is shown in Fig. 2, where a_k is the probability that there are k arrivals during MAC service time T_{MAC} which is defined as the interval between the time that the packet becomes a Head-of-Line (HOL) packet in the transmission queue and the time that the packet is acknowledged for correct reception by the destination.

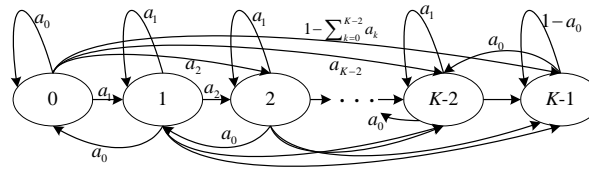


Fig. 2. State transition of Markov process X_k

The K -dimensional matrix of state transition probability \mathbf{P} is given by,

$$\mathbf{P} = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{K-2} & 1 - \sum_{k=0}^{K-2} a_k \\ a_0 & a_1 & a_2 & \cdots & a_{K-2} & 1 - \sum_{k=0}^{K-2} a_k \\ 0 & a_0 & a_1 & \cdots & a_{K-3} & 1 - \sum_{k=0}^{K-3} a_k \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_0 & 1 - a_0 \end{bmatrix} \tag{11}$$

Let $\boldsymbol{\eta} = [\eta_0 \ \eta_1 \ \cdots \ \eta_{K-1}]$, we have,

$$\boldsymbol{\eta} \cdot \mathbf{P} = \boldsymbol{\eta}, \quad \sum_{k=0}^{K-1} \eta_k = 1 \tag{12}$$

From which we can obtain $\boldsymbol{\eta}$ including the probability η_0 .

According to M/G/1/K queuing model, we get,

$$\begin{cases} p_0 = \frac{\eta_0}{\eta_0 + \rho}, p_k = \frac{\eta_k}{\eta_0 + \rho} & (k \in [0, K-1]) \\ p_K = 1 - \frac{1}{\eta_0 + \rho} \end{cases} \tag{13}$$

Where ρ is the traffic intensity and $\rho = \lambda \cdot E[T_{MAC}]$.

2.3 Probability Generating Function (PGF) of MAC Layer Service Time

Note that the MAC service time T_{MAC} consists of multiple “real” slot times (i.e., $E[slot]$), possible collision periods (i.e., T_c) and one successfully transmission time T_s . From the state transition process in Fig. 1, we can derive the PGF of MAC service time $F(Z)$ as follows:

$$\left\{ \begin{array}{l} FW_i(Z) = \sum_{j=0}^{2^i W - 1} (Z^{E[slot]})^j / (2^i W), \quad i \in [0, m] \\ F(Z) = (1-p)Z^{T_s} \sum_{i=0}^{m-1} (pZ^{T_c})^i \prod_{j=0}^i FW_j(Z) \\ \quad + \frac{(1-p)Z^{T_s} (pZ^{T_c})^m \prod_{j=0}^m FW_j(Z)}{1 - (pZ^{T_c})FW_m(Z)} \end{array} \right. \quad (14)$$

$F(Z)$ can be expanded as:

$$F(Z) = \sum_{i=0}^{\infty} \Pr(T_{MAC} = i \cdot E[slot]) Z^{(i \cdot E[slot])} \quad (15)$$

Note that $F(Z)$ completely characterizes the discrete probability distribution of T_{MAC} and we can get the arbitrary n th moment of T_{MAC} by differentiation (hence the mean value and standard deviation),

$$\mu^{-1} = E[T_{MAC}] = \left. \frac{dF(Z)}{dZ} \right|_{Z=1} \quad (16)$$

Since Poisson arrivals of packets is assumed, we get,

$$a_k = \sum_{i=0}^{\infty} \frac{e^{-\lambda i E[slot]} (\lambda i E[slot])^k}{k!} \Pr\{T_{MAC} = i E[slot]\} = \frac{\lambda^k}{(-1)^k k!} \frac{\partial^k F(e^{-\lambda})}{\partial \lambda^k} \quad (17)$$

It is clear that η_0 can be seen as a function of p , i.e.,

$$\eta_0 = f(p, \lambda, E[slot], T_s, T_c, W, m) \quad (18)$$

2.4 Model Solution

Equations (4), (5), (10) and (18) represent a nonlinear system with four unknown parameters τ , p , q and η_0 , which can be solved using numerical methods.

Let S be the total throughput, defined as the ratio of payload information transmitted in a randomly chosen generalized slot time whose average length is computed by

$$E[slot]' = P_s' \cdot T_s + P_{idle}' \cdot \sigma + (1 - P_s' - P_{idle}') T_c \quad (19)$$

where P_{idle}' and P_s' is the probability that the channel is idle or busy because of a successfully transmission among the n stations, respectively:

$$P_{idle}' = (1 - \tau)^n, \quad P_s' = n\tau(1 - \tau)^{n-1} \quad (20)$$

Note that $E[slot]' > E[slot]$ and $E[slot] \approx E[slot]'$ for large n . The throughput can be expressed as

$$S = \frac{P_s' \cdot L}{E[slot]'} = \frac{P_s' \cdot L}{P_s' \cdot T_s + P_{idle}' \cdot \sigma + (1 - P_s' - P_{idle}') T_c} \quad (21)$$

The throughput also can be computed by

$$S = load \cdot R \cdot (1 - p_B), \quad load = n \cdot \lambda \cdot L / R \quad (22)$$

where $load$ is normalized total load, R is channel bit rate and p_B is packet blocking probability due to limited buffer capacity at steady state. Note that $T_s = T_c$ and the average MAC service time is given by

$$E[T_{MAC}] = F'(Z) \Big|_{Z=1} = \left[\frac{(1-2p)(W-1) + pW(1-(2p)^m)}{2(1-2p)(1-p)} \right] E[slot] + \frac{T_s}{1-p} \quad (23)$$

If $T_s \approx E[slot]$ and $E[slot] \approx E[slot]'$, (23) becomes the same as that in [5]. Note that (23) is more accurate because $T_s \square E[slot]$ in unsaturated cases. The standard deviation of MAC service

time is

$$\begin{aligned}\sigma_{T_{MAC}} &= \sqrt{E[(T_{MAC})^2] - \{E[T_{MAC}]\}^2} \\ &= \sqrt{F''(Z) + F'(Z) - [F'(Z)]^2} \Big|_{Z=1}\end{aligned}\quad (24)$$

The main queuing metrics such as average queue length, packet blocking probability, average waiting time including MAC service time and average queuing delay are given by

$$\begin{aligned}E[L_q] &= \sum_{k=0}^K k \cdot p_k, \quad p_B = p_K = 1 - \frac{1}{\eta_0 + \rho} \\ E[T_w] &= \frac{E[L_q]}{\lambda(1 - p_B)}, \quad E[T_q] = E[T_w] - E[T_{MAC}]\end{aligned}\quad (25)$$

3. Performance Evaluation

3.1 Model Validation

The simulations are developed in OPNET where the Direct Spread Sequence Spectrum (DSSS) technique is used in 802.11b standard and the system parameters are summarized in **Table 1**. Since we focus on MAC layer issues in a single-hop 802.11 ad hoc networks, in the simulation, the MAC layer of each station receives packets from upper layer based on Poisson process with arrival rate λ and the randomly distributed nodes can communicate with each other. The total normalized load varies from zero to 1.5 and different buffer size K or number of stations n is used. The simulation time is five minutes. The performance metrics obtained from both analytical model and simulations are shown in **Fig. 3, Fig. 4, Fig. 5, Fig. 6, Fig. 7, Fig. 8, Fig. 9, Fig. 10, Fig. 11** and **Fig.12**. The simulation results are with 95% confidence interval between $(\hat{\theta} - 0.01 \cdot \hat{\theta}, \hat{\theta} + 0.01 \cdot \hat{\theta})$, where $\hat{\theta}$ denote the simulation results (averaged from over 20 runs) and the percentage error for simulation results is below 1%. The simulation results show that the analytical model is highly accurate.

Table 1. System parameters

channel bit rate(R)	11Mbps	control rate (CR)	1Mbps
System slot time (σ)	20 μ s	W	32
MAC Header (H_{MAC})	224bit	m	5
PHY Header (H_{PHY})	192bit	Packet size (L)	8000bits
Distributed interframe space (DIFS)	50 μ s	Size of ACK frame (ACK)	112bit+ H_{PHY}
Propagation delay (δ)	2 μ s	Short interframe space (SIFS)	10 μ s

It's shown that 802.11 ad hoc networks can achieve the optimal performance by controlling the traffic load. Under the optimal total load, the system can obtain the maximum throughput, almost the lowest MAC service delay and very low standard deviation of MAC service delay, furthermore, the average queuing delay and packet blocking probability tends to zero. In the case of overload, the throughput decreases smoothly, however, the MAC service delay, MAC delay variation, queuing delay and packet blocking probability, increases very quickly as the total load continues to increase. In the case of overload, the standard deviation of MAC service time is much larger than its mean value.

3.2 Impact of Buffer Size

We set $n=30$ and $K=1,2,3$, respectively. The results are shown in Fig. 3, Fig. 4, Fig. 5, Fig. 6, and Fig. 7. When $K=1$, the MAC layer queuing delay is zero but the packet blocking probability is larger than that of $K>1$ in the case of underload. As K becomes larger, the network tends to saturated status more rapidly, in consequence, for $K>1$, the optimal total load is obviously less than that of $K=1$. When $K>1$ and the load is no more than the optimal value, the packet blocking probability tends to zero and the buffer size K almost has no influence on all the performance metrics.

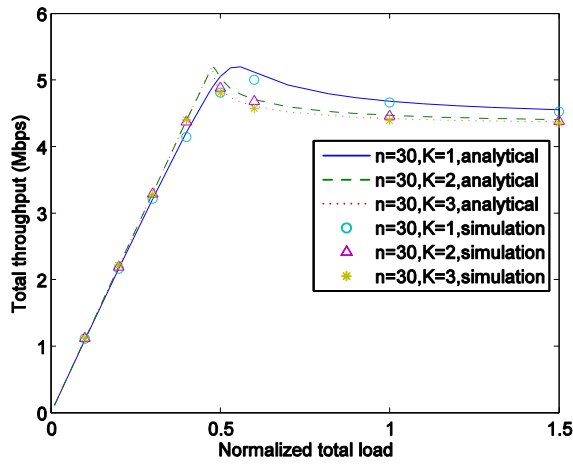


Fig. 3. Throughput versus offered load ($n=30, K=1,2,3$)

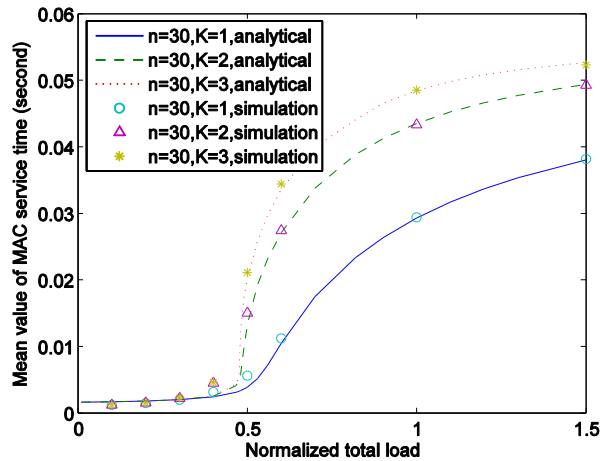


Fig. 4. Mean MAC service time versus offered load ($n=30, K=1,2,3$)

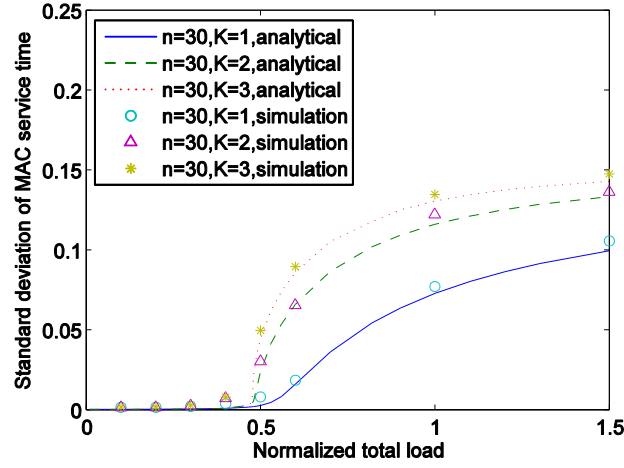


Fig. 5. Standard deviation of MAC service time versus load ($n=30$, $K=1,2,3$)

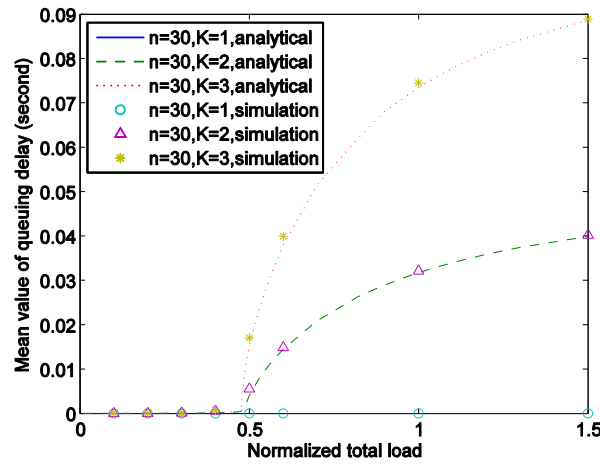


Fig. 6. Mean queuing delay versus offered load ($n=30$, $K=1,2,3$)

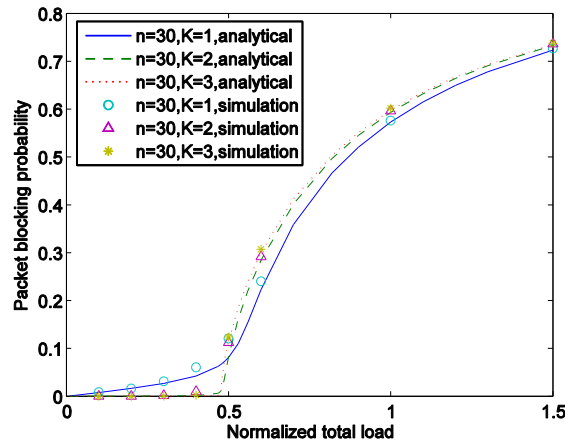


Fig. 7. Packet blocking probability versus offered load ($n=30$, $K=1,2,3$)

3.3 Comparison of M/G/1/K and M/M/1/K

If we can approximate the distribution of MAC service time T_{MAC} by exponential distribution, the steady-state probability η_0 for M/M/1/K queuing model [3] is given by,

$$\eta_0 = \left[\sum_{k=0}^{K-1} \rho^k \right]^{-1} = \begin{cases} \frac{1-\rho}{1-\rho^K}, & \rho \neq 1 \\ 1/K, & \rho = 1 \end{cases} \quad (26)$$

where $\rho = \lambda \cdot E[T_{MAC}]$ and the solution of the nonlinear system (4), (5), (10) and (26) becomes considerably simple.

In the case of $K=1$ (i.e., $\eta_0=1$) or saturated conditions (i.e., $\eta_0=0$), no queuing model is needed, we compare the two queuing models for 802.11ad hoc networks with $K>1$ in unsaturated conditions. The results are shown in Fig. 8. Unfortunately, M/M/1/K model leads to large prediction error while M/G/1/K model is very accurate, which is quite different from the previous work in the literatures. The results also indicate that exponential distribution does not provide good approximation for the MAC layer service time in most cases of unsaturated conditions for the specified network scenarios in this paper.

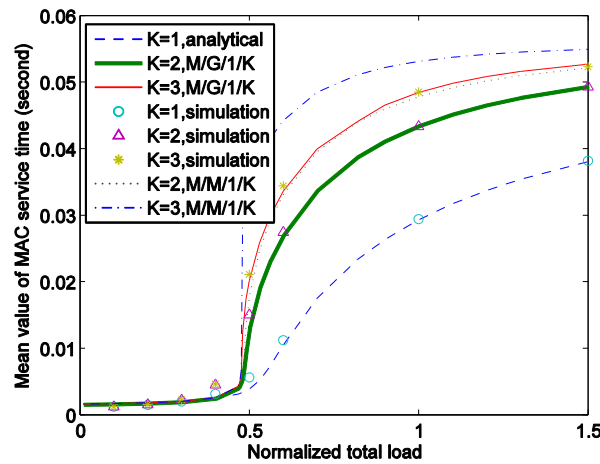


Fig. 8. Comparison of M/G/1/K and M/M/1/K model ($n=30$, $K=1,2,3$)

4. Model Applications

4.1 Performance Optimization

Since the network will achieve maximum throughput under the optimal total load, we firstly derive the expression of maximum achievable throughput. Let us rewrite (21) as

$$S = \frac{L}{T_s - T_c + \frac{(1 - P_{idle}^i) T_c + P_{idle}^i \sigma}{P_s}} \quad (27)$$

S is maximized when the following quantity is maximized:

$$\frac{P_s}{(1 - P_{idle}^i) T_c / \sigma + P_{idle}^i} = \frac{n\tau(1-\tau)^{n-1}}{T_c^* - (1-\tau)^n (T_c^* - 1)} = G(\tau) \quad (28)$$

where $T_c^* = T_c / \sigma$. Let $G'(\tau) = 0$, using the similar method in [3], we obtain the optimal

transmission probability as,

$$\tau = \frac{\sqrt{[n+2(n-1)(T_c^* - 1)]/n-1}}{(n-1)(T_c^* - 1)} \approx \frac{1}{n\sqrt{T_c^*}/2} \quad (29)$$

In [3], the optimal τ is obtained by tuning system parameters W and m . In this paper we provide a flexible way to achieve the optimal τ by controlling the total load. We can find that the transmission probability τ is function of offered load.

Let $K' = \sqrt{T_c^*}/2$ and use the approximate solution $\tau = 1/(nK')$, for n sufficiently large

$$\begin{cases} P_{idle} = (1-\tau)^n = (1 - \frac{1}{nK'})^n \approx e^{-1/K'} \\ P_s = n\tau(1-\tau)^{n-1} \approx \frac{1/K'}{1-1/(nK')} e^{-1/K'} \approx \frac{1}{K'} e^{-1/K'} \\ p = 1 - (1-\tau)^{n-1} \approx 1 - \frac{e^{-1/K'}}{1-\tau} \approx 1 - e^{-1/K'} \end{cases} \quad (30)$$

Then $E[slot]$ is given as

$$E[slot] = e^{-1/K'} \sigma + \frac{1}{K'} e^{-1/K'} T_s + \left[1 - e^{-1/K'} \left(\frac{K'+1}{K'}\right)\right] T_c \quad (31)$$

The maximum achievable throughput can be approximated as

$$S_{max} = \frac{P_s L}{E[slot]} \approx \frac{L}{T_s + \sigma K' + T_c [K' (e^{1/K'} - 1) - 1]} \quad (32)$$

Since $E[slot] \approx E[slot]$ and p is approximated in (30), the average MAC service time is given by

$$E[T_{MAC}] = \left[e^{\frac{1}{K'}} \frac{W-1}{2} + \frac{W}{2} e^{\frac{1}{K'}} (1 - e^{-\frac{1}{K'}}) \sum_{i=0}^{m-1} [2(1 - e^{-\frac{1}{K'}})]^i \right] E[slot] + e^{\frac{1}{K'}} T_s \quad (33)$$

The standard deviation of MAC service time $\sigma_{T_{MAC}}$ can be computed by (24).

It is surprising that S_{max} , $E[T_{MAC}]$ and $\sigma_{T_{MAC}}$ is independent of n and K . Now we try to derive the exact expression of the optimal total load. From (22), the optimal load can be obtained

$$Load = \frac{S_{max} / R}{1 - p_B} \quad (34)$$

As shown in Fig. 7, when $K=2$ or 3, the packet blocking probability p_B is very small (lower than 0.01) under the optimal total load. Since p_B is caused due to limited buffer size, p_B may become smaller as K increases. Hence the approximate optimal load is

$$Load \approx S_{max} / R, \quad K \geq 2 \quad (35)$$

For arbitrary K , p_B can be proved to be zero as $n \rightarrow \infty$. From (13), (22) and (25) we obtain

$$0 \leq p_B = 1 - \frac{1}{\eta_0 + \rho} \leq 1 - \frac{1}{1 + \lambda \cdot E[T_{MAC}]} = \frac{1}{1 + \frac{(n \cdot D)}{\{Load \cdot R \cdot E[T_{MAC}]\}}} \quad (36)$$

As $E[T_{MAC}]$ is limited value, $Load$ is a given value and R, L are constants, we get

$$\lim_{n \rightarrow \infty} p_B = 0 \quad (37)$$

Using the same parameters in Table 1, the optimal achievable performance metrics are reported in Table 2, where the results are obtained from (29) without approximations, the case $n = \infty$ is obtained from (30), (32) and (33). The results are also highly consistent with the that in Section 3.

Table 2. Optimal achievable performance

n	S_{\max} (Mbps)	$Load$	$E[T_{MAC}](s)$	$\sigma_{T_{MAC}}(s)$
5	5.2765	0.47968	0.0056634	0.0053222
20	5.2066	0.47332	0.0061002	0.0061111
40	5.1956	0.47232	0.0061709	0.0062428
60	5.1919	0.47199	0.0061943	0.0062868
200	5.1869	0.47153	0.0062270	0.0063483
∞	5.1837	0.47124	0.0067583	0.0073815

Furthermore, the approximations for $n = \infty$ provide lower bound of maximum achievable throughput, upper bound of average MAC service time and its standard deviation. For a large range of n , the standard deviation of MAC service time, as well as its mean value, is much lower than that in saturated conditions. Therefore, the analytical model provides a simple method for accurate and quantitative QoS guarantee in practical IEEE 802.11 ad hoc networks. The performance optimization including derivation of the number of optimal stations for voice communications over 802.11 ad hoc networks was given in [26], however, the method needs the M/G/1/K queuing analysis in this paper, although both the methods are similar.

4.2 Performance Analysis of 802.11e Ad Hoc Networks

The analytical model is also suitable for performance analysis of 802.11e in unsaturated conditions. Since the relative service differentiation can be achieved by varying the initial contention window alone [6], we consider the service differentiation schemes only by using W . We assumed that there is only one queue in each station, which can be looked at as a special case of IEEE 802.11e. We assume that all the stations are divided into N service classes and the number of stations in the i th service class is n_i , the initial contention window of stations in service class i is W_i , $i \in [1, N]$. Let τ_i , p_i , η_0^i , q_i denote the transmission probability, packet collision probability, the stationary probability that the transmission queue is empty upon a departure, the probability that at least one packet arrives during a virtual slot time for stations in class i , respectively. By the generalized Markov chain for stations in class i , we have

$$\tau_i = \frac{2(1-2p_i)q_i}{(1-2p_i)[(W_i+1)q_i+2\eta_0^i(1-p_i)]+p_iq_iW_i[1-(2p_i)^m]} \quad (38)$$

$$p_i = 1 - (1-\tau_i)^{n_i-1} \prod_{j=1, j \neq i}^N (1-\tau_j)^{n_j} \quad (39)$$

When n_i ($i \in [1, N]$) is large, for stations in class i , the average length of real slot time $E[slot]_i \approx E[slot]^i$, i.e., the average length of a generalized slot time, we get

$$E[slot]^i = \sum_{i=1}^N P_s^i(i)T_s + P_{idle}^i \sigma + [1 - P_{idle}^i - \sum_{i=1}^N P_s^i(i)]T_c \quad (40)$$

$$\begin{cases} P_s^i(i) = n_i \cdot \tau_i (1-\tau_i)^{n_i-1} \prod_{j=1, j \neq i}^N (1-\tau_j)^{n_j} \\ P_{idle}^i = \prod_{i=1}^N (1-\tau_i)^{n_i} \end{cases} \quad (41)$$

Then we obtain

$$q_i = 1 - \exp(-\lambda \cdot E[slot]_i) \quad (42)$$

And

$$\eta_0^i = f(p_i, \lambda, E[slot]_i, T_s, T_c, W_i, m) \quad (43)$$

The set of equations (38) (39) (42) and (43) ($i=1, 2, \dots, N$) represent a nonlinear system with $4N$

unknowns τ_i , p_i , η_0^i and q_i , which can be solved using numerical methods. The throughput for each class and total throughput is given by

$$S_i = \frac{P_s(i) \cdot L/R}{E[slot]}, \quad S = \sum_{i=1}^N S_i = \frac{P_s \cdot L/R}{E[slot]} \quad (44)$$

The mean MAC service time can be easily obtained as:

$$E[T_{MAC}]_i = \left[\frac{(1-2p_i)(W_i-1) + p_i W_i [1-(2p_i)^m]}{2(1-2p_i)(1-p_i)} \right] E[slot]_i + \frac{T_s}{1-p_i} \quad (45)$$

In simulations, we set $K=1$ and $N=2$, i.e., there are two service classes. Let $n_1=n_2=20$ and $W_1=32, W_2=64$, the other system parameters are the same as that in **Table 1**. The analytical and simulation results including throughput and mean MAC service time are shown in **Fig. 9** and **Fig. 10**, which indicate the accuracy of the model. As the total load tends to saturated status, the analytical results tend to the same as that in [6]. However, for the specific MAC settings, the service differentiation scheme could hardly achieve performance differentiation such as throughput or MAC service delay when the load is less than the optimal value. The scheme could just achieve relative service differentiation in saturated conditions.

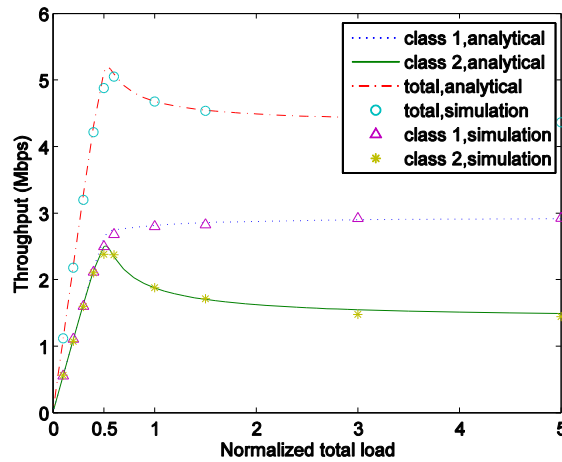


Fig. 9. Throughput versus offered load for 802.11e

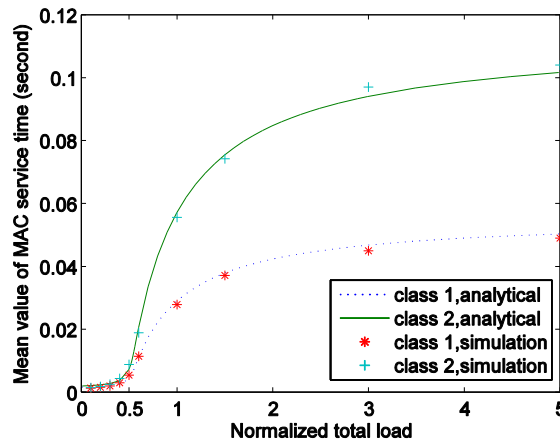


Fig. 10. Mean MAC service time versus offered load for 802.11e

The probability P'_{idle} is given by:

$$P'_{idle} = \prod_{i=1}^N P'_{idle}(i) = \prod_{i=1}^N (1 - \tau_i)^{n_i} \tag{53}$$

The time duration T_{si} and T_{ci} is expressed as:

$$\begin{cases} T_{si} = \frac{H_{PHY}}{CR} + \frac{H_{MAC} + L_i}{R} + SIFS + \delta + \frac{ACK}{CR} + \delta + DIFS \\ T_{ci} = \frac{H_{PHY}}{CR} + \frac{H_{MAC} + L_i}{R} + SIFS + \delta + EIFS = T_{si} \end{cases} \tag{54}$$

In addition, the probability η_0^i is given by:

$$\eta_0^i = f(p_i, \lambda_i, E[slot], T_{si}, T_{ci}, W, m) \tag{55}$$

The set of equations (46) (47) (49) and (55) ($i=1,2,\dots,N$) represent a nonlinear system with $4N$ unknowns τ_i, p_i, η_0^i and q_i , which can be solved using numerical methods. The throughput for each class and total throughput is given by

$$S_i = \frac{P'_s(i) \cdot L_i}{E[slot]}, \quad S = \sum_{i=1}^N S_i \tag{56}$$

The mean MAC service time can be easily obtained as

$$E[T_{MAC}]_i = \left[\frac{(1 - 2p_i)(W - 1) + p_i W [1 - (2p_i)^m]}{2(1 - 2p_i)(1 - p_i)} \right] E[slot]' + \frac{T_{si}}{1 - p_i} \tag{57}$$

In simulations we set $K = 1, N=2, n_1=10, n_2=20, L_1=4000\text{bits}, L_2=8000\text{bits}, \lambda_1 = 2\lambda_2$. The performance results versus total load are shown in Fig. 11, where solid lines denote throughput (S) and dashed lines denote average packet delay (D). Since a station's load equals its packet arriving rate multiplied by packet size, the analytical results exhibit a traffic load based scheme for service differentiation for IEEE 802.11 wireless networks. The new scheme can achieve both throughput differentiation and statistical QoS guarantee by controlling the total load.

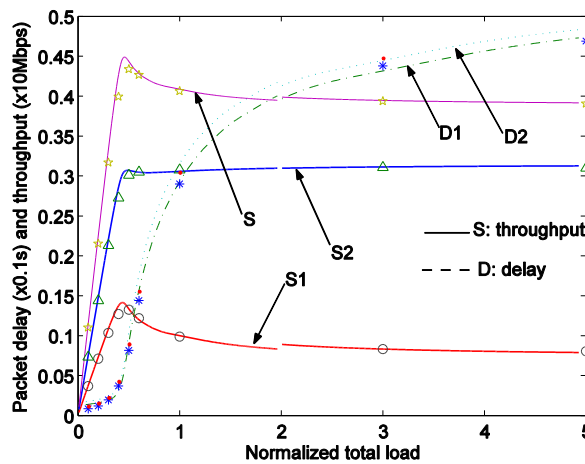


Fig. 11. Performance results ($N=2, n_1=10, n_2=20, L_1=4000\text{bits}, L_2=8000\text{bits}, \lambda_1 = 2\lambda_2$)

5. Discussions

There are some assumptions in the analytical model. Firstly, a station is modeled by the generalized Markov chain on the assumption that the station is in steady state (i.e., $t \rightarrow \infty$), otherwise the bidimensional stochastic process $(s(t), b(t))$ of the backoff counter is non-Markovian because over two packets may arrive during the transmission process of the previous packet. In fact, in the model η_0 is the stationary probability that the transmission queue is empty upon a departure in steady state. This assumption considerably simplifies modeling and we have seen producing reasonable predictions. In addition, the extra state transition for modeling of backoff counter freezing in [6] should be ignored in Fig.1 because the similar treatments can't be used for a virtual slot time. This ignoring is also reasonable for the stations in steady state. The last but not the least, the size of virtual slot time is equal to that of real slot time, i.e., the backoff counter is seen as still "active" when the station is empty. This is the key point in the extension of Bianchi's model in unsaturated cases in this paper.

6. Conclusions

We have developed a novel mathematical model for performance analysis of 802.11 MAC protocol in unsaturated conditions. Extensive simulations show that the analytical model is highly accurate. By the model, we give the accurate queuing analysis of 802.11 MAC layer and obtain the lower bound of maximum achievable throughput, the upper bound of average MAC service time and its standard deviation. Moreover, the optimal total load is proved to be equal to the maximum achievable throughput. Hence we can accurately provide quantitative QoS guarantee in practical IEEE 802.11 ad hoc networks. Furthermore, the model can be easily extended for performance analysis of 802.11e in unsaturated conditions and 802.11 ad hoc networks with traffic flow diversity. The analytical model has its merit for both designing call admission schemes and cross-layer QoS routing protocols for real multimedia transmission over IEEE 802.11 wireless ad hoc networks.

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