Bandwidth-efficient Cooperative Diversity with Rotated Constellations and Its Performance Analysis

Weijia Lei[1], Xianzhong Xie[1] and Xiangming Li[2]
[1] Institute of Personal Communications, Chongqing University of Posts and Telecommunications
Chongqing, 400065 - China
[e-mail: leiweijia@gmail.com, xiexzh@cqupt.edu.cn]
[2] School of Information and Electronics, Beijing Institute of Technology
Beijing, 100081 - China
[e-mail: xmli@bit.edu.cn]
*Corresponding author: Xiangming Li

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Abstract

Cooperative diversity is a technique with which a virtual multiple antenna array is established among the single antenna users of the wireless network to realize space diversity. Signal space diversity (SSD) is a bandwidth-efficient diversity technique, which uses constellation rotation and interleaving techniques to achieve diversity gain. A new cooperative diversity scheme with rotated constellations (RCCD) is proposed in this paper. In this scheme, data are modulated by using a rotated constellation, and the source and the relays transmit different components of the modulated symbols. Since any one of the components contains full information of the symbols, the destination can obtain multiple signals conveying the same information from different users. In this way, space diversity is achieved. The RCCD scheme inherits the advantage of SSD – being bandwidth-efficient but without the delay problem of SSD brought by interleaving. The symbol error rate of the RCCD scheme is analyzed and simulated. The analysis and simulation results show that the RCCD scheme can achieve full diversity order of two when the inter-user channel is good enough, and, with the same bandwidth efficiency, has a better performance than amplify-and-forward and detect-and-forward methods.

Keywords: Cooperative diversity, signal space diversity, rotated constellation, bandwidth efficiency

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1. Introduction

Diversity is an effective technique to mitigate the effects of fading on wireless channels. There are several diversity techniques available such as space diversity, time diversity, and frequency diversity. The idea behind the diversity is to send the data over independent fading paths. All these diversity techniques need more space, time or frequency resources to realize the multiple independent fading paths. Compared with these diversity techniques, signal space diversity (SSD) is a bandwidth-efficient diversity technique [1][2]. The basic idea of SSD is that multidimensional signal constellations are used and the components of modulated symbols are transmitted over independent fading paths. The independence of the fading paths can easily be achieved by interleaving. Two-dimensional signal constellation for SSD can be obtained by applying a certain rotation to a classical signal constellation. An example of the rotation of a QPSK constellation is shown in **Fig. 1**. In the classical QPSK constellation, the numbers of the projections of the 4 constellation points on both I axis and Q axis are 2. **Fig. 1-(b)** is a rotated constellation in which an anticlockwise rotation has been applied to the classical constellation. Different from the classical constellation, the projections of the 4 points on I axis and Q axis do not overlap. So the points of the constellation can be recognized by the in-phase component or quadrature component. Since the Euclidean distances between any two points in the rotated constellation remain the same as those in the original constellation, their performances are identical in the AWGN channel. In the fading channel, since each component of the modulated symbols contains full information of the symbols, the diversity gain is obtained if the fading experienced by the in-phase component and quadrature component are independent. SSD does not require additional time, frequency or space, though the receiver needs more complicated algorithm to detect the received symbols. Although there are some more effective receiver structures and detecting algorithms proposed [3][4], the receiver complexity is rather high when the constellation has a relatively large number of dimensions. Furthermore, SSD needs an interleaver/deinterleaver pair to make each component experience independent fading. This brings additional delay, which can be very large if the channel changes slowly.

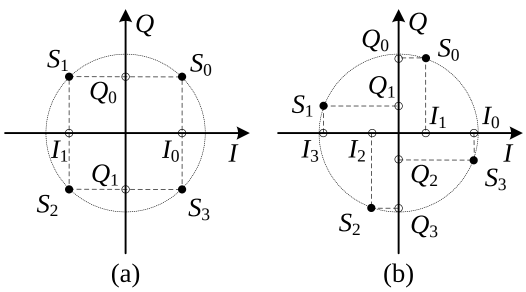

Fig. 1. The rotation of signal constellation.
(a) Classical QPSK constellation.
(b) Rotated QPSK constellation.

Cooperative diversity is a kind of space diversity technique in which the users of wireless networks collaborate with each other, share their antennas, and build virtual multiple transmission antenna array [5][6]. The schemes of cooperative diversity are usually classified into three categories: amplify-and-forward (AF) [7], detect-and-forward (DF, also called decode-and-forward) [7], and coded cooperation (CC) [8]. In AF and DF schemes, the relays repeatedly transmit the data of the source. For CC protocol, the relays help the source to transmit parts of the codeword. Because the repetition in the AF and DF schemes degrades the bandwidth efficiency, and CC may need to decrease the code rate of data to obtain good performance, the application of AF, DF or CC may degrade the communication efficiency to some degree.

Recently, some cooperative diversity schemes adopting the idea of SSD are discussed in [9][10][11][12]. A bandwidth efficient cooperation diversity scheme among three users based on constellation rotation is proposed in [12]. In this scheme, each user has two partners. It firstly transmits to the destination its own symbols without constellation rotation. Then it modulates its partners' data by using a rotated constellation, and forms a new symbol by combining the real part of one partner's symbol and the imaginary part of the other's symbol, and then sends it out. This scheme can achieve diversity order of three while the bandwidth efficiency is decreased by a half compared with a non-cooperation scheme.

The basic idea behind our scheme is to modulate users' data using a rotated constellation, and then make different users send different components of the modulated symbols. Because each component of the modulated symbols contains complete information of the symbols, the source just needs to send one of the components, and other users (i.e. relays) can recover the original data from this component, re-modulate the data and send other components of the symbols to the destination. We call this scheme cooperative diversity with rotated constellations (RCCD). RCCD inherits the advantages of SSD: no additional time, bandwidth, frequency or space resources are needed. But unlike SSD, there is no requirement of interleaver/deinterleaver pair, thus no additional delay. In theory, the number of the dimensions of the rotated constellation is also the upper limit of the number of the relays. But the algorithm complexity to detect the symbols restricts the number of relays. Since the system performance improvement is the most substantial when the system goes from no diversity to two-order diversity, and in this case the complexity of detection algorithm is acceptable, we focus our work on the scheme of RCCD between two users (single relay). Our RCCD scheme has better bandwidth efficiency than [12]. The bandwidth efficiency of the RCCD scheme is kept the same as that of a non-cooperation system, while the scheme offers diversity order of two. Furthermore, the combining and detecting algorithm in the RCCD scheme is simpler than those adopted in [9][10][11][12].

The rest of this paper is organized as follows. In section II, we describe the scheme of RCCD in detail, and introduce the design criterion of rotated constellation. The combining and detecting criterion for received signals at the destination is introduced in this section too. In section III, the upper bound on symbol error rate of the RCCD scheme is derived. In section IV, Monte-Carlo simulation is performed, and the performance of the RCCD scheme is compared with those of AF and DF. Section V is the conclusion of this paper.

2. The Scheme of Cooperative Diversity with Rotated Constellations

2.1 The Cooperative Scheme with Rotated Constellations

In this paper, we assume that the access method is time division multiple accesses (TDMA) and the users work in half duplex mode. **Fig. 2** is the transmission model of the cooperation. The labels beside the arrow-headed lines are channel fading coefficients. User 1 and 2 collaborate to send data to the destination. They form a partnership and help to relay other's data. We call each user source and its partner its relay. A cooperation transmission process is split into two phases. In the first phase, two users modulate their data by using a rotated constellation and get K symbols (for convenience, it is presumed that K is even). Every symbol has two components – in-phase component and quadrature component. We denote the l-th symbol of user i as s_{il}, and s_{il} = Re(s_{il}) + jIm(s_{il}), where Re(s_{il}) means the real part, i.e. the in-phase component of s_{il}, and Im(s_{il}) means the imaginary part, i.e. the quadrature component of s_{il}. Each component of s_{il} contains full information of s_{il}. The users re-construct the K modulated symbols before transmission. The symbols for transmission are composed as:

$$\tilde{s}_{il} = \mathrm{Re}(s_{i,2\times l-1}) + j\,\mathrm{Im}(s_{i,2\times l}),\ i=1,2,\ l=1,2,\ldots,K/2 \tag{1}$$

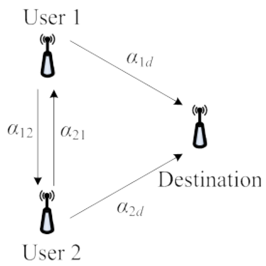

Fig. 2. The transmission model of cooperation communication

Fig. 3 illustrates the transmission processes of two users in a non-cooperation system **Fig. 3-(a)** and those in a RCCD system **Fig. 3-(b)**. During a timeslot (TS) in the non-cooperation system, a user can transmit K symbols. In the RCCD system, a timeslot with K symbols is divided into 2 sub-timeslots (STS) with K/2 symbols. User 1 and user 2 send their K/2 re-constructed symbols in these two sub-timeslots. Because the transmission from the users to the destination is broadcast, user 2 can receive and detect user 1's data during user 1's transmission process, and vice versa. This is the first phase. Next two sub-timeslots are the second phase of cooperation. Each user recovers the data of its partner's, which they subsequently re-modulate, and then re-constructs the modulated symbols in a similar way to that in the first phase except that the positions of the two symbols in an original symbol pair are exchanged:

$$\tilde{s}_{il} = \mathrm{Re}(s_{i,2\times l}) + j\,\mathrm{Im}(s_{i,2\times l-1}),\ i=1,2,\ l=1,2,\ldots,K/2 \tag{2}$$

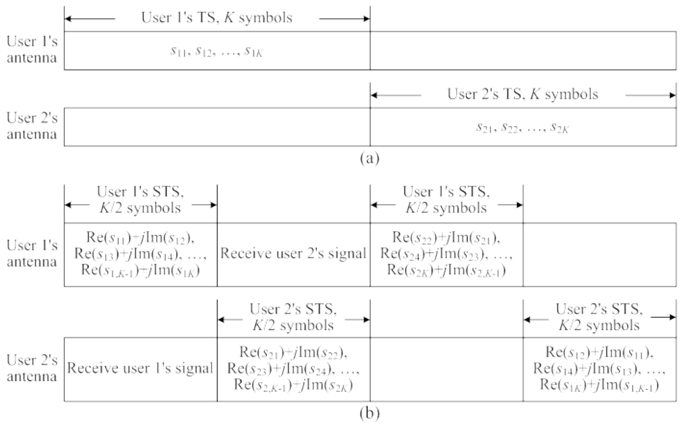

Fig. 3. Data transmission in TDMA channel
(a) Non-cooperation. **(b)** Cooperation with rotated constellations

These symbols are sent to the destination. After all symbols have been received, the destination restores the in-phase and quadrature component pair of the symbols to the original form, and detects them.

To avoid error propagation in the second phase, the relays check the validity of the recovered data. According to the different responses of the relays at the time when an error is found, we develop two cooperation modes:

1) Joint error-checking mode (RCCD-JEC). The user does error checking for the received symbols. When an error is found, the user sends a notification to its partner and the destination. Only when both users have correctly received their partners' data do they send out the data of their partner's. Otherwise, both users will deal with their own data and send them to the destination. The transmission process for the first two symbols is illustrated in **Fig. 4-(a)**.

2) Independent error-checking mode (RCCD-IEC). The user checks the validity of the received data. If no error is found, the user relays its partner's data. Otherwise, it transmits its own data. The transmission process for the first two symbols is illustrated in **Fig. 4-(b)**. Four cases may occur in the process of cooperation. In Case 2-3 and Case 2-4, one user receives data wrongly. So its data will be transmitted by both users in the second phase, while the data of the other are not transmitted by any user. The users need to transmit a notification together with the transmitted data to the destination for the identification of the data's owner in the second phase in this mode.

		Case 1-1	Case 1-2
User 1	Phase 1	$\mathrm{Re}(s_{11})+j\mathrm{Im}(s_{12})$	
	Phase 2	$\mathrm{Re}(s_{22})+j\mathrm{Im}(s_{21})$	$\mathrm{Re}(s_{12})+j\mathrm{Im}(s_{11})$
User 2	Phase 1	$\mathrm{Re}(s_{21})+j\mathrm{Im}(s_{22})$	
	Phase 2	$\mathrm{Re}(s_{12})+j\mathrm{Im}(s_{11})$	$\mathrm{Re}(s_{22})+j\mathrm{Im}(s_{21})$

(a)

		Case 2-1	Case 2-2	Case 2-3	Case 2-4
User 1	Phase 1	$\mathrm{Re}(s_{11})+j\mathrm{Im}(s_{12})$			
	Phase 2	$\mathrm{Re}(s_{22})+j\mathrm{Im}(s_{21})$	$\mathrm{Re}(s_{12})+j\mathrm{Im}(s_{11})$	$\mathrm{Re}(s_{22})+j\mathrm{Im}(s_{21})$	$\mathrm{Re}(s_{12})+j\mathrm{Im}(s_{11})$
User 2	Phase 1	$\mathrm{Re}(s_{21})+j\mathrm{Im}(s_{22})$			
	Phase 2	$\mathrm{Re}(s_{12})+j\mathrm{Im}(s_{11})$	$\mathrm{Re}(s_{22})+j\mathrm{Im}(s_{21})$	$\mathrm{Re}(s_{22})+j\mathrm{Im}(s_{21})$	$\mathrm{Re}(s_{12})+j\mathrm{Im}(s_{11})$

(b)

Fig. 4. Data transmission of two RCCD modes. **(a)** RCCD-JEC **(b)** RCCD-IEC
Case 1-1: both users receive correctly. Case 1-2: one or both of the users receive wrongly.
Case 2-1: both users receive correctly. Case 2-2: both users receive wrongly.
Case 2-3: user 1 receives correctly, while user 2 does not.
Case 2-4: user 2 receives correctly, while user 1 does not.

2.2 The Design of Rotated Constellations

In the SSD system, one of the criteria for constellation design is maximizing the minimal product distance between any two points of the constellation. In the RCCD system, data recovery in the first phase relies only on the in-phase component or quadrature component of the symbols. Further more, in Case 2-3 and Case 2-4, only one component of symbols can be used to recover them by the destination. So the priority lies in optimizing the performance when only one component can be used to recover the original data. In-phase component or quadrature component of a symbol is an M-PAM signal, whose performance is determined by the minimal distance between any two points on I axis or Q axis. The minimal distance is maximized when the M points are evenly scattered on I axis or Q axis in a rotated QPSK constellation **Fig. 5**. We denote the rotation angle as θ. Four projections of the rotated points on I axis or Q axis should be $+3a$, $+a$, $-a$, $-3a$, and the space between any two adjacent points is $2a$. To realize this, θ must satisfy

$$\begin{cases} a = R\cos(\pi/4 + \theta) \\ 3a = R\cos(\pi/4 - \theta) \end{cases} \tag{3}$$

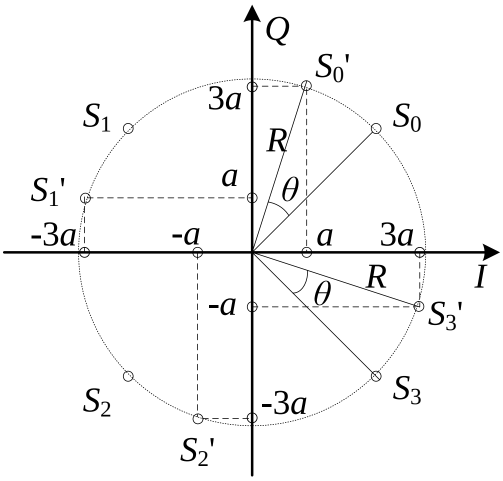

Fig. 5. Rotation angle of constellation.

Solve the equation system, and we get $\theta = 0.4636$ rad $= 26.565°$. Similarly, the rotation angle for a high-level constellation can also be got.

From **Fig. 5**, it is easy to find the relationship between the original point S_n and the rotated point S_n', which is

$$\begin{cases} S_{nI}' = S_{nI} \times \cos(\theta) - S_{nQ} \times \sin(\theta) \\ S_{nQ}' = S_{nI} \times \sin(\theta) + S_{nQ} \times \cos(\theta) \end{cases}, \quad n = 0, 1, \cdots, M-1 \tag{4}$$

where subscripts I and Q represent in-phase and quadra- ture component of the point. The matrix form of (4) is

$$\begin{bmatrix} S_{nI}' \\ S_{nQ}' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} S_{nI} \\ S_{nQ} \end{bmatrix}, \quad n = 0, 1, \cdots, M-1 \tag{5}$$

The rotated constellation can be generated by using (5).

2.3 The Detection of Signal

We introduce the processing of the first two symbols to explain the combination and detection of signals. Assume receivers know the characteristics of the channels, but the transmitter does not. The symbols transmitted by user 1 are s_{11}, s_{12}, and those by user 2 are s_{21}, s_{22}. All channels are frequency-nonselective Rayleigh channel. The channel gain keeps constant during four $K/2$-symbols sub-timeslots. The fading coefficient of the channel from user i to user l and that from user i to the destination are denoted as $\alpha_{il} = |\alpha_{il}| e^{j\varphi_{il}}$ and $\alpha_{id} = |\alpha_{id}| e^{j\varphi_{id}}$ respectively **Fig. 2**. $E\left[|\alpha_{il}|^2\right] = E\left[|\alpha_{id}|^2\right] = 1$.

The signals received by user 1, 2 and the destination in the first phase are

$$\begin{aligned} r_{sr1}' &= \alpha_{21}\left[\mathrm{Re}(s_{21}) + j\,\mathrm{Im}(s_{22})\right] + z_{21} \\ r_{sr2}' &= \alpha_{12}\left[\mathrm{Re}(s_{11}) + j\,\mathrm{Im}(s_{12})\right] + z_{12} \\ r_{sd1}' &= \alpha_{1d}\left[\mathrm{Re}(s_{11}) + j\,\mathrm{Im}(s_{12})\right] + z_{sd1} \\ r_{sd2}' &= \alpha_{2d}\left[\mathrm{Re}(s_{21}) + j\,\mathrm{Im}(s_{22})\right] + z_{sd2} \end{aligned} \tag{6}$$

where r_{sri} is the signal received by user i; r_{sdi} is the signal sent by user i and received by the destination; z_{il} and z_{sdi} are white Gaussian noises added to the signal from user i to user l and the signal from user i to the destination respectively.

Suppose the receiver can compensate for the phase in channel. The signals for detecting are

$$\begin{aligned} r_{sr1} &= |\alpha_{21}|\left[\mathrm{Re}(s_{21}) + j\,\mathrm{Im}(s_{22})\right] + z_{21} \\ r_{sr2} &= |\alpha_{12}|\left[\mathrm{Re}(s_{11}) + j\,\mathrm{Im}(s_{12})\right] + z_{12} \\ r_{sd1} &= |\alpha_{1d}|\left[\mathrm{Re}(s_{11}) + j\,\mathrm{Im}(s_{12})\right] + z_{sd1} \\ r_{sd2} &= |\alpha_{2d}|\left[\mathrm{Re}(s_{21}) + j\,\mathrm{Im}(s_{22})\right] + z_{sd2} \end{aligned} \tag{7}$$

Users 1 and 2 apply maximum likelihood (ML) detection to the signal coming from their partner to detect the original data. The in-phase component and quadrature component of a symbol are detected separately. The detecting criterion is

$$\begin{aligned} \hat{s}_{21} &= \arg\min_{S_n} \left|\mathrm{Re}(r_{sr1}) - |\alpha_{21}|\mathrm{Re}(S_n)\right|^2 \\ \hat{s}_{22} &= \arg\min_{S_n} \left|\mathrm{Im}(r_{sr1}) - |\alpha_{21}|\mathrm{Im}(S_n)\right|^2 \\ \hat{s}_{11} &= \arg\min_{S_n} \left|\mathrm{Re}(r_{sr2}) - |\alpha_{12}|\mathrm{Re}(S_n)\right|^2 \\ \hat{s}_{12} &= \arg\min_{S_n} \left|\mathrm{Im}(r_{sr2}) - |\alpha_{12}|\mathrm{Im}(S_n)\right|^2 \end{aligned} \tag{8}$$

In the second phase, the transmitted data and the detecting criterion in the destination are different between the two modes.

1) Joint error-checking mode.

When both users receive their partner's data correctly in the first phase (Case 1-1), they transmit their partner's data in the second phase. The data received by the destination for detecting are

$$\begin{aligned} r_{rd1} &= |\alpha_{1d}|\left[\mathrm{Re}(s_{22}) + j\,\mathrm{Im}(s_{21})\right] + z_{rd1} \\ r_{rd2} &= |\alpha_{2d}|\left[\mathrm{Re}(s_{12}) + j\,\mathrm{Im}(s_{11})\right] + z_{rd2} \end{aligned} \tag{9}$$

The destination combines the signals received in two phases and does ML detection:

$$\begin{aligned} \hat{s}_{11} &= \arg\min_{S_n} \left[\left|\mathrm{Re}(r_{sd1}) - |\alpha_{1d}|\mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd2}) - |\alpha_{2d}|\mathrm{Im}(S_n)\right|^2\right] \\ \hat{s}_{12} &= \arg\min_{S_n} \left[\left|\mathrm{Im}(r_{sd1}) - |\alpha_{1d}|\mathrm{Im}(S_n)\right|^2 + \left|\mathrm{Re}(r_{rd2}) - |\alpha_{2d}|\mathrm{Re}(S_n)\right|^2\right] \\ \hat{s}_{21} &= \arg\min_{S_n} \left[\left|\mathrm{Re}(r_{sd2}) - |\alpha_{2d}|\mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd1}) - |\alpha_{1d}|\mathrm{Im}(S_n)\right|^2\right] \\ \hat{s}_{22} &= \arg\min_{S_n} \left[\left|\mathrm{Im}(r_{sd2}) - |\alpha_{2d}|\mathrm{Im}(S_n)\right|^2 + \left|\mathrm{Re}(r_{rd1}) - |\alpha_{1d}|\mathrm{Re}(S_n)\right|^2\right] \end{aligned} \tag{10}$$

If any one of the users receives data wrongly (Case 1-2), both users will transmit their own data. The received data at the destination are

$$\begin{aligned} r_{rd1} &= |\alpha_{1d}|\left[\mathrm{Re}(s_{12}) + j\,\mathrm{Im}(s_{11})\right] + z_{rd1} \\ r_{rd2} &= |\alpha_{2d}|\left[\mathrm{Re}(s_{22}) + j\,\mathrm{Im}(s_{21})\right] + z_{rd2} \end{aligned} \tag{11}$$

The detecting criterion is

$$\begin{aligned} \hat{s}_{11} &= \arg\min_{S_n}\left[\left|\mathrm{Re}(r_{sd1}) - |\alpha_{1d}|\mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd1}) - |\alpha_{1d}|\mathrm{Im}(S_n)\right|^2\right] \\ \hat{s}_{12} &= \arg\min_{S_n}\left[\left|\mathrm{Im}(r_{sd1}) - |\alpha_{1d}|\mathrm{Im}(S_n)\right|^2 + \left|\mathrm{Re}(r_{rd1}) - |\alpha_{1d}|\mathrm{Re}(S_n)\right|^2\right] \\ \hat{s}_{21} &= \arg\min_{S_n}\left[\left|\mathrm{Re}(r_{sd2}) - |\alpha_{2d}|\mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd2}) - |\alpha_{2d}|\mathrm{Im}(S_n)\right|^2\right] \\ \hat{s}_{22} &= \arg\min_{S_n}\left[\left|\mathrm{Im}(r_{sd2}) - |\alpha_{2d}|\mathrm{Im}(S_n)\right|^2 + \left|\mathrm{Re}(r_{rd2}) - |\alpha_{2d}|\mathrm{Re}(S_n)\right|^2\right] \end{aligned} \tag{12}$$

2) Independent error-checking mode.

When both users receive their partner’s data correctly in the first phase (Case 2-1), the transmitted data in the second phase and the detecting criterion of the destination are the same as those in Case 1-1, i.e. (9) and (10).

When both users receive their partner’s data wrongly in the first phase (Case 2-2), the transmitted data in the second phase and the detecting criterion of the destination are the same as those in Case 1-2, i.e. (11) and (12).

When user 1 receives its partner’s data correctly, while user 2 does not (Case 2-3), both users transmit the data of user 2 in the second phase. The data received by the destination are

$$\begin{aligned} r_{rd1} &= |\alpha_{1d}|\left[\mathrm{Re}(s_{22}) + j\,\mathrm{Im}(s_{21})\right] + z_{rd1} \\ r_{rd2} &= |\alpha_{2d}|\left[\mathrm{Re}(s_{22}) + j\,\mathrm{Im}(s_{21})\right] + z_{rd2} \end{aligned} \tag{13}$$

The detecting criterion is

$$\begin{aligned} \hat{s}_{11} &= \arg\min_{S_n}\left|\mathrm{Re}(r_{sd1}) - |\alpha_{1d}|\mathrm{Re}(S_n)\right|^2 \\ \hat{s}_{12} &= \arg\min_{S_n}\left|\mathrm{Im}(r_{sd1}) - |\alpha_{1d}|\mathrm{Im}(S_n)\right|^2 \\ \hat{s}_{21} &= \arg\min_{S_n}\left[\left|\mathrm{Re}(r_{sd2}) - |\alpha_{2d}|\mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd1}) - |\alpha_{1d}|\mathrm{Im}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd2}) - |\alpha_{2d}|\mathrm{Im}(S_n)\right|^2\right] \\ \hat{s}_{22} &= \arg\min_{S_n}\left[\left|\mathrm{Im}(r_{sd2}) - |\alpha_{2d}|\mathrm{Im}(S_n)\right|^2 + \left|\mathrm{Re}(r_{rd1}) - |\alpha_{1d}|\mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Re}(r_{rd2}) - |\alpha_{2d}|\mathrm{Re}(S_n)\right|^2\right] \end{aligned} \tag{14}$$

When user 2 receives its partner’s data correctly, while user 1 does not (Case 2-4), both users transmit the data of user 1 in the second phase. The data received by the destination are

$$\begin{aligned} r_{rd1} &= |\alpha_{1d}|\left[\mathrm{Re}(s_{12}) + j\,\mathrm{Im}(s_{11})\right] + z_{rd1} \\ r_{rd2} &= |\alpha_{2d}|\left[\mathrm{Re}(s_{12}) + j\,\mathrm{Im}(s_{11})\right] + z_{rd2} \end{aligned} \tag{15}$$

The detecting criterion is

$$\begin{aligned} \hat{s}_{11} &= \arg\min_{S_n}\left[\left|\mathrm{Re}(r_{sd1}) - |\alpha_{1d}|\mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd1}) - |\alpha_{1d}|\mathrm{Im}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd2}) - |\alpha_{2d}|\mathrm{Im}(S_n)\right|^2\right] \\ \hat{s}_{12} &= \arg\min_{S_n}\left[\left|\mathrm{Im}(r_{sd1}) - |\alpha_{1d}|\mathrm{Im}(S_n)\right|^2 + \left|\mathrm{Re}(r_{rd1}) - |\alpha_{1d}|\mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Re}(r_{rd2}) - |\alpha_{2d}|\mathrm{Re}(S_n)\right|^2\right] \\ \hat{s}_{21} &= \arg\min_{S_n}\left|\mathrm{Re}(r_{sd2}) - |\alpha_{2d}|\mathrm{Re}(S_n)\right|^2 \\ \hat{s}_{22} &= \arg\min_{S_n}\left|\mathrm{Im}(r_{sd2}) - |\alpha_{2d}|\mathrm{Im}(S_n)\right|^2 \end{aligned} \tag{16}$$

3. Performance Analysis

We use the symbol error rate (SER) to evaluate the performance of the RCCD scheme which uses rotated QPSK constellation and the rotation angle is $\theta = 26.565°$. The channels from user 1 and user 2 to the destination exhibit the same characteristic, though they are independent. Because of the symmetry between the two users, they have the same performance. So we only analyze the performance of user 1. Suppose the channel gain keeps constant during four $K/2$-symbols sub-timeslots.

3.1 The SER of the Transmission between Two Users in the First Phase

In the transmission between two users, the in-phase component and quadrature component of every symbol convey different information, and they are 4-PAM signals. Denote the received signal-to-noise power ratio (SNR) per symbol at the relay as γ_{ssr}. The SNR per bit is $\gamma_{bsr} = \gamma_{ssr}/2$.[1] Since the in-phase component and quadrature component each carries half of all energy, the average SNR per symbol of 4-PAM signal is $\gamma_{ssr}/2 = \gamma_{bsr}$. The SER of 4-PAM signal from the source to the relay in AWGN channel is [13]

$$P_{esr} = \frac{3}{2} Q\left(\sqrt{\frac{2}{5}\left(\frac{\gamma_{ssr}}{2}\right)}\right) = \frac{3}{2} Q\left(\sqrt{\frac{2}{5}\gamma_{bsr}}\right) \tag{17}$$

Here, $Q(x)$ is the Gaussian Q-function, defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt \tag{18}$$

In Rayleigh fading channel, the average SER is [13]

$$\bar{P}_{esr} = \int_0^{\infty} P_{esr}\, p(\gamma_{bsr})\, d\gamma_{bsr} = \frac{3}{4}\left(1 - \sqrt{\frac{\bar{\gamma}_{bsr}}{5 + \bar{\gamma}_{bsr}}}\right) \tag{19}$$

where γ_{bsr} is a random variable with exponential distribution. The probability density function (PDF) of γ_{bsr} is

$$p(\gamma_{bsr}) = \frac{1}{\bar{\gamma}_{bsr}} e^{-\gamma_{bsr}/\bar{\gamma}_{bsr}}, \quad \gamma_{bsr} \geq 0 \tag{20}$$

where $\bar{\gamma}_{bsr} = \frac{E_b}{N_0}\Big|_{sr} \cdot E\left[\left|\alpha_{ij}\right|^2\right] = \frac{E_b}{N_0}\Big|_{sr}$ is the average SNR per bit.

3.2 Upper Bound on the SER of JEC Mode

We firstly derive SER when the system is in Case 1-1 and Case 1-2 respectively, and then derive the probabilities when the system is in Case 1-1 and Case 1-2. Due to the symmetry among the four points of the constellation, we assume the value of the symbol is S_0 in our analysis.

1) Upper bound on SER when system is in Case 1-1

We analyze the SER of the symbol transmitted in s_{11}. The SER remains the same when the symbol is transmitted in s_{12}. Substitute s_{11} in (8) and (11) with S_0, and we get

[1] In this paper, we use γ_s (γ_b) to denote SNR per symbol (bit), and $\bar{\gamma}_s$ ($\bar{\gamma}_b$) to denote average SNR per symbol (bit) in fading channel. E_s (E_b) is receiving energy per symbol (bit) in Gaussian channel. In Gaussian channel, $\gamma_s = \bar{\gamma}_s = E_s/N_0$, $\gamma_b = \bar{\gamma}_b = E_b/N_0$. In Rayleigh channel, $\bar{\gamma}_s = E_s/N_0$, $\bar{\gamma}_b = E_b/N_0$. $N_0/2$ is the PSD of AWGN.

$$\begin{aligned} r_{sd1} &= |\alpha_{1d}|\left[\mathrm{Re}(S_0) + j\,\mathrm{Im}(s_{12})\right] + z_{sd1} \\ r_{rd2} &= |\alpha_{2d}|\left[\mathrm{Re}(s_{12}) + j\,\mathrm{Im}(S_0)\right] + z_{rd2} \end{aligned} \tag{21}$$

We define

$$d_n = \left|\mathrm{Re}(r_{sd1}) - |\alpha_{1d}| \cdot \mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd2}) - |\alpha_{2d}| \cdot \mathrm{Im}(S_n)\right|^2,\ n = 0,1,2,3 \tag{22}$$

An error will occur in the detection if $d_0 > d_1$ or $d_0 > d_2$ or $d_0 > d_3$. The probability of wrong detection is

$$P_{es(1\text{-}1)} = P\left(d_0 > d_1 \cup d_0 > d_2 \cup d_0 > d_3 \middle| S_0\right) \le P\left(d_0 > d_1 \middle| S_0\right) + P\left(d_0 > d_2 \middle| S_0\right) + P\left(d_0 > d_3 \middle| S_0\right) \tag{23}$$

Here,

$$\begin{aligned} P\left(d_0 > d_n \middle| S_0\right) = P\Big(&\left|\mathrm{Re}(r_{sd1}) - |\alpha_{1d}| \cdot \mathrm{Re}(S_0)\right|^2 + \left|\mathrm{Im}(r_{rd2}) - |\alpha_{2d}| \cdot \mathrm{Im}(S_0)\right|^2 \\ &> \left|\mathrm{Re}(r_{sd1}) - |\alpha_{1d}| \cdot \mathrm{Re}(S_n)\right|^2 + \left|\mathrm{Im}(r_{rd2}) - |\alpha_{2d}| \cdot \mathrm{Im}(S_n)\right|^2\Big) \\ = P\Big(&2|\alpha_{1d}| \cdot \mathrm{Re}(S_n - S_0) \cdot \mathrm{Re}(z_{sd1}) + 2|\alpha_{2d}| \cdot \mathrm{Im}(S_n - S_0) \cdot \mathrm{Im}(z_{rd2}) \\ &> |\alpha_{1d}|^2 \cdot \left[\mathrm{Re}(S_0 - S_n)\right]^2 + |\alpha_{2d}|^2 \cdot \left[\mathrm{Im}(S_0 - S_n)\right]^2\Big) \end{aligned} \tag{24}$$

where $2|\alpha_{1d}| \cdot \mathrm{Re}(S_n - S_0) \cdot \mathrm{Re}(z_{sd1}) + 2|\alpha_{2d}| \cdot \mathrm{Im}(S_n - S_0) \cdot \mathrm{Im}(z_{rd2})$ is a Gaussian random variable whose mean is zero and its variance is $\left\{4|\alpha_{1d}|^2\left[\mathrm{Re}(S_n - S_0)\right]^2 + 4|\alpha_{2d}|^2\left[\mathrm{Im}(S_n - S_0)\right]^2\right\} \cdot \frac{N_0}{2}$. So the $P\left(d_0 > d_n \middle| S_0\right)$ can be written as

$$\begin{aligned} P\left(d_0 > d_n \middle| S_0\right) &= Q\left(\sqrt{\frac{|\alpha_{1d}|^2\left[\mathrm{Re}(S_0 - S_n)\right]^2 + |\alpha_{2d}|^2\left[\mathrm{Im}(S_0 - S_n)\right]^2}{2N_0}}\right) \\ &\le \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{|\alpha_{1d}|^2\left[\mathrm{Re}(S_0 - S_n)\right]^2 + |\alpha_{2d}|^2\left[\mathrm{Im}(S_0 - S_n)\right]^2}{4N_0}\right) \end{aligned} \tag{25}$$

where we use a bound on Q-function [14]:

$$Q(x) \le \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{x^2}{2}\right) \quad \text{when } x >> 1 \tag{26}$$

Then we get

$$\begin{aligned} P\left(d_0 > d_1 \middle| S_0\right) &\le \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{|\alpha_{1d}|^2\left[\mathrm{Re}(S_0 - S_1)\right]^2 + |\alpha_{2d}|^2\left[\mathrm{Im}(S_0 - S_1)\right]^2}{4N_0}\right) \\ &= \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{\left(4|\alpha_{1d}|^2 + |\alpha_{2d}|^2\right)a^2}{N_0}\right) \end{aligned} \tag{27}$$

where a is half the distance between any two points on I axis or Q axis **Fig. 5** a^2 can be substituted by E_b:

$$\frac{a^2 + (3a)^2}{2} = \frac{E_s}{2} = E_b \Rightarrow a^2 = \frac{E_s}{10} = \frac{E_b}{5} \tag{28}$$

Substitute (28) into (27), and we obtain

$$P\left(d_0>d_1\middle|S_0\right)\le\frac{1}{2\sqrt{2\pi}}\exp\left(-\frac{4\left|\alpha_{1d}\right|^2+\left|\alpha_{2d}\right|^2}{5}\cdot\frac{E_b}{N_0}\right)=\frac{1}{\sqrt{\ }}\exp\left(-\frac{4\left|\alpha_{1d}\right|^2+\left|\alpha_{2d}\right|^2}{5}\cdot\bar{\gamma}_b\right) \tag{29}$$

Similarly, we obtain

$$\begin{aligned} P\left(d_0>d_2\middle|S_0\right)&\le\frac{1}{2\sqrt{\pi}}\exp\left(-\frac{\left|\alpha_{1d}\right|^2+9\left|\alpha_{2d}\right|^2}{5}\cdot\bar{\gamma}_b\right)\\ P\left(d_0>d_3\middle|S_0\right)&\le\frac{1}{2\sqrt{\pi}}\exp\left(-\frac{\left|\alpha_{1d}\right|^2+4\left|\alpha_{2d}\right|^2}{5}\cdot\bar{\gamma}_b\right)\end{aligned} \tag{30}$$

So the SER is

$$\begin{aligned} P_{e(1\text{-}1)}&\le P\left(d_0>d_1\middle|S_0\right)+P\left(d_0>d_2\middle|S_0\right)+P\left(d_0>d_3\middle|S_0\right)\\ &\le\frac{1}{2\sqrt{\pi}}\left[\exp\left(-\frac{4\left|\alpha_{1d}\right|^2+\left|\alpha_{2d}\right|^2}{5}\cdot\bar{\gamma}_b\right)+\exp\left(-\frac{\left|\alpha_{1d}\right|^2+9\left|\alpha_{2d}\right|^2}{5}\cdot\bar{\gamma}_b\right)+\exp\left(-\frac{\left|\alpha_{1d}\right|^2+4\left|\alpha_{2d}\right|^2}{5}\cdot\bar{\gamma}_b\right)\right]\end{aligned} \tag{31}$$

where $\left|\alpha_{1d}\right|^2$ and $\left|\alpha_{2d}\right|^2$ are random variables with exponential distribution. For convenience, we denote them as β_{1d} and β_{2d}. Their PDFs are

$$p\left(\beta_{id}\right)=e^{-\beta_{id}},\ \ \beta_{id}\ge 0,\ \ i=1,2 \tag{32}$$

The average SER in Rayleigh fading channel is

$$\bar{P}_{e(1\text{-}1)}=\int_0^\infty\int_0^\infty P_{e(1\text{-}1)}\,p\left(\beta_{1d}\right)d\beta_{1d}\cdot p\left(\beta_{2d}\right)d\beta_{2d}\le\frac{25}{2\sqrt{\pi}}\cdot\frac{1}{5+\bar{\gamma}_b}\cdot\left(\frac{2}{5+4\bar{\gamma}_b}+\frac{1}{5+9\bar{\gamma}_b}\right) \tag{33}$$

2) Upper bound on SER when system is in Case 1-2

In this case, the users transmit their own data in the second phase, so the SER is identical to that of the standard QPSK. The SER of QPSK in the AWGN channel is [13]

$$P_{e(1\text{-}2)}=1-\left[1-Q\left(\sqrt{2\gamma_b}\right)\right]^2\le\frac{1}{\sqrt{\pi}}\exp\left(-\gamma_b\right)-\frac{1}{4\pi}\exp\left(-2\gamma_b\right) \tag{34}$$

The average SER in Rayleigh channel is

$$\bar{P}_{e(1\text{-}2)}=\int_0^\infty P_{e(1\text{-}2)}\,p\left(\gamma_b\right)d\gamma_b\le\frac{1}{\sqrt{\pi}}\cdot\frac{1}{1+\bar{\gamma}_b}-\frac{1}{24}\cdot\frac{1}{\ +\ \bar{\gamma}_b} \tag{35}$$

3) The probabilities that the system is in the two cases

In the first phase, a user is considered to have received its partner's data correctly only when all the K symbols have been received without error. Because the channel gain keeps constant during this phase, so the received SNR per bit at the relay (γ_{bsr}) is not changed. The probability that all the K symbols are correctly received is

$$P_{cK}=\left(1-P_{esr}\right)^K=\left[1-\frac{3}{2}Q\left(\sqrt{\frac{2}{5}\gamma_{bsr}}\right)\right]^K\ge\left[1-\frac{3}{4\sqrt{\pi}}\exp\left(-\frac{\gamma_{bsr}}{5}\right)\right]^K \tag{36}$$

The average value of the probability in Rayleigh channel is

$$\begin{aligned}\bar{P}_{cK}&=\int_0^\infty P_{cK}\,p\left(\gamma_{bsr}\right)d\gamma_{bsr}\ge\int_0^\infty\left(1-\frac{3}{4\sqrt{\pi}}\cdot e^{-\frac{\gamma_{ssr}}{5}}\right)^K\frac{1}{\bar{\gamma}_{bsr}}e^{-\gamma_{bsr}/\bar{\gamma}_{bsr}}d\gamma_{bsr}\\ &=\sum_{k=0}^{K}\binom{K}{k}\left(-\frac{3}{4\sqrt{\pi}}\right)^k\cdot\frac{5}{5+k\cdot\bar{\gamma}_{bsr}}\end{aligned} \tag{37}$$

If both users receive their partner's data correctly, the system is in Case 1-1. The probability for this is

$$P_{C(1-1)} = \left(\overline{P}_{cK}\right)^2 \geq \left[\sum_{k=0}^{K}\binom{K}{k}\left(-\frac{3}{4\sqrt{\pi}}\right)^k \cdot \frac{5}{5+k\cdot\overline{\gamma}_{bsr}}\right]^2 \tag{38}$$

The probability for the system to be in Case 1-2 is

$$P_{C(1-2)} = 1 - P_{C(1-1)} \leq 1 - \left[\sum_{k=0}^{K}\binom{K}{k}\left(-\frac{3}{4\sqrt{\pi}}\right)^k \cdot \frac{5}{5+k\cdot\overline{\gamma}_{bsr}}\right]^2 \tag{39}$$

The average SER of RCCD-JEC is

$$\overline{P}_{eJEC} = P_{C(1-1)}\overline{P}_{e(1\text{-}1)} + P_{C(1-2)}\overline{P}_{e(1\text{-}2)} \tag{40}$$

3.3 Upper Bound on the SER of IEC Mode

In IEC mode, 4 cases may happen. In Case 2-1 and Case 2-2, so are their average SERs, i.e. (33) and (35). Now we derive the upper bound on SER when the system is in Case 2-3 and Case 2-4.

1) Case 2-3

In this case, both users transmit user 2's data in the second phase. The only signal conveying user 1's data is

$$r_{sd1} = \left|\alpha_{1d}\right|\left[\mathrm{Re}\left(s_{11}\right) + j\,\mathrm{Im}\left(s_{12}\right)\right] + z_{sd1} \tag{41}$$

Only its in-phase component conveys the information of s_{11}. The average SER is

$$\overline{P}_{e(2\text{-}3)} = \frac{3}{4}\left(1 - \sqrt{\frac{\overline{\gamma}_b}{5+\overline{\gamma}_b}}\right) \tag{42}$$

2) Case 2-4

In this case, both users send user 1's data in the second phase, so the quadrature component of symbol s_{11} and the in-phase component of s_{12} are transmitted by two users. Because the in-phase component and quadrature component of a symbol are asymmetric, so s_{11} and s_{12} have different SERs. We analyze the SER of s_{11} firstly. We define

$$d_{n(S_{11})} = \left|\mathrm{Re}\left(r_{sd1}\right) - \left|\alpha_{1d}\right|\cdot\mathrm{Re}\left(S_n\right)\right|^2 + \left|\mathrm{Im}\left(r_{rd1}\right) - \left|\alpha_{1d}\right|\cdot\mathrm{Im}\left(S_n\right)\right|^2 + \left|\mathrm{Im}\left(r_{rd2}\right) - \left|\alpha_{2d}\right|\cdot\mathrm{Im}\left(S_n\right)\right|^2,\ n=0,1,2,3 \tag{43}$$

Undertaking a derivation process similar to that in Case 1-1, we obtain

$$\begin{aligned} P\left(d_{0(S_{11})} > d_{n(S_{11})} \middle| S_0\right) &= Q\left(\sqrt{\frac{\left|\alpha_{1d}\right|^2\left[\mathrm{Re}\left(S_0-S_n\right)\right]^2 + \left|\alpha_{1d}\right|^2\left[\mathrm{Im}\left(S_0-S_n\right)\right]^2 + \left|\alpha_{2d}\right|^2\left[\mathrm{Im}\left(S_0-S_n\right)\right]^2}{2N_0}}\right) \\ &\leq \frac{1}{2\sqrt{\pi}}\exp\left(-\frac{\left|\alpha_{1d}\right|^2\left[\mathrm{Re}\left(S_0-S_n\right)\right]^2 + \left|\alpha_{1d}\right|^2\left[\mathrm{Im}\left(S_0-S_n\right)\right]^2 + \left|\alpha_{2d}\right|^2\left[\mathrm{Im}\left(S_0-S_n\right)\right]^2}{4N_0}\right), n=1,2,3 \end{aligned} \tag{44}$$

The SER is

$$\begin{aligned} P_{e(S_{11})} &\le P\left(d_{0(S_{11})} > d_{1(S_{11})} \left|S_0\right.\right) + P\left(d_{0(S_{11})} > d_{2(S_{11})} \left|S_0\right.\right) + P\left(d_{0(S_{11})} > d_{3(S_{11})} \left|S_0\right.\right) \\ &\le \frac{1}{2\sqrt{2\pi}} \exp\left(-\frac{5\left|\alpha_{1d}\right|^2 + \left|\alpha_{2d}\right|^2}{5} \cdot \frac{E_b}{N_0}\right) + \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{10\left|\alpha_{1d}\right|^2 + 9\left|\alpha_{2d}\right|^2}{5} \cdot \frac{E_b}{N_0}\right) \\ &+ \frac{1}{2\sqrt{2\pi}} \exp\left(-\frac{5\left|\alpha_{1d}\right|^2 + 4\left|\alpha_{2d}\right|^2}{5} \cdot \frac{E_b}{N_0}\right) \end{aligned} \tag{45}$$

The average SER in Rayleigh channel is

$$\overline{P}_{e(S_{11})} \le \frac{25}{2\sqrt{2\pi}}\left(\frac{1}{5+5\overline{\gamma}_b} \cdot \frac{1}{5+\overline{\gamma}_b} + \frac{1}{5+10\overline{\gamma}_b} \cdot \frac{1}{5+9\overline{\gamma}_b} + \frac{1}{5+5\overline{\gamma}_b} \cdot \frac{1}{5+4\overline{\gamma}_b}\right) \tag{46}$$

For symbol s_{12}, we define

$$d_{n(S_{12})} = \left|\mathrm{Im}\left(r_{sd1}\right) - \left|\alpha_{1d}\right| \cdot \mathrm{Im}\left(S_n\right)\right|^2 + \left|\mathrm{Re}\left(r_{rd1}\right) - \left|\alpha_{1d}\right| \cdot \mathrm{Re}\left(S_n\right)\right|^2 + \left|\mathrm{Re}\left(r_{rd2}\right) - \left|\alpha_{2d}\right| \cdot \mathrm{Re}\left(S_n\right)\right|^2, \ n = 0,1,2,3 \tag{47}$$

and obtain

$$\begin{aligned} P\left(d_{0(S_{12})} > d_{n(S_{12})} \left|S_0\right.\right) &= Q\left(\sqrt{\frac{\left|\alpha_{1d}\right|^2 \left[\mathrm{Im}\left(S_0 - S_n\right)\right]^2 + \left|\alpha_{1d}\right|^2 \left[\mathrm{Re}\left(S_0 - S_n\right)\right]^2 + \left|\alpha_{2d}\right|^2 \left[\mathrm{Re}\left(S_0 - S_n\right)\right]^2}{2N_0}}\right) \\ &\le \frac{1}{2\sqrt{2\pi}} \exp\left(-\frac{\left|\alpha_{1d}\right|^2 \left[\mathrm{Im}\left(S_0 - S_n\right)\right]^2 + \left|\alpha_{1d}\right|^2 \left[\mathrm{Re}\left(S_0 - S_n\right)\right]^2 + \left|\alpha_{2d}\right|^2 \left[\mathrm{Re}\left(S_0 - S_n\right)\right]^2}{4N_0}\right), n = 1,2,3 \end{aligned} \tag{48}$$

The SER is

$$\begin{aligned} P_{e(S_{12})} &\le P\left(d_{0(S_{12})} > d_{1(S_{12})} \left|S_0\right.\right) + P\left(d_{0(S_{12})} > d_{2(S_{12})} \left|S_0\right.\right) + P\left(d_{0(S_{12})} > d_{3(S_{12})} \left|S_0\right.\right) \\ &\le \frac{1}{2\sqrt{2\pi}} \exp\left(-\frac{5\left|\alpha_{1d}\right|^2 + 4\left|\alpha_{2d}\right|^2}{5} \cdot \frac{E_b}{N_0}\right) + \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{10\left|\alpha_{1d}\right|^2 + \left|\alpha_{2d}\right|^2}{5} \cdot \frac{E_b}{N_0}\right) \\ &+ \frac{1}{2\sqrt{2\pi}} \exp\left(-\frac{5\left|\alpha_{1d}\right|^2 + \left|\alpha_{2d}\right|^2}{5} \cdot \frac{E_b}{N_0}\right) \end{aligned} \tag{49}$$

The average SER in Rayleigh channel is

$$\overline{P}_{e(S_{12})} \le \frac{25}{2\sqrt{2\pi}}\left(\frac{1}{5+5\overline{\gamma}_b} \cdot \frac{1}{5+4\overline{\gamma}_b} + \frac{1}{5+10\overline{\gamma}_b} \cdot \frac{1}{5+\overline{\gamma}_b} + \frac{1}{5+5\overline{\gamma}_b} \cdot \frac{1}{5+\overline{\gamma}_b}\right) \tag{50}$$

The average SER in Case 2-4 is

$$\overline{P}_{e(2\text{-}4)} = \frac{\overline{P}_{e(S_{11})} + \overline{P}_{e(S_{12})}}{2} \le \frac{5}{2\sqrt{2\pi}}\left[\frac{1}{1+\overline{\gamma}_b} \cdot \left(\frac{1}{5+4\overline{\gamma}_b} + \frac{1}{5+\overline{\gamma}_b}\right) + \frac{1}{2+4\overline{\gamma}_b} \cdot \left(\frac{1}{5+9\overline{\gamma}_b} + \frac{1}{5+\overline{\gamma}_b}\right)\right] \tag{51}$$

3) The probabilities that the system is in the four cases

The system is in Case 2-1 when both users receive data correctly in the first phase. The probability is

$$P_{C(2-1)} = \overline{P}_{cK}^{\,2} \tag{52}$$

The system is in Case 2-2 when both users receive data wrongly in the first phase. The probability is

$$P_{C(2-2)} = \left(1-\bar{P}_{cK}\right)^2 \tag{53}$$

The system is in Case 2-3 or Case 2-4 when only one of the users receives data correctly in the first phase. The probability is

$$P_{C(2-3)} = P_{C(2-4)} = \bar{P}_{cK}\left(1-\bar{P}_{cK}\right) \tag{54}$$

The average SER of RCCD-IEC is

$$\bar{P}_{eIEC} = P_{C(2-1)}\bar{P}_{e(2\text{-}1)} + P_{C(2-2)}\bar{P}_{e(2\text{-}2)} + P_{C(2-3)}\bar{P}_{e(2\text{-}3)} + P_{C(2-4)}\bar{P}_{e(2\text{-}4)} \tag{55}$$

4. Simulation Results

We perform Monte-Carlo simulations for both RCCD modes. In the simulation, K = 32; all channels are frequency-nonselective Rayleigh channel; the channel gain keeps constant during four K/2-symbols sub-timeslots, i.e. 16×4=64 symbols. Suppose the two uplink channels exhibit the same performance. We also perform simulation for AF and DF with joint error-checking (the same as that used in the RCCD-JEC mode) cooperation method. To keep the bandwidth efficiency identical to that in the RCCD system, the modulation scheme employed in the simulation for AF and DF is 16-QAM. No channel code is employed.

Fig. 6 and **Fig. 7** are the SERs of the two modes of the RCCD system when the SNR per bit of uplink channel varies from 0 dB to 30 dB. In these figures, the marked solid lines are the simulation results of the SER of the RCCD system when the SNR per bit of the inter-user channel (E_b/N_0_iu) is from 10 dB to 50 dB, and the dotted lines with the same markers are the corresponding upper bounds. Between the two RCCD modes, the JEC mode has a better performance. The performance of the inter-user channel greatly influences the diversity gain. The results show that both two modes of RCCD can achieve full diversity order of two if the SNR per bit of the inter-user channel is greater than 40 dB.

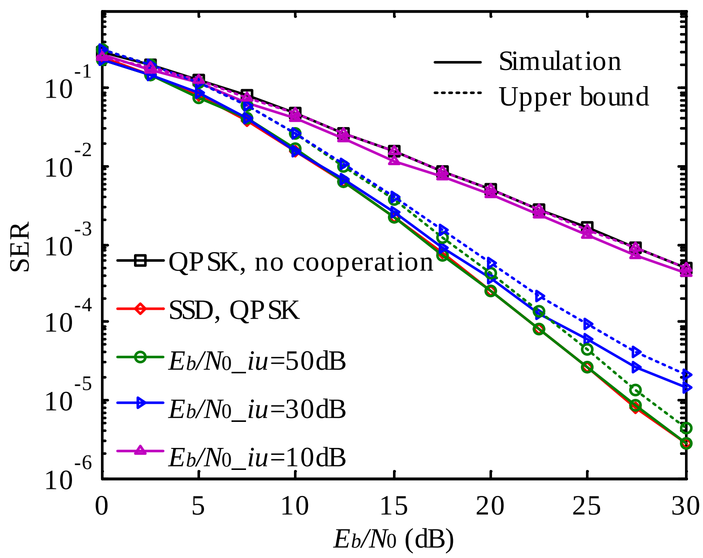

Fig. 6. The SER of RCCD-JEC

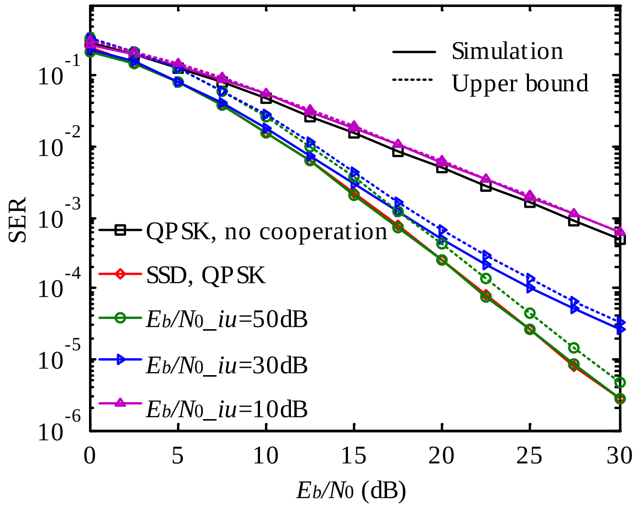

Fig. 7. The SER of RCCD-IEC

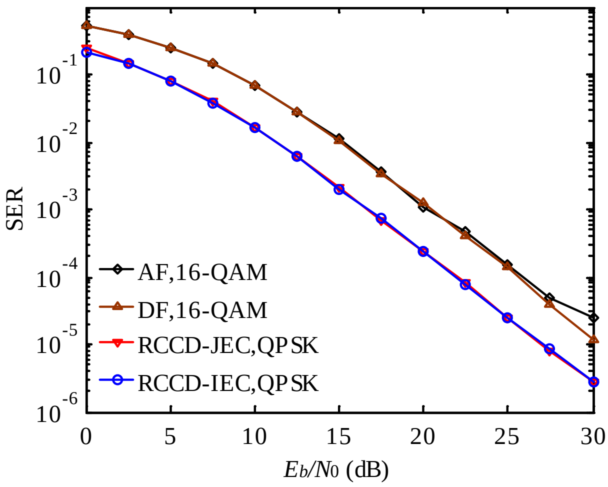

Fig. 8. Comparison of SERs among RCCD, AF and DF. E_b/N_0_iu = 50 dB.

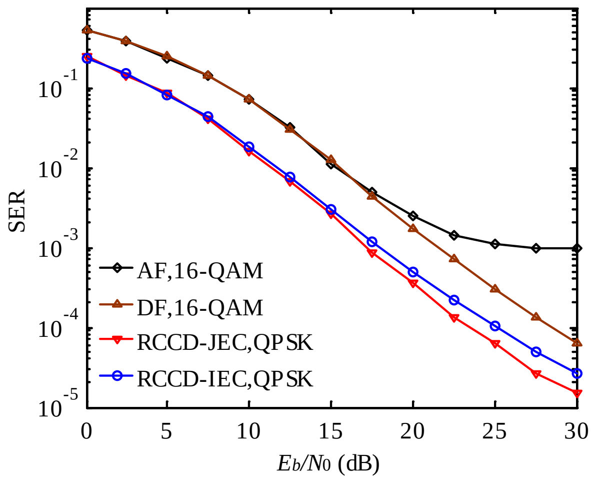

Fig. 9. Comparison of SERs among RCCD, AF and DF. E_b/N_0_iu = 30 dB.

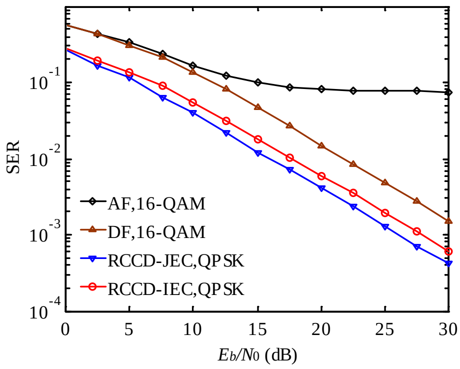

Fig. 10. Comparison of SERs among RCCD, AF and DF. E_b/N_{0}_iu = 10 dB.

Fig. 8, **Fig. 9**, and **Fig. 10** show the performance comparison between the two modes of RCCD, AF and DF systems. All results are simulation results. Because of the adoption of error checking, the error propagation problem does not exist in DF. From the results we can find that the performances of both two RCCD modes are always better than those of AF and DF. It is because AF and DF systems must employ a higher level modulation scheme to make sure they have the same bandwidth efficiency as does the RCCD system.

5. Conclusions

In this paper, a bandwidth-efficient diversity technique – cooperative diversity with rotated constellations (RCCD) is proposed. The bandwidth efficiency of the RCCD scheme is kept the same as that of a non-cooperation system, while the scheme offers diversity order of two. We have also analyzed and simulated the performances of the two modes of the RCCD system. The results of the theory analysis and simulation show that it can achieve full diversity order if the inter-user channel is good enough. In the two modes of the RCCD system, the JEC mode has a better performance; but it requires that the two users exchange their error-checking results. The simulation results also show that the performances of the JEC and IEC modes of the RCCD system are always better than those of the AF and DF systems when they have the same bandwidth efficiency.

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Weijia Lei received his Ph.D. degree in signal and information processing from University of Electronic Science and Technology of China, Chengdu, China in 2010, M.S. degree in communications and information system from Beijing University of Posts and Telecommunications, Beijing, China in 1999 and B.S. degree in communication engineering from Chongqing University of Posts and Telecommunications (CQUPT), Chongqing, China in 1992. He is a professor in School of Communication and Information Engineering, CQUPT. His main research field is in wireless communications technology.

Xianzhong Xie received the Ph.D. degree in communication and information system from Xidian University, Xi'an, China in 2000. He is currently with School of Computer Science and Technology at CQUPT, where he is a professor and director of Institute of Personal Communications. His research interestists include cooperative diversity, cognitive radio networks, MIMO and precoding, and signal processing for wireless communications.

Xiangming Li received the Ph.D. degree in communication and information system from Beijing University of Posts and Telecommunications, Beijing, China, in 2000 and the B.S. degree in communication engineering from Chongqing University of Posts Telecommunications, Chongqing, China, in 1992. He is now working in the School of Information and Electronics, Beijing Institute of Technology as an Associate Professor. His current research interests include wireless and mobile communications, MIMO-OFDM, channel coding, applications of coding theory to wireless communication systems.