

# Cooperative Multi-relay Scheme for Secondary Spectrum Access

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## Abstract

In this paper, we propose a cooperative multi-relay scheme for a secondary system to achieve spectrum access along with a primary system. In the primary network, a primary transmitter (PT) transmits the primary signal to a primary receiver (PR). In the secondary network,  $N$  secondary transmitter-receiver pairs (ST-SR) selected by a centralized control unit (CCU) are ready to assist the primary network. In particular, in the first time slot, PT broadcasts the primary signal to PR, which is also received by STs and SRs. At STs, the primary signal is regenerated and linearly combined with the secondary signal by assigning fractions of the available power to the primary and secondary signals respectively. The combined signal is then broadcasted by STs in a predetermined order. In order to achieve diversity gain, STs, SRs and PT will combine received replicas of the primary signal, using selection combining technique (SC). We derive the exact outage probability for the primary network as well as the secondary network. The simulation results are presented to verify the theoretical analyses.

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**Keywords:** Cognitive radios, spectrum sharing, cooperative communication, decode-and-forward relaying, selection combining, outage probability

## 1. Introduction

Nowadays, due to rapid increase in the number of wireless devices, spectrum scarcity becomes a critical issue. To overcome this problem, Mitola [1] first presented a concept, called cognitive radio (CR), to improve spectrum utilization. In CR networks [2][3][4][5][6], the primary users (PUs) have the right to access the licensed bands at any time, while secondary users (SUs) can only access these bands without causing interference to the primary users. Due to the lower spectrum access priority, SUs must opportunistically sense the presence and absence of the primary users. Once there is no transmission/reception activity of primary users, SUs can use the licensed spectrum. An alternative model, called Property-rights or spectrum leasing, has been proposed in [4][5][6]. In this model, PUs lease a part of the spectrum resources to SUs in exchange for appropriate remuneration.

For the remuneration, the primary link attempts to obtain a better quality of service in terms of achievable rate (or outage probability). It is well-known that cooperative communication can improve the channel capacity and achieve higher diversity gain under Rayleigh fading environment [7][8]. In conventional cooperative protocols [7][8][9][10][11], relay(s) is used to enhance the reliability of data transmission for source-destination link. Thus, in CR networks, the secondary users can play the role as relays and exploit a fraction of leased spectrum to improve the quality of service for primary link as well as to transmit their own data.

Various cooperative relaying schemes for secondary spectrum access have been proposed in [12][13]. In [12], the primary link leases the owned bandwidth for a fraction of time to achieve the benefit from enhanced quality of service. A subset of secondary transmitters cooperates to transmit the primary signal to primary receiver via distributed space-time coding. However, in this proposal, the primary system fully controls the spectrum sharing mechanism. The implementation of this model, using the game theory to optimize the achievable rate and requiring instantaneous channel state information (CSI) of both primary and secondary systems, is a difficult work. In [13], Yang Han *et al* proposed a cooperative decode-and-forward relaying for secondary system. In this model, the secondary transmitter, after decoding successfully the primary signal received from primary transmitter, will combine linearly the primary signal and secondary signal by assigning fractions of the available power. The authors of [13] also derived the approximate expressions to determine the outage probability of the primary and secondary systems.

In this paper, we extend the single-relay model proposed in [13] to multi-relay model. Also, in this proposal, each secondary transmitter will combine linearly the primary signal with its own signal and then broadcasts the combined signal in a predetermined order. Each ST, SR and PR can thus receive many replicas of the primary signal and then attempts to decode it using selection combining technique (SC). The benefit of SC technique, compared with maximal combining technique (MRC), is reduced hardware complexity at each receiver. It just chooses the best signal among all received replicas for further processing and neglects the remaining ones. In this paper, we derive the exact expressions to evaluate the performance of primary network as well as secondary network in terms of outage probability. The simulation results are then presented to validate the theoretical analyses.

The rest of the paper is organized as follows. The system model is described in Section 2 and performance analysis is discussed in Section 3. In Section 4, we will show the simulation results and Section 5 concludes the paper.

## 2. System Model

In this paper, we assume that the secondary network has a centralized control unit (CCU) to control the secondary network operations. It is also assumed that the CCU is aware of the positions of SUs and it can select several STs from the secondary network to help the data transmission of primary network. In Fig. 1, we consider a single pair of primary transmitter (PT) and receiver (PR) and a group of  $N$  secondary transmitter-receiver pairs selected by the CCU. Assume that the channels over links  $PT \rightarrow PR$ ,  $PT \rightarrow ST$ ,  $PT \rightarrow SR$  and  $ST \rightarrow SR$  are Rayleigh flat fading. Each node has a single half duplex radio and a single antenna. Due to the half duplex constraint, each transmitter must transmit on separate channels. Hence, for medium access, a time-division channel allocation including  $N + 1$  time slots is occupied in order to realize orthogonal channels. The CCU will assign these time slots for the selected secondary transmitters as follows.

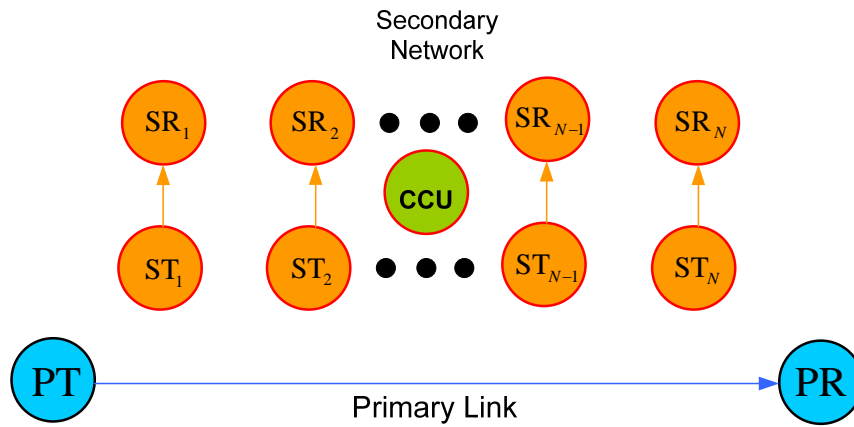


Fig. 1. Cooperative multi-relay scheme for secondary network.

Without loss of generality, we assume that  $ST_1$  is nearest to the PT and  $ST_N$  is furthest to PT. In the first time slot, PT transmits the primary signal  $x_p$  to PR, which is also received by  $N$  STs and  $N$  SRs. If  $ST_1$  decodes  $x_p$  unsuccessfully, it will keep silent in the second time slot. Otherwise,  $ST_1$  will combine linearly the primary signal  $x_p$  and its signal  $x_{s,1}$  by assigning fractions  $\alpha_1$  and  $1 - \alpha_1$  of the available power to  $x_p$  and  $x_{s,1}$ , respectively. Then the combined signal will be broadcasted at the second time slot. Generally, for  $ST_j$  ( $1 \leq j \leq N$ ), it will combine all received replicas from PT and the previous STs by using selection combining technique (SC). If it decodes correctly, it will combine the primary signal  $x_p$  and its signal  $x_{s,j}$  with fractions  $\alpha_j$  and  $1 - \alpha_j$  of the available power to  $x_p$  and  $x_{s,j}$ , respectively.  $ST_j$  will then broadcast the combined signal in the  $(j + 1)^{\text{th}}$  time slot. At PR, SC technique is also used to decode the primary signal. Similar to [13], at each SR, interference cancellation is first applied to cancel the primary component and then its secondary signal is retrieved.

In this paper, we assume that all the nodes (primary and secondary) are synchronized. Such synchronization can be achieved through MAC layer control signals. However, the detail of the medium access control policy is beyond the scope of the paper.

### 3. Performance Analysis

The signal received at node  $j$  due to the transmission of node  $i$  is given by

$$r_{i,j} = \sqrt{P} h_{i,j} x_i + \eta_j \quad (1)$$

where AWGN noise  $\eta_j$  at the receiver  $j$  has variance  $N_0$ ,  $h_{i,j}$  is fading coefficient between node  $i$  and node  $j$ ,  $x_i$  is the signal transmitted by node  $i$  and  $P$  is transmit power. In this paper, we assume that the transmit power is same for PT and STs.

From (1), the instantaneous signal to noise ratio (SNR) is determined as follows:

$$\gamma_{i,j} = \frac{P |h_{i,j}|^2}{N_0} = \bar{\gamma} |h_{i,j}|^2 \quad (2)$$

where  $\bar{\gamma} = P / N_0$  is average SNR.

In (2),  $|h_{i,j}|^2$  has exponential distribution with parameter  $\lambda_{i,j}$ . To take path loss into account, we can model the variance of channel coefficient between node  $i$  and node  $j$  as a function of distance between two nodes [14]. Therefore, the parameter  $\lambda_{i,j}$  can be expressed as

$$\lambda_{i,j} = d_{i,j}^{-\beta} \quad (3)$$

where  $\beta$  is path loss exponent that varies from 2 to 6 and  $d_{i,j}$  is the distance between node  $i$  and node  $j$ .

#### 3.1 Outage Analysis

We consider the secondary transmitter-receiver pair  $ST_j$ - $SR_j$  ( $1 < j \leq N$ ). Let us denote  $D_j$  as set of secondary transmitters  $ST_i$  ( $1 \leq i < j$ ) decoding successfully the primary signal. Note that  $D_j$  is a random set and the number of nodes in set  $D_j$  is a random variable  $n$ , i.e.,  $0 \leq n \leq j-1$ . Assume that  $D_j = \{ST_{k_1}, ST_{k_2}, \dots, ST_{k_n}\}$ , where  $1 \leq k_1 < k_2 < \dots < k_n \leq j-1$ ; and set of secondary transmitters decoding incorrectly the primary signal is  $F_j = \{ST_{l_1}, ST_{l_2}, \dots, ST_{l_{j-1-n}}\}$  with  $1 \leq l_1 < l_2 < \dots < l_{j-1-n} \leq j-1$ . For each value of  $n$ , there are  $\binom{j-1}{n}$  possible sets of size  $n$  and hence, we have total  $2^{j-1}$  possible sets of  $D_j$ . Now, we will calculate the probability for each set  $D_j$ .

Consider node  $ST_{k_m}$  ( $1 \leq m \leq n$ ) belonging to set  $D_j$ ; it is obvious that  $ST_{k_m}$  receives  $m$  replicas of the primary signal: one from PT and  $(m-1)$  from  $ST_{k_1}, ST_{k_2}, \dots, ST_{k_{m-1}}$ . Because  $ST_{k_g}$  ( $1 \leq g \leq m-1$ ) decodes successfully the primary signal, it combines linearly the primary signal  $x_p$  and its own signal  $x_{s,k_g}$ . Therefore, the signal transmitted by  $ST_{k_g}$  is given as

$$z_{ST_{k_g}} = \sqrt{\alpha_{k_g}} P x_p + \sqrt{(1 - \alpha_{k_g})} P x_{s,k_g} \quad (4)$$

The signal received at  $ST_{k_m}$  due to the transmission of  $ST_{k_g}$  is

$$r_{ST_{k_g}, ST_{k_m}} = h_{ST_{k_g}, ST_{k_m}} z_{ST_{k_g}} + \eta_{ST_{k_m}} = \sqrt{\alpha_{k_g} P} h_{ST_{k_g}, ST_{k_m}} x_p + \sqrt{(1 - \alpha_{k_g}) P} h_{ST_{k_g}, ST_{k_m}} x_{s, k_g} + \eta_{ST_{k_m}} \quad (5)$$

Assume that the channel  $h_{ST_{k_g}, ST_{k_m}}$  can be estimated at the primary transmitter  $ST_{k_m}$  by using standard preamble-aided channel estimation technique. From (5), the instantaneous SNR can be calculated as

$$\gamma_{ST_{k_g}, ST_{k_m}} = \frac{\alpha_{k_g} P |h_{ST_{k_g}, ST_{k_m}}|^2}{(1 - \alpha_{k_g}) P |h_{ST_{k_g}, ST_{k_m}}|^2 + N_0} = \frac{\bar{\gamma} \alpha_{k_g} |h_{ST_{k_g}, ST_{k_m}}|^2}{\bar{\gamma} (1 - \alpha_{k_g}) |h_{ST_{k_g}, ST_{k_m}}|^2 + 1} \quad (6)$$

In (6), it is noted that  $ST_{k_m}$  must have explicit knowledge of the factor  $\alpha_{k_g}$ . Now, the instantaneous SNR at the output of the selection combiner in  $ST_{k_m}$  is given by

$$\gamma_{ST_{k_m}} = \max_{g=1,2,\dots,m-1} (\gamma_{PT, ST_{k_m}}, \gamma_{ST_{k_g}, ST_{k_m}}) \quad (7)$$

Therefore, the achievable rate between PT and  $ST_{k_m}$  is determined as follows:

$$R_{ST_{k_m}} = \frac{1}{N+1} \log_2 (1 + \gamma_{ST_{k_m}}) \quad (8)$$

where the factor of  $N+1$  accounts for the fact that the overall transmission is split into  $N+1$  time slots.

Because decoding at  $ST_{k_m}$  is successful,  $R_{ST_{k_m}}$  is larger than target rate  $R$  of the system.

Consequently, the probability of this case is calculated by

$$\begin{aligned} \Pr(R_{ST_{k_m}} \geq R) &= \Pr(\gamma_{ST_{k_m}} \geq \tau) = 1 - \Pr\left(\max_{g=1,2,\dots,m-1} (\gamma_{PT, ST_{k_m}}, \gamma_{ST_{k_g}, ST_{k_m}}) < \tau\right) \\ &= 1 - \Pr(\gamma_{PT, ST_{k_m}} < \tau) \prod_{g=1}^{m-1} \Pr(\gamma_{ST_{k_g}, ST_{k_m}} < \tau) \end{aligned} \quad (9)$$

where  $\tau = 2^{(N+1)R} - 1$ .

Relying on (2) and (3), the probability  $\Pr(\gamma_{PT, ST_{k_m}} < \tau)$  in (9) can be given as

$$\Pr(\gamma_{PT, ST_{k_m}} < \tau) = \Pr(|h_{PT, ST_{k_m}}|^2 < \rho) = 1 - \exp(-\lambda_{PT, ST_{k_m}} \rho) \quad (10)$$

where  $\rho = \tau / \bar{\gamma}$ .

Now, in order to calculate  $\Pr(\gamma_{ST_{k_g}, ST_{k_m}} < \tau)$  in (9), we must find the cumulative density function (CDF) of the random variable  $\gamma_{ST_{k_g}, ST_{k_m}}$ . Indeed, using the definition of CDF, we have

$$F_{\gamma_{ST_{k_g}, ST_{k_m}}}(\chi) = \Pr \left( \frac{\bar{\gamma} \alpha_{k_g} |h_{ST_{k_g}, ST_{k_m}}|^2}{\bar{\gamma} (1 - \alpha_{k_g}) |h_{ST_{k_g}, ST_{k_m}}|^2 + 1} < \chi \right)$$

$$= \begin{cases} 0; & \text{if } \chi < 0 \\ 1 - \exp \left( -\frac{\lambda_{ST_{k_g}, ST_{k_m}} \chi}{\bar{\gamma} (\alpha_{k_g} - (1 - \alpha_{k_g}) \chi)} \right); & \text{if } 0 \leq \chi < \frac{\alpha_{k_g}}{(1 - \alpha_{k_g})} \\ 1; & \text{if } \chi \geq \frac{\alpha_{k_g}}{(1 - \alpha_{k_g})} \end{cases} \quad (11)$$

From (10) and (11), (9) is written as

$$\Pr(R_{ST_{k_m}} \geq R) = 1 - \left( 1 - \exp(-\lambda_{PT, ST_{k_m}} \rho) \right) \prod_{g=1}^{m-1} F_{\gamma_{ST_{k_g}, ST_{k_m}}}(\tau) \quad (12)$$

We should note that for the case  $m = 1$ , (12) reduces to  $\Pr(R_{ST_{k_1}} \geq R) = \exp(-\lambda_{PT, ST_{k_1}} \rho)$ .

Therefore, the probability for each set  $D_j$  can be calculated as follows:

$$P(D_j) = \prod_{m=1}^n \Pr(R_{ST_{k_m}} \geq R)$$

$$= \exp(-\lambda_{ST_{k_m}, PT} \rho) \prod_{m=2}^n \left[ 1 - \left( 1 - \exp(-\lambda_{PT, ST_{k_m}} \rho) \right) \prod_{g=1}^{m-1} F_{\gamma_{ST_{k_g}, ST_{k_m}}}(\tau) \right] \quad (13)$$

As mentioned above, corresponding to a set  $D_j$ , we have a set  $F_j$ . In the following, the probability of each set  $F_j$  will be derived.

Consider node  $ST_{l_b}$  ( $1 \leq b \leq j-1-n$ ) belonging to the set  $F_j$ ; it is assumed that  $k_{m-1} < l_b < k_m$  ( $1 < m < n$ ). In this case,  $ST_{l_b}$  receives  $m$  replicas of the primary signal: one from PT and  $(m-1)$  from  $PT_{k_1}, PT_{k_2}, \dots, PT_{k_{m-1}}$ . Using a similar method as above, the probability for the unsuccessful decoding at  $ST_{l_b}$  is given by

$$\Pr(R_{ST_{l_b}} < R) = \Pr \left( \max_{g=1,2,\dots,m-1} (\gamma_{PT, ST_{l_b}}, \gamma_{ST_{k_g}, ST_{l_b}}) < \tau \right) = \left( 1 - \exp(-\lambda_{PT, ST_{l_b}} \rho) \right) \prod_{g=1}^{m-1} F_{\gamma_{ST_{k_g}, ST_{l_b}}}(\tau) \quad (14)$$

In case that  $l_b < k_1$ ,  $ST_{l_b}$  only receives the primary signal from PT, (14) reduces to

$$\Pr(R_{ST_{l_b}} < R) = 1 - \exp(-\lambda_{PT, ST_{l_b}} \rho) \quad (15)$$

In addition, in case that  $l_b > k_n$ , node  $ST_{l_b}$  receives replicas of primary signal from all nodes belonging to the set  $D_j$ , hence (14) can be rewritten as

$$\Pr(R_{ST_{l_b}} < R) = \left(1 - \exp(-\lambda_{PT,ST_{l_b}} \rho)\right) \prod_{g=1}^n F_{\gamma_{ST_{k_g},ST_{l_b}}}(\tau) \quad (16)$$

From (14)-(16), the probability for each set  $F_j$  can be expressed generally as follows:

$$P(F_j) = \prod_{b=1}^{j-n-1} \Pr(R_{ST_{l_b}} < R) = \prod_{b=1}^{j-n-1} \left[ \left(1 - \exp(-\lambda_{PT,ST_{l_b}} \rho)\right) \prod_{g=1}^{m-1} F_{\gamma_{ST_{k_g},ST_{l_b}}}(\tau) \right] \quad (17)$$

Similar to the case of  $l_b > k_n$ , it is obvious that  $ST_j$  and  $SR_j$  also receive signals from all nodes belonging to the set  $D_j$ , hence the probability that the decoding at nodes  $ST_j$  and  $SR_j$  is unsuccessful is calculated respectively as

$$P_{ST_j}^{out,x_p} = \left(1 - \exp(-\lambda_{PT,ST_j} \rho)\right) \prod_{g=1}^n F_{\gamma_{ST_{k_g},ST_j}}(\tau) \quad (18)$$

$$P_{SR_j}^{out,x_p} = \left(1 - \exp(-\lambda_{PT,SR_j} \rho)\right) \prod_{g=1}^n F_{\gamma_{ST_{k_g},SR_j}}(\tau) \quad (19)$$

Note that with  $j=1$ , (18) and (19) reduce to  $P_{ST_1}^{out,x_p} = 1 - \exp(-\lambda_{PT,ST_1} \rho)$  and  $P_{SR_1}^{out,x_p} = 1 - \exp(-\lambda_{PT,SR_1} \rho)$ , respectively. Once  $ST_j$  decodes correctly the primary signal, the combined signal of the primary signal and its own signal is transmitted and given by

$$z_{ST_j} = \sqrt{\alpha_j} P x_p + \sqrt{(1-\alpha_j)} P x_{s,j} \quad (20)$$

The received signal at node  $SR_j$  due to the transmission of  $ST_j$  is

$$r_{ST_j,SR_j} = h_{ST_j,SR_j} z_{ST_j} + n_{SR_j} = \sqrt{\alpha_j} P h_{ST_j,SR_j} x_p + \sqrt{(1-\alpha_j)} P h_{ST_j,SR_j} x_{s,j} + n_{SR_j} \quad (21)$$

As discussed in [13], if  $SR_j$  also decodes successfully the primary signal, the component  $\sqrt{\alpha_j} P h_{ST_j,SR_j} x_p$  can be canceled out from (21) and we have

$$r'_{ST_j,SR_j} = \sqrt{(1-\alpha_j)} P h_{ST_j,SR_j} x_{s,j} + n_{SR_j} \quad (22)$$

Thus, the achievable rate between  $ST_j$ - $SR_j$  link is written as follows:

$$R_{ST_j,SR_j} = \frac{1}{N+1} \log_2 \left( 1 + \gamma (1-\alpha_j) |h_{ST_j,SR_j}|^2 \right) \quad (23)$$

From (23), the probability that node  $SR_j$  retrieves  $x_{s,j}$  incorrectly is calculated by

$$P_{SR_j}^{out,x_{s,j}} = \Pr(R_{ST_j,SR_j} < R) = 1 - \exp\left(-\frac{\lambda_{ST_j,SR_j} \rho}{(1-\alpha_j)}\right) \quad (24)$$

Note that  $SR_j$  can not receive  $x_{s,j}$  successfully if  $SR_j$  or  $ST_j$  is not able to decode  $x_p$  correctly or  $ST_j$ - $SR_j$  link is in outage. Therefore, from (13), (17), (18), (19) and (24), the total outage probability at  $SR_j$  is given by

$$P_{SR_j}^{out} = \sum_{D_j} P(D_j) \times P(F_j) \times \left[ 1 - \left(1 - P_{ST_j}^{out,x_p}\right) \left(1 - P_{SR_j}^{out,x_p}\right) \left(1 - P_{SR_j}^{out,x_{s,j}}\right) \right] \quad (25)$$

Now, defining the outage probability of secondary system as the probability that all secondary pairs are in outage, we have

$$P_{\text{Secondary}}^{\text{out}} = \prod_{j=1}^N P_{\text{SR}_j}^{\text{out}} \quad (26)$$

Consider the primary link; we denote set of STs having the successful decoding and unsuccessful decoding of the primary signal by  $D$  and  $F$ , respectively. Assume that  $D = \{\text{ST}_{k_1}, \text{ST}_{k_2}, \dots, \text{ST}_{k_n}\}$  and  $F = \{\text{ST}_{l_1}, \text{ST}_{l_2}, \dots, \text{ST}_{l_{N-n}}\}$ , where  $0 \leq n \leq N$ ,  $1 \leq k_1 < \dots < k_n \leq N$  and  $1 \leq l_1 < \dots < l_{N-n} \leq N$ . Similarly to (13), (17) and (18), we obtain

$$P(D) = \prod_{m=1}^n \left[ 1 - \left( 1 - \exp(-\lambda_{\text{ST}_{k_m}, \text{PR}} \rho) \right) \prod_{g=1}^{m-1} F_{\gamma_{\text{ST}_{k_g}, \text{PR}}}(\tau) \right] \quad (27)$$

$$P(F) = \prod_{b=1}^{N-n} \left[ \left( 1 - \exp(-\lambda_{\text{PT}, \text{ST}_{l_b}} \rho) \right) \prod_{g=1}^{m-1} F_{\gamma_{\text{ST}_{k_g}, \text{ST}_{l_b}}}(\tau) \right] \quad (28)$$

$$P_{\text{PR}}^{\text{out}} = \left( 1 - \exp(-\lambda_{\text{PT}, \text{PR}} \rho) \right) \prod_{g=1}^n F_{\gamma_{\text{ST}_{k_g}, \text{PR}}}(\tau) \quad (29)$$

Using the theorem on total probability, the average outage probability of primary system is given by

$$P_{\text{Primary}}^{\text{out}} = \sum_D P(D) \times P(F) \times P_{\text{PR}}^{\text{out}} \quad (30)$$

### 3.2 Diversity Order

**Proposition 1:** Node  $\text{ST}_j$  ( $1 \leq j \leq N$ ) assigns fractions  $\alpha_j$  and  $1 - \alpha_j$  of the transmit power  $P$  to primary signal and its own signal. If  $\alpha_j$  satisfies the condition  $\frac{\alpha_j}{1 - \alpha_j} \leq \tau$ ,  $\text{ST}_j$  is called a

non-diversity relay. Among  $N$  selected STs, if there are  $q$  ( $0 \leq q \leq N$ ) non-diversity relays, the achievable diversity gain of the primary network is  $N + 1 - q$ .

**Proof:** We assume that  $D = \{\text{ST}_{k_1}, \text{ST}_{k_2}, \dots, \text{ST}_{k_n}\}$ ,  $F = \{\text{ST}_{l_1}, \text{ST}_{l_2}, \dots, \text{ST}_{l_{N-n}}\}$ , and there are  $w$  ( $0 \leq w \leq n$ ) non-diversity relays in set  $D$  and  $q - w$  non-diversity relays in set  $F$ . Without loss of generality, assume that  $w$  non-diversity relays belonging to set  $D$  and  $q - w$  non-diversity relays belonging to set  $F$  are  $\{\text{ST}_{k_1}, \text{ST}_{k_2}, \dots, \text{ST}_{k_w}\}$  and  $\{\text{ST}_{l_1}, \text{ST}_{l_2}, \dots, \text{ST}_{l_{q-w}}\}$ , respectively. From (11) and (27), at high SNR regime  $\bar{\gamma}$ ,  $P(D)$ ,  $P(F)$  and  $P_{\text{PR}}^{\text{out}}$  in (27), (28) and (29) can be approximated as

$$P(D) \approx 1 \quad (31)$$

$$P(F) \approx \begin{cases} c_{1,D} (\bar{\gamma})^{-(N-n)}; & \text{if } m \leq w+1 \\ c_{2,D} (\bar{\gamma})^{-(N-n)(m-w)}; & \text{if } m > w+1 \end{cases} \quad (32)$$

$$P_{\text{PR}}^{\text{out}} \approx c_{3,D} (\bar{\gamma})^{-(n-w+1)} \quad (33)$$



where  $c_{1,D} = \prod_{b=1}^{N-n} \lambda_{\text{PT}, \text{ST}_{i_b}} \tau$ ,  $c_{2,D} = \prod_{b=1}^{N-n} \lambda_{\text{PT}, \text{ST}_{i_b}} \tau \prod_{g=w+1}^{m-1} \frac{\lambda_{\text{ST}_{k_g}, \text{ST}_{i_b}} \tau}{\alpha_{k_g} - (1 - \alpha_{k_g}) \tau}$  and

$$c_{3,D} = \lambda_{\text{PT}, \text{PR}} \tau \prod_{g=w+1}^n \frac{\lambda_{\text{ST}_{k_g}, \text{PR}} \tau}{\alpha_{k_g} - (1 - \alpha_{k_g}) \tau}.$$

Relying on (31)-(33), we can approximate the outage probability of primary link for each possible set of  $D$  as

$$P(D) \times P(F) \times P_{\text{PR}}^{\text{out}} \approx \begin{cases} c_{1,D} c_{3,D} (\bar{\gamma})^{-(N-w+1)}; & \text{if } m \leq w+1 \\ c_{1,D} c_{3,D} (\bar{\gamma})^{-(N-n)(m-w)-(n-w+1)}; & \text{if } m > w+1 \end{cases} \quad (34)$$

Furthermore, in case that all secondary transmitters decode successfully or  $D = \{\text{ST}_1, \text{ST}_2, \dots, \text{ST}_N\}$  and  $F = \{\emptyset\}$ , the outage probability of this case is given by

$$P(D) \times P(F) \times P_{\text{PR}}^{\text{out}} = (1 - \exp(-\lambda_{\text{PT}, \text{PR}} \rho)) \prod_{j=1}^N (1 - \exp(-\lambda_{\text{ST}_j, \text{PR}} \rho)) \quad (35)$$

At high SNR  $\bar{\gamma}$ , (35) can be approximated by

$$P(D) \times P(F) \times P_{\text{PR}}^{\text{out}} \approx c_{4,D} (\bar{\gamma})^{-(N-q+1)} \quad (36)$$

where  $c_{4,D} = \lambda_{\text{PT}, \text{PR}} \tau \prod_{j=q+1}^N (\lambda_{\text{ST}_j, \text{PR}} \tau)$ .

Futhermore, it is easy to see that  $(N-n)(m-w) + n - w + 1 \geq N - w + 1 \geq N - q + 1$ , hence, from (30), (34) and (36), we can approximate  $P_{\text{Primary}}^{\text{out}}$  at high SNR  $\bar{\gamma}$  as

$$P_{\text{Primary}}^{\text{out}} = \sum_D P(D) \times P(F) \times P_{\text{PR}}^{\text{out}} \approx c (\bar{\gamma})^{-(N-q+1)} \quad (37)$$

where  $c$  is a constant.

Finally, the diversity order is determined by [11]

$$\text{Diversity order} = \lim_{\bar{\gamma} \rightarrow \infty} - \frac{\log(P_{\text{Primary}}^{\text{out}})}{\log(\bar{\gamma})} = \lim_{\bar{\gamma} \rightarrow \infty} - \frac{\log(c (\bar{\gamma})^{-(N-q+1)})}{\log(\bar{\gamma})} = N - q + 1 \quad (38)$$

**Proposition 2:** For each  $\text{ST}_j$ - $\text{SR}_j$  link, the achivable diversity order is 1 and hence the diversity gain of the secondary network is  $N$ .

**Proof:** As in Section 3.1, we define  $D_j = \{\text{ST}_{k_1}, \text{ST}_{k_2}, \dots, \text{ST}_{k_n}\}$  and  $F_j = \{\text{ST}_{l_1}, \text{ST}_{l_2}, \dots, \text{ST}_{l_{j-1-n}}\}$ . It is assumed that there are  $w$  ( $0 \leq w \leq n$ ) non-diversity relays in set  $D_j$  and without loss of generality, we can assume that they are  $\{\text{ST}_{k_1}, \text{ST}_{k_2}, \dots, \text{ST}_{k_w}\}$ . At first, we approximate (13), (17) at high  $\bar{\gamma}$  as

$$P(D_j) \approx 1 \quad (39)$$

$$P(F_j) \approx \begin{cases} c_{1,D_j} (\bar{\gamma})^{-(j-n-1)}; & \text{if } m \leq w+1 \\ c_{2,D_j} (\bar{\gamma})^{-(j-n-1)(m-w)}; & \text{if } m > w+1 \end{cases} \quad (40)$$

where  $c_{1,D_j} = \prod_{b=1}^{j-n-1} \lambda_{PT,ST_{lb}} \tau$  and  $c_{2,D_j} = \prod_{b=1}^{j-n-1} \lambda_{PT,ST_{lb}} \tau \prod_{g=w+1}^n \frac{\lambda_{ST_{kg},ST_{lg}} \tau}{\alpha_{kg} - (1 - \alpha_{kg}) \tau}$ .

Next, from (18), (19) and (24), the expression  $1 - (1 - P_{ST_j}^{\text{out},x_p})(1 - P_{SR_j}^{\text{out},x_p})(1 - P_{SR_j}^{\text{out},x_{s,j}})$  in (25) can be calculated approximately by

$$1 - (1 - P_{ST_j}^{\text{out},x_p})(1 - P_{SR_j}^{\text{out},x_p})(1 - P_{SR_j}^{\text{out},x_{s,j}}) \approx \begin{cases} c_{3,D_j} (\bar{\gamma})^{-1}; & \text{if } m \leq w+1 \\ c_{4,D_j} (\bar{\gamma})^{-1}; & \text{if } m > w+1 \end{cases} \quad (41)$$

where  $c_{3,D_j} = \left( \lambda_{PT,ST_j} + \lambda_{PT,SR_j} + \frac{\lambda_{ST_j,SR_j}}{1 - \alpha_j} \right) \tau$  and  $c_{4,D_j} = \frac{\lambda_{ST_j,SR_j} \tau}{1 - \alpha_j}$ .

From (39)-(41), the outage probability for each possible set of  $D_j$  is approximated as

$$P(D_j)P(F_j) \left[ 1 - (1 - P_{ST_j}^{\text{out},x_p})(1 - P_{SR_j}^{\text{out},x_p})(1 - P_{SR_j}^{\text{out},x_{s,j}}) \right] \approx \begin{cases} c_{1,D_j} c_{3,D_j} (\bar{\gamma})^{-(j-n)}; & \text{if } m \leq w+1 \\ c_{2,D_j} c_{4,D_j} (\bar{\gamma})^{-(j-n-1)(m-w)-1}; & \text{if } m > w+1 \end{cases} \quad (42)$$

Note that when all STs, i.e.,  $ST_1, ST_2, \dots, ST_{j-1}$  decode the primary signal successfully, (42) is calculated as follows:

$$P(D_j)P(F_j) \left[ 1 - (1 - P_{ST_j}^{\text{out},x_p})(1 - P_{SR_j}^{\text{out},x_p})(1 - P_{SR_j}^{\text{out},x_{s,j}}) \right] \approx \begin{cases} c_{1,D_j} c_{3,D_j} (\bar{\gamma})^{-1}; & \text{if } m \leq w+1 \\ c_{2,D_j} c_{4,D_j} (\bar{\gamma})^{-1}; & \text{if } m > w+1 \end{cases} \quad (43)$$

For all remaining cases of set  $D_j$ , we have  $(j-n-1)(m-w)+1 \geq j-n \geq 1$ , hence the outage probability for  $ST_j$ - $SR_j$  link  $P_{SR_j}^{\text{out}}$  in (25) can be approximated at high  $\bar{\gamma}$  by

$$P_{SR_j}^{\text{out}} \approx c_j (\bar{\gamma})^{-1} \quad (44)$$

where  $c_j$  is a constant.

Therefore, the outage probability of secondary system in (26) can be calculated approximately as

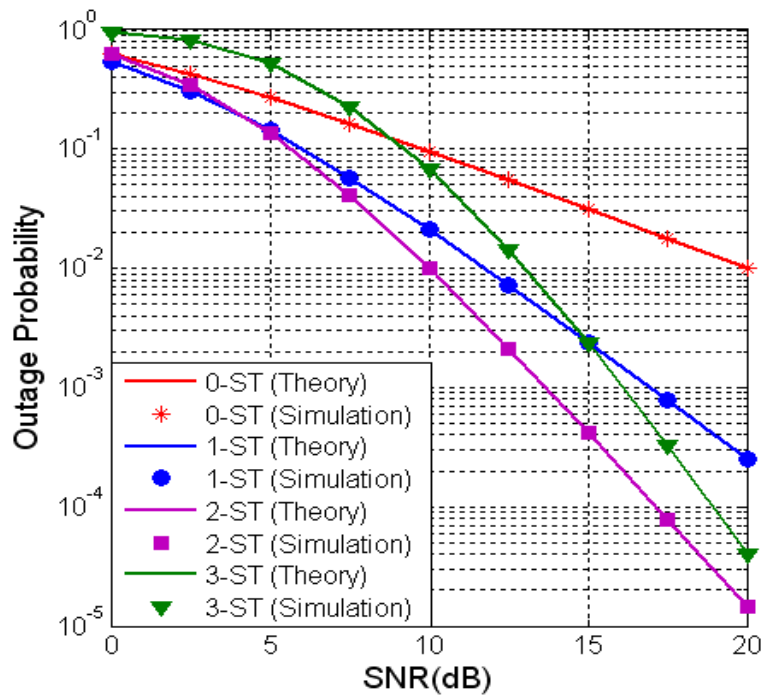
$$P_{\text{Secondary}}^{\text{out}} = \prod_{j=1}^N P_{SR_j}^{\text{out}} \approx (\bar{\gamma})^{-N} \prod_{j=1}^N c_j \quad (45)$$

Applying the definition of diversity order [11], the diversity gains of  $ST_j$ - $SR_j$  link and secondary system are easily determined by 1 and  $N$ , respectively.

#### 4. Simulation Results

In this section, we use the Monte-Carlo simulation to verify theoretical results. We assume that the distance between PT and PR, and the distances between  $ST_j$  and  $SR_j$  are normalized to 1. In addition, STs are placed on the line between PT and PR such that the distance between  $ST_j$  and PT equals  $\frac{j}{N+1}$ . We set the path loss exponent  $\beta$  to 3 and the target rate  $R$  to 1 in all simulations.

**Fig. 2** shows the outage probability of the primary network as a function of average SNR  $\bar{\gamma}$  in dB. In this simulation, the number of selected STs  $N$  varies from 0 to 3 and the fractions of transmit power to primary signal are set to 0.95 for all STs. In case of  $N=0$ , PT transmits the primary signal to PR directly, without the help of STs. It can be seen from the figure that the simulation and theoretical results match very well with each other. In addition, for all cases, the performance of primary link employing cooperative transmission from STs is better than that of direct transmission ( $N=0$ ) at high SNR regime.



**Fig. 2.** The outage probability for primary network.

In **Fig. 3**, we investigate the effect of the fraction of transmit power to primary signal assigned by STs on the performance of primary network. In this figure, the results are presented by theoretical calculations. It can be observed that in case that  $[\alpha_1 \ \alpha_2] = [0.8 \ 0.8]$ , the performance is worst. It is due to the fact that in this case  $\tau > \frac{\alpha_1}{1-\alpha_1}$  and  $\tau > \frac{\alpha_2}{1-\alpha_2}$ , hence the achievable diversity order is equal to 1, follows the statement of proposition 1. For the case

that  $[\alpha_1 \ \alpha_2] = [0.9 \ 0.8]$  (or  $[\alpha_1 \ \alpha_2] = [0.8 \ 0.9]$ ), due to  $\frac{\alpha_1}{1-\alpha_1} > \tau$  (or  $\frac{\alpha_2}{1-\alpha_2} > \tau$ ), the diversity gain increases 1 from the help of  $ST_1$  ( $ST_2$ ). In the last case  $[\alpha_1 \ \alpha_2] = [0.9 \ 0.9]$ , the performance is best because the diversity gain of 3 can be achievable.

In Fig. 4 and 5, the outage performances of the secondary network are evaluated and compared. In Fig. 4, we assume the CCU selects 3 STs for cooperation. It is also assumed that each selected  $ST_j, j=1,2,3$ , uses the same fraction  $\alpha$  of transmit power to primary signal. As we can see, when we decrease the value of  $\alpha$ , the performance of secondary network increases. It is because the fraction  $1-\alpha$  of transmit power assigned to secondary signals increases with decreasing of  $\alpha$ . In addition, the simulation and theoretical results again match very well.

The theoretical results are presented in Fig. 5 to determine the diversity order of the secondary network. In this figure, the number of selected STs is set to 4, while fraction  $\alpha$  changes. It can be observed that for all values of  $\alpha$ , the diversity order does not change and equals to 4. This is in accordance to the proposition 2.

In Fig. 6, 7, and 8, we present the outage probability of each ST-SR pair. In particular, we assume that 3 STs are selected to help PT to transmit the primary signal to PR. As expected, the results from simulation and theory are in excellent agreement. It can be seen from these figures that the performance of  $ST_j$ -SR $_j$  pair increases with decreasing  $\alpha_j$  ( $1 \leq j \leq 3$ ). Furthermore, the performance of  $ST_1$ -SR $_1$  pair does not depend on  $\alpha_2$  and  $\alpha_3$  while that of  $ST_2$ -SR $_2$  pair just depends on  $\alpha_1$  and that of  $ST_3$ -SR $_3$  pair depends on  $\alpha_1$  and  $\alpha_2$ .

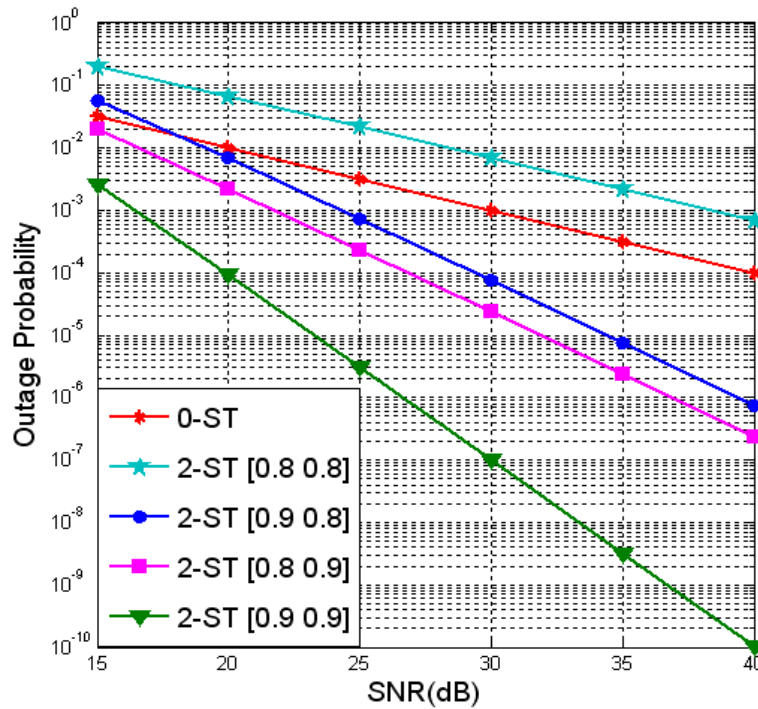


Fig. 3. The outage probability for primary network.

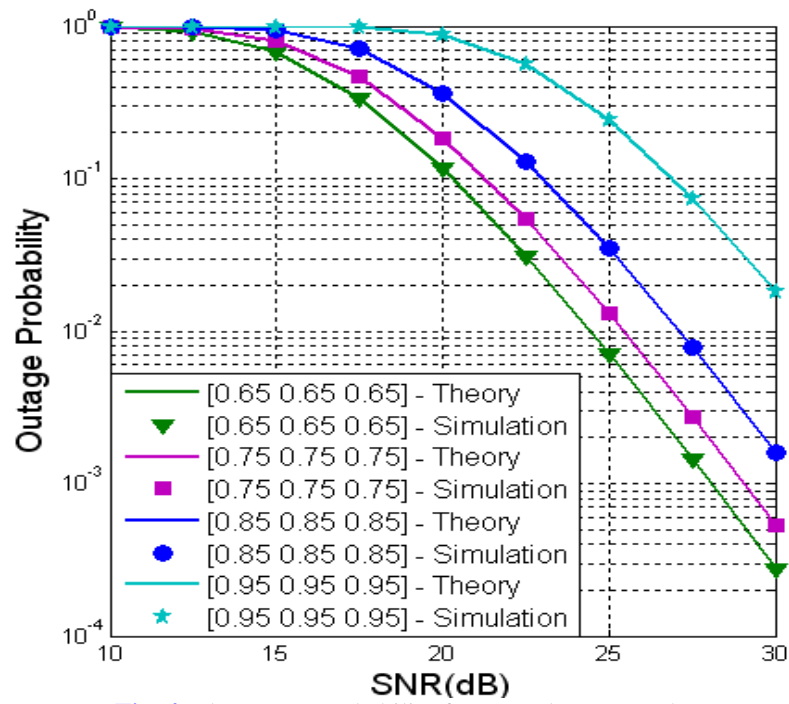


Fig. 4. The outage probability for secondary network.

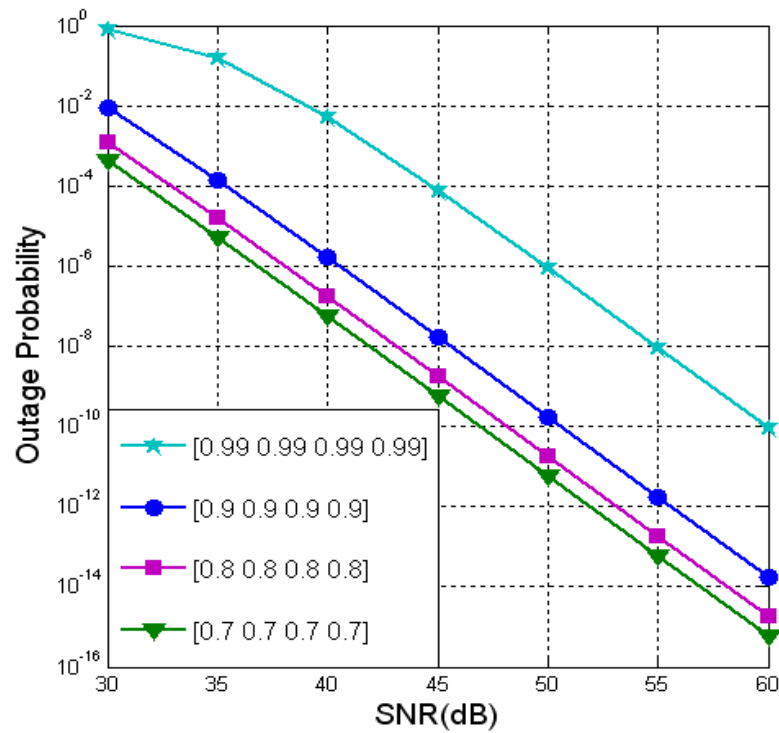


Fig. 5. The outage probability for secondary network.

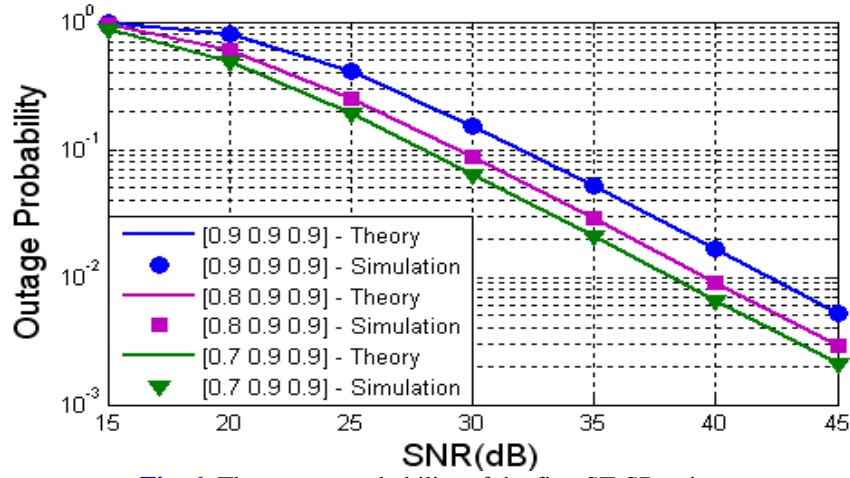


Fig. 6. The outage probability of the first ST-SR pair.

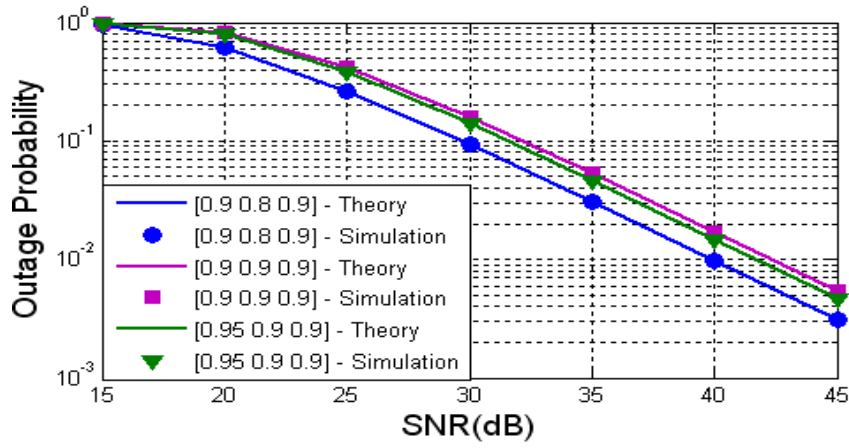


Fig. 7. The outage probability of the second ST-SR pair.

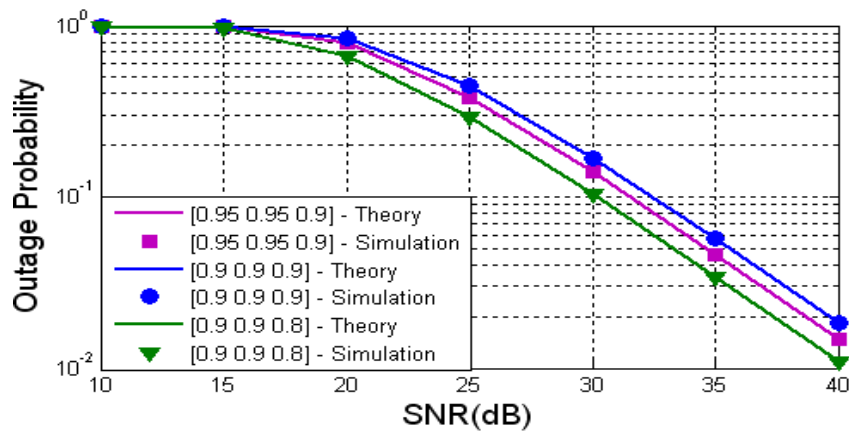


Fig. 8. The outage probability of the third ST-SR pair.

## 5. Conclusions

In this paper, a cooperative multi-relay scheme for the secondary system was proposed. We

also presented the exact outage probability expressions for the primary network as well as secondary network. Then, the validity was verified by a variety of Monte-Carlo simulations. The simulation results showed that the performance of the primary network employing the proposed protocol was enhanced significantly when compared with the system that does not cooperate with secondary nodes. Furthermore, cooperation of secondary users not only improves the outage performance but also achieves higher diversity order for secondary system.

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